Theoretical Framework and Applications of Inputâ€“output Models Disaggregation

Topic: Input-Output Theory and Methodology (5)
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In constructing input-output models, two fundamental issues frequently arise: sector aggregation and sector splitting. Sector aggregation involves combining multiple detailed sectors into broader composite categories, thereby reducing dimensionality and enhancing model solvability. In contrast, sector disaggregation entails subdividing one or more sectors in the original input-output table into multiple sub-sectors, often driven by the need for analytical precision or to capture heterogeneity within a sector. These procedures critically affect data accuracy, the economic interpretability of the model, and the reliability of empirical results. Disaggregationâ€”even when relying on limited or proxy dataâ€”is generally more accurate and policy-relevant than aggregation, particularly in cases where sectors exhibit heterogeneous economic or environmental characteristics. The problem of sector disaggregation can generally be viewed as the inverse process of aggregation. Its central objective is to disaggregate an aggregated sectoral structure into finer sub-sectoral components under conditions of incomplete or aggregated data, by imposing a set of structural and economically meaningful constraints and utilizing limited external information (such as the weight output ratios of the new sectors).

This paper aims to address the following core issues: Given an existing aggregated input-output table, how can it be decomposed into more technologically homogeneous detailed sectors? When the information on the intermediate consumption structure of the detailed sectors is severely lacking (only the output weights are known), how can the technical coefficients of multiple new sectors be estimated? What impact will the structural uncertainty introduced after disaggregation have on the conclusions of traditional input-output analysis?

Firstly, this paper re-examines and formalizes the three fundamental assumptions of the input-output model through the axiomatic expression of matrix algebra: (1) The pure department assumption (homogeneity assumption); (2) The square system assumption; (3) The fixed proportion production function and constant returns to scale assumption. Based on these axioms, this paper derives a general mathematical model for multi-department disaggregation, revealing the linear constraint relationship between aggregation coefficients and refinement coefficients.

Secondly, in response to the common problem of insufficient data in reality - when the detailed information of newly divided departments is limited to output weights and the intermediate consumption structure is unknown - this paper proposes a probabilistic solution method based on high-dimensional constrained sampling. By constructing the feasible technical coefficient set as a convex polyhedron and using the random walk algorithm to conduct high-dimensional uniform sampling in the constrained space, a large number of intermediate input coefficient matrices that satisfy the conditions of aggregation compatibility and non-negativity are generated, thereby obtaining the probability distribution of intermediate consumption/input for each new department. This method extends the disaggregation problem from the traditional point estimation to distribution inference, achieving quantitative analysis of structural uncertainty.

Thirdly, this paper conducts a systematic random matrix analysis on the intermediate flow matrix generated after de-aggregation, including spectral distribution characteristics, the fluctuation range of multiplier effects, and sensitivity tests of key department indicators (influence coefficient, sensitivity coefficient), to assess the robustness impact of department heterogeneity on the traditional input-output analysis conclusions.

The novelty of the research mainly lies in: (1) At the theoretical level, the three fundamental assumptions of the input-output model are strictly expressed through matrix algebra axioms, providing a unified formal basis for sector disaggregation; (2) At the methodological level, high-dimensional constrained sampling techniques are introduced to address the coefficient estimation problem under incomplete information, and the framework of random matrix analysis is applied to the intermediate flow matrix after disaggregation, achieving quantification of structural uncertainty and robustness testing; (3) At the empirical level, for the first time, a systematic sector disaggregation study is conducted based on the 2023 Chinese input-output table, revealing the potential impact of sector heterogeneity on the conclusions of China's industrial linkage analysis, and providing methodological support for the refined application of input-output tables.