The construction of commodity-by-commodity input-output tables: A theoretical and empirical review of Almonâ€™s algorithm

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For the construction of input-output tables in the dimension commodities to commodities the commodity technology assumption (CTA) is the natural starting point. The well-known problem of negatives in the resulting input-output (IO) table when using this assumption has led to many propositions of solving this problem. Among them the algorithm developed by Almon (see the essential description in his article in Economic Systems Research, 2000) is most widely used, both by statistical offices and IO practitioners. However, analyses of the precise workings of this algorithm have been rare.
In this paper we review the algorithm of Almon both from a theoretical and an empirical perspective. One attractive feature of the algorithm is its economic interpretation, which assumes an accountant in every industry who fulfils the task of allocating input uses according to the knowledge about technology presently available to him/her and at the same time avoiding negatives. Following the arguments of Almon we illustrate the resulting system of accounting transfers by way of an accounting matrix Z=(z_ij), whose elements denote accounting transfers of the input considered from industry i to commodity j. In the case of the precise validity of the CTA this matrix is given as Z=Mr Ì‚, where, in the notation of Almon (2000), M is the well-known market share matrix and r is the vector of flows of the input considered in the dimension of commodities; the hat-symbol denotes diagonalisation of a vector. In the case of negatives occurring when applying the CTA, the Almon algorithm provides a solution that implies Z=s Ì‚M ÌŒr Ì‚+z Ì‚, where s,0â‰¤sâ‰¤1, is a vector that reduces the transfers such that no negatives are created and z is the vector of inputs used in industry i for production of commodity i (i.e. for â€œcharacteristic productionâ€•); the check-symbol denotes off-diagonalisation of a matrix, i.e. M ÌŒ is the matrix derived from M where the elements on the main diagonal have been set to 0.
Using the matrix Z and a simple example like the one used by Almon (2000) we illustrate the workings of the algorithm. One main result is to show that the algorithm provides exact information on how the used technologies to produce commodity j differ across the various industries i with respect to the input considered. We further explore the consequences and make several propositions on the inner workings of the algorithm.
Furthermore, we explore the possibility that there exists an optimisation model that delivers the same result as the algorithm of Almon (2000). This is indeed the case. We describe the optimisation model and show its motivation. This model furnishes additional economic interpretation of the algorithm and allows the calculation of a quality measure of the resulting transformed input vector, by answering the question which proportion of the inputs can be allocated according to the CTA.
We review the question whether the algorithm can be generalized in a meaningful way. An earlier paper by the author (Koller, 2007) explored this possibility. We review this attempt in the light of the accounting matrix concept.
We compare the algorithm with an alternative approach proposed by Bohlin/Widell (2006).
An empirical application compares the performance of the original and generalized Almon algorithm and the algorithm by Bohlin/Widell (2006) on actual data from the Austrian input-output and make-use tables. We consider the following inputs: i) selected intermediate inputs, ii) value added

components (labour compensation, depreciation) and iii) employment. The differences between the various approaches are documented with appropriate measures and illustrated graphically.

References

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