Jens Kammerath
Königswinter / Germany

Königswinter, May 2026

32nd IIOA Conference, Seville, Spain, June 22-26, 2026

Visual Network Decomposition of the Leontief Inverse: TiVA, Environmental Footprint, Ownership, Endogenized Capital

Contents

Summary

Global Value Chain indicators based on the Leontief inverse such as TiVA or carbon footprints offer a highly condensed view of complex global and national supply chain networks. For a better understanding of these indicators it is often important to look at the underlying value chains.

Such a value chain decomposition can help to explain results of Input-Output research to certain stakeholders. Policy measures intended to affect these summary indicators often need to be targeted at specific stages of the global value chain in particular countries. For Input-Output analysts, such a GVC detail view helps to validate results, including the bias due to aggregation and certain simplifying assumptions. It is a valuable source for intuitive in-depth analysis of the results of complex matrix operations.

The **Visual Network Decomposition approach** was developed precisely for this purpose. It has already been implemented for several MRIO such as FIGARO or OECD ICIO and for national IO data like the detailed US BEA table.

In the MRIO case, the analysis usually starts with a country-by-country table, e.g., of value added contribution or carbon footprints, followed by a product-by-product table for a country pair selected in the first chart. For any pair of input and output products in this table, the linking value chains can be displayed immediately as an interactive flowchart.

A distinctive feature of this method is the close connection between visualization and calculation. This enables the tracking of the calculation along the value chains and the evaluation of the extent to which the selected paths cover the total multiplier.

The concept combines ideas of Structural Path Analysis (SPA), network analysis based on graph theory and Material Flow Analysis (MFA). The first implementation, "Product Chain Analysis", dates back as far as the late 1980s, when some key research outcomes in these areas were still to come.

Various use cases demonstrate the feasibility of this approach for a range of extensions to input-output analysis, such as carbon footprint, decomposition by type of supply chain activity, by foreign and domestic ownership or the endogenization of capital consumption.

List of figures, tables, and abbreviations

Figures

Tables

Abbreviations

BFS	Breadth-First Search (graph traversal)
CO_2	Carbon dioxide
DFS	Depth-First Search (graph traversal)
FD	Final Demand
FIGARO	Full International and Global Accounts for Research in Input-Output Analysis
GHG	Greenhouse Gas
GVC	Global Value Chain
IIOA	International Input-Output Association
JRC	Joint Research Center of the European Commission
MNE	Multinational Enterprise
MRIO	Multiregional Input-Output table or model
NAM	National Accounting Matrix
OECD	Organization of Economic Cooperation and Development
SAM	Social Accounting Matrix
SCA	Supply Chain Activity
SPA	Structural Path Analysis
SUT	Supply and Use Tables
SWIOT	Sample World Input-Output Table
TiVA	Trade in Value Added
US BEA	United States Bureau of Economic Analysis
VA	Value Added
VACM	Value-Added Contribution Matrix
VBA	Microsoft(c) Visual Basic for Applications
VND	Visual Network Decomposition of the Leontief inverse

1 Why visualization of input-output networks matters?

In the 21st century, almost 100 years after the opening of a new world of economic analysis by later Nobel price laureate Wassily Leontief, the international Input-Output community with its work on key challenges of global environmental, social and economic development is addressing an even broader audience of politicians, decision makers, analysts and the interested public across the world. The content focus of IIOA conferences and related events clearly demonstrates, that Input-Output is alive and is making an active contribution far beyond pure academic research.

In this environment, the toolbox of Input-Output methods is continuously being improved and expanded to meet these requirements. Latest developments in economic and mathematical science as well as in computer technology facilitate this process. At the same time, the complexity and size of models is increasing, covering global and regional details and endogenizing more and more factors.

Why this paper starts with such general statements? In order to have a real impact, the way of communication of modeling results to stakeholders beyond the Input-Output community and the world of academic journals is key.

The approach presented in this paper goes back as far as four decades. It was born out of the necessity to **explain Input-Output results to decision makers to win their trust in the analysis, get their buy-in and discuss options for action**. For many of them, economic models are "black boxes", abstract and simplified reflections of economic reality with often missing transparency about all their assumptions and the impact of the latter on calculation results.

Hence the main challenge is to **translate the dry language of mathematical models into the conceptual world and the terms of the recipients**. Input-Output analysts must understand this world to a degree that empowers them to become "business partners" as it is used to say in the corporate business.

In this context, expectation management is important. Models should be seen as tools to understand causalities rather than as the source of figures to be directly used in budgets or investment decisions. Real action has to be prepared using the decision making tools and policies of the area of application, supported by in-depth research by the respective scientific disciplines.

Models can only provide the initial impetus for this, like scenarios as decision options.

For this "translation" process, the **key causal chains from inputs and model assumptions to final outcomes have to be revealed in an understandable and reproducible way**. This should be done on an appropriate level of detail to **make things "tangible" for the addressees**.

In terms of Input-Output, this is, first of all, about the decomposition of "multipliers", not just as a set of driving factors, again described as matrix terms and still summarizing a whole set of numerous paths of inter-industry linkages, but referring to single paths.

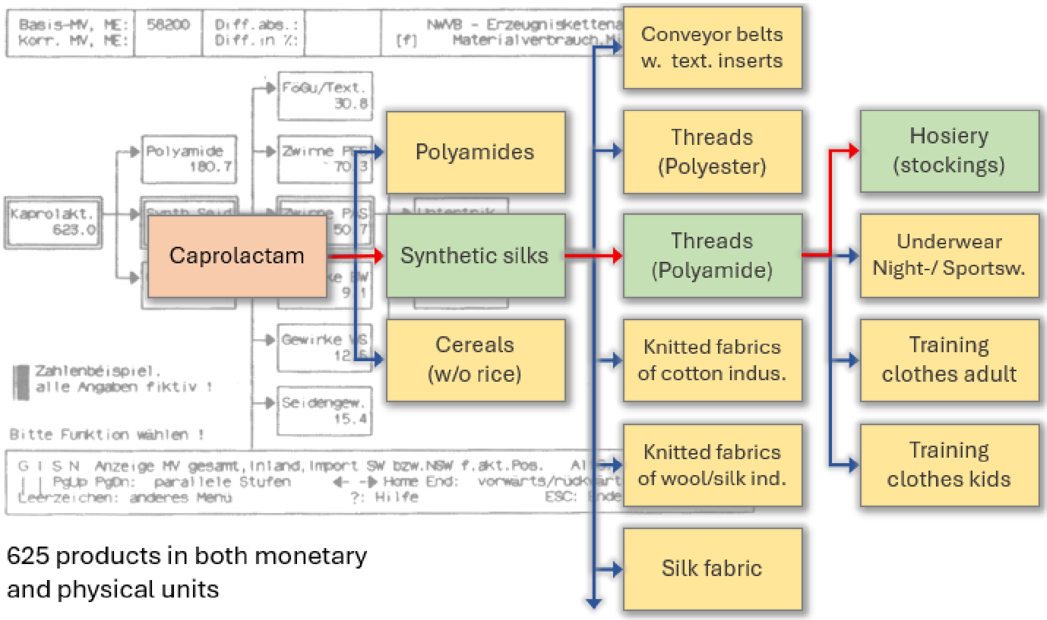
Finding the Paths. Product Chain Analysis. IOPathFinder.

The transparency regarding the driving paths behind the Input-Output multipliers brings a number of benefits for the communication with stakeholders but also for Input-Output researchers, developers and analysts themselves:

- **Explaining:**
  - "bringing light into the black box" - tracing calculation, understanding the principal logics of an Input-Output framework
- **Analyzing:**
  - understanding contents, causality and dynamics, transmission of impacts referring to real-world processes
- **Addressing:**
  - assigning responsibility and measures to specific political, economic and civil society agents along the chain of inter-industry linkages
- **Validating:**
  - checking, identifying the bias due to model assumptions, including SUT methodology and aggregation level, eliminating data errors and outliers

This example from the early days of visual network decomposition illustrates the principal idea of the approach.

Figure 1 *- Product Chain Analysis 1989: Caprolactam for Nylon stockings*



625 products in both monetary and physical units

Source: Kammerath (1989), p. 13 (flowchart translated from German)

The flowchart shows a "Product Chain Analysis" report for the indirect use of caprolactam, a major basic organic chemical, for the production of Nylon stockings. This is still an important supply chain of the textile industry today.

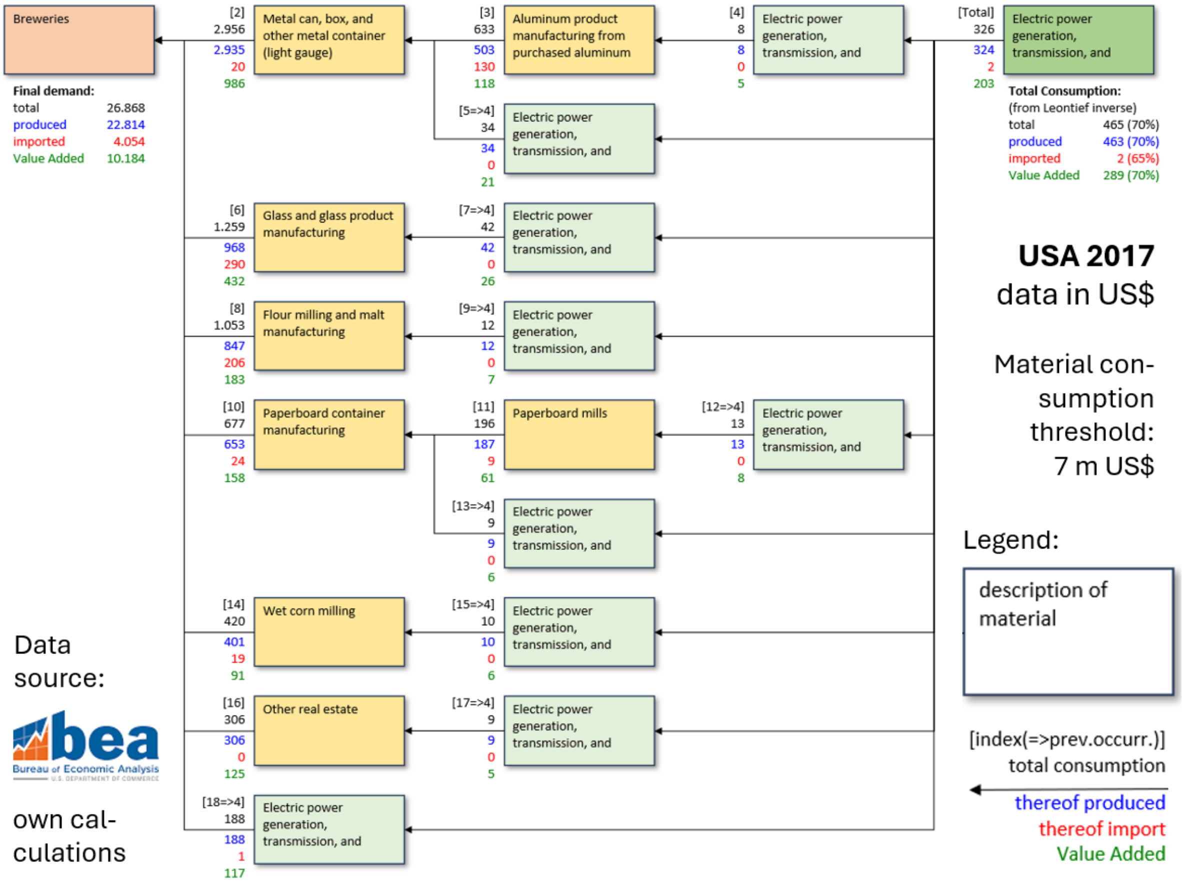
The chart is based on a two-level national Input-Output framework with 16 aggregated sectors and 625 products in both monetary and physical units. The products covered around 75% of gross output (Köhler et al., 1981, chapter 5, p. 164).

One might be surprised by the direct use of caprolactam for the production of cereals. A byproduct of the caprolactam production is ammonium sulfate, amounting to several tons per ton of caprolactam depending on the technology used. Ammonium sulfate is a nitrogen fertilizer. As it was not included in the high-coverage, but still selective list of products of the Input-Output model, a direct link to cereals was set up.

To do so, substantial knowledge of the affected industries was required.

The second example shows a result of the first implementation of the visual decomposition approach in 2023. The analysis of **direct and indirect use of energy for the production of beer in the US** was based on the detailed commodity by commodity Input-Output table of the US Bureau of Economic Analysis for 2017 (402 commodities).

Figure 2 *- Electric energy for US beer production in 2017*



Source: own calculations based on U.S. Bureau of Economic Analysis (2026)

On the top left, the product under investigation is displayed - "Breweries". As the data actually refers to commodities, the proper description would be "Brewery products" [1].

The report demonstrates the "Find Linking Chains" mode (see segment 2.1 below). The light green boxes show all cases where "Electric energy" was found in the backward links.

A threshold of 7 M US$ is used to stop search, as soon as material consumption of a certain node is lower than this value. With this criteria in place, 9 relevant supply chains have been found.

For example, the first chain shows the use of electric energy for the production of aluminum products, which then are used to produce aluminum cans [2]. Finally, cans are used as packaging for beer.

The flowchart is aligned with the typical structure of a national IO table with distinction of domestic and import materials. The calculation can be traced along the chains with the figures to the left of the yellow and green boxes. More calculation detail (like direct and cumulated coefficients) is provided in a hierarchical table which will be discussed later in segment 2.2.

The figures to the left of the dark green box on the top right show the total values for energy consumption represented in the chart: total, domestic and imported energy plus the value added embodied in the domestic part.

Below the dark green box, a comparison to the total respective values according to the Leontief inverse is provided. The 9 selected chains represent already 70% of total and domestic energy consumption according to the Leontief multiplier.

A key message of the flowchart is the high share of energy consumption for packaging via aluminum cans (8 + 34 = 42 M US$) , glass bottles (42 M US$) and paperboard containers (13 + 9 = 22 M US$), amounting to 106 M US$ or 33% of the total consumption shown in the chart and 23% of the total from the inverse.

The energy footprint of packaging for food and beverages has been discussed for a long time. A recent study on "sustainable beer packaging" (Marrucci et al., 2024) has provided deeper insights using Life Cycle Assessment (LCA), including recycling and waste management from a circular economy perspective. This study shows, that Input-Output results provide important signals, but due to their aggregate and simplified nature first of all give impulses for the necessary in-depth research by the respective scientific disciplines.

---

[1] Data files available on the BEA website in the case of the detailed Input-Output tables use the same descriptions for industries and commodities. In tables, e.g., of eurostat, there is a distinction between NACE and CPA codes and their respective descriptions.

[2] Possibly also aluminum pallets or crates. Beer barrels and bottle caps nowadays are mainly made of steel resp. tinplate.

2 The concept of visual network decomposition

Wassily Leontief was, of course, entirely aware of the chains hidden behind his magic tables:

> "With this brief introduction" (*to the basics of input-output tables in the article, note from JK*) "the lay economist is now qualified to turn around and trace his way back into the table via whatever **chain of interindustry relationships** engages his interest." (Leontief, 1951, p. 19)

Let's help the "lay economist" to cope with this task with least manual effort.

The approach of visual network decomposition presented here is connected to some concepts that are part of or closely related to Input-Output analysis.

a. **Structural Path Analysis (SPA)** is used to break down aggregate items like Leontief multipliers and derived values into contributing downstream or upstream paths (see, e.g., "8.5 Paths" in Miller/ Blair, 2022). A systematic overview of literature can be found in Xie et al. (2020). Defourny/ Thorbecke (1984) is often referenced as a key source for the concept, applied to Social Accounting Matrices (SAM).

b. Looking at **Input-Output frameworks as networks from a graph theory perspective** has long been common practice. A survey of approaches can be found in Sargent and Stachurski (2024), chapter "2 - Production". According to the authors, a weighted digraph (directed graph) can be derived from the production coefficient matrix $A = (a_{ij})$ with the sectors being the nodes and the edges representing the backward linkages with the coefficients a_{ij} as the weights (see p. 60).

c. Input-Output research is also closely linked to **Material Flow Analysis (MFA).** The foundations of this approach were laid by Ayres and Kneese (1969). A recent example is a study by the JRC of the EC on "Plastics materials flows in the EU-27 and their environmental impacts" as part of the EC's RMIS Raw Material Information System (Amadei et al., 2025). This article contains, e.g., a number of Sankey diagrams showing flows of plastic materials for different using sectors, including recycling and waste disposal from a circular economy perspective.

From the beginning, Visual Network decomposition was aiming at an **intuitive, yet mathematically precise visual approach** to reveal the driving paths behind Leontief multipliers and related indicators. Economists should be enabled **to interactively explore the network of a real Input-Output system**, preferably with a high level of detail.

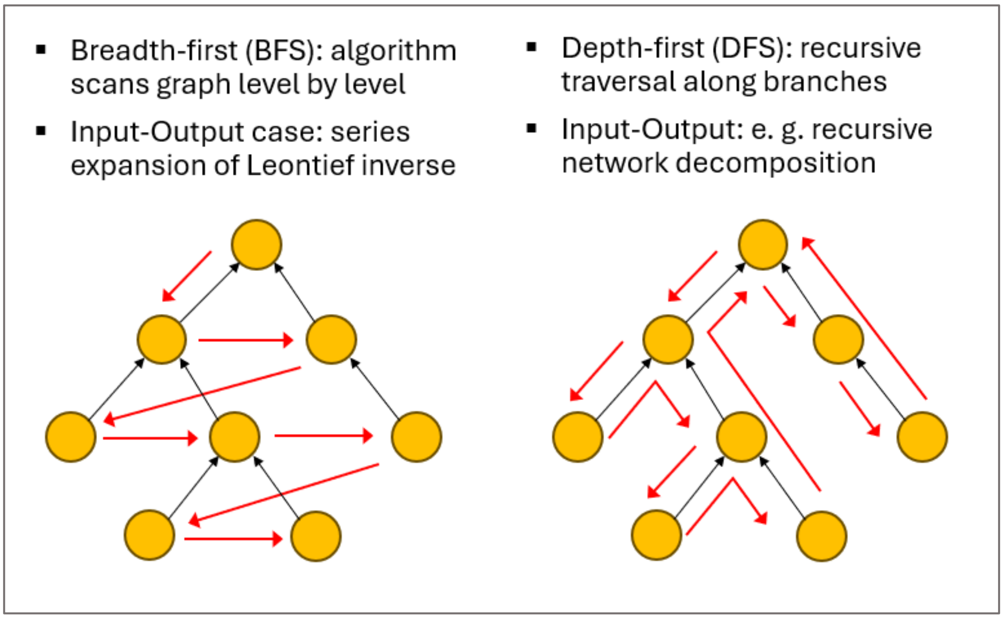
The focus was on "moving" along the real paths, still keeping the link to a level-by-level perspective as represented by the series expansion of the Leontief inverse.

In the late 1980ies, the early days of the concept presented here, several of the abovementioned research outcomes were still to come.

2.1 Recursive traversal of an Input-Output network

When looking at the Input-Output table as a directed graph, there are two principal ways to scan, or "traverse" this graph to obtain a list of all possible paths:

Figure 3 *- Methods of graph traversal: BFS and DFS*



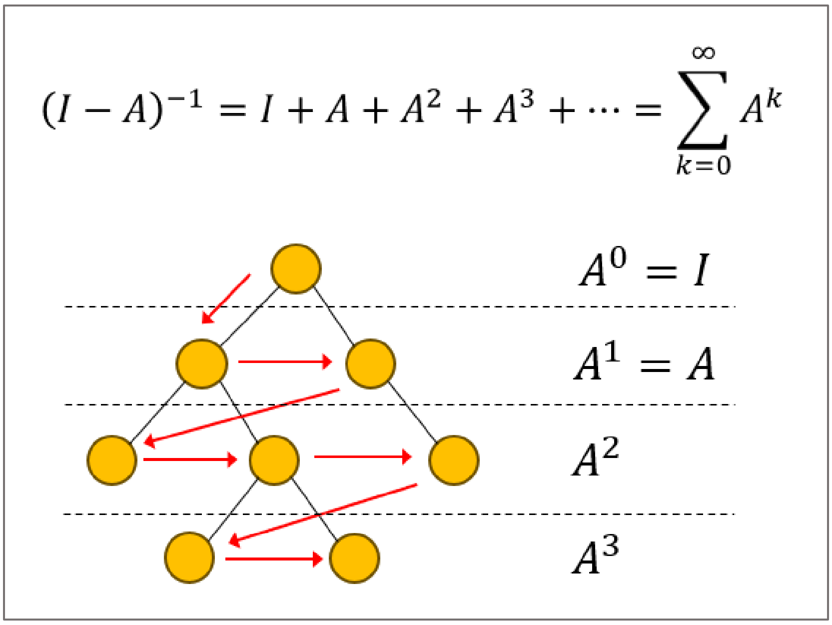
- Breadth-first (BFS): algorithm scans graph level by level
- Input-Output case: series expansion of Leontief inverse

- Depth-first (DFS): recursive traversal along branches
- Input-Output: e. g. recursive network decomposition

The top nodes in the two graphs represent products intended for final use, while the black arrows represent backward linkages of materials used directly or indirectly for the production of this product.

The direction of the arrows $A \to B$ means that material A is used in the production of product B.

Figure 4 *- Breadth First Search: Series expansion of the Leontief inverse*



$$(I - A)^{-1} = I + A + A^2 + A^3 + \cdots = \sum_{k=0}^{\infty} A^k$$

Visual network decomposition is based on the Depth First Search approach (DFS).

At the same time, it provides the link to the Breadth First Search (BFS) by grouping paths by the number of stages involved.

There are three modes of operation of visual network decomposition:

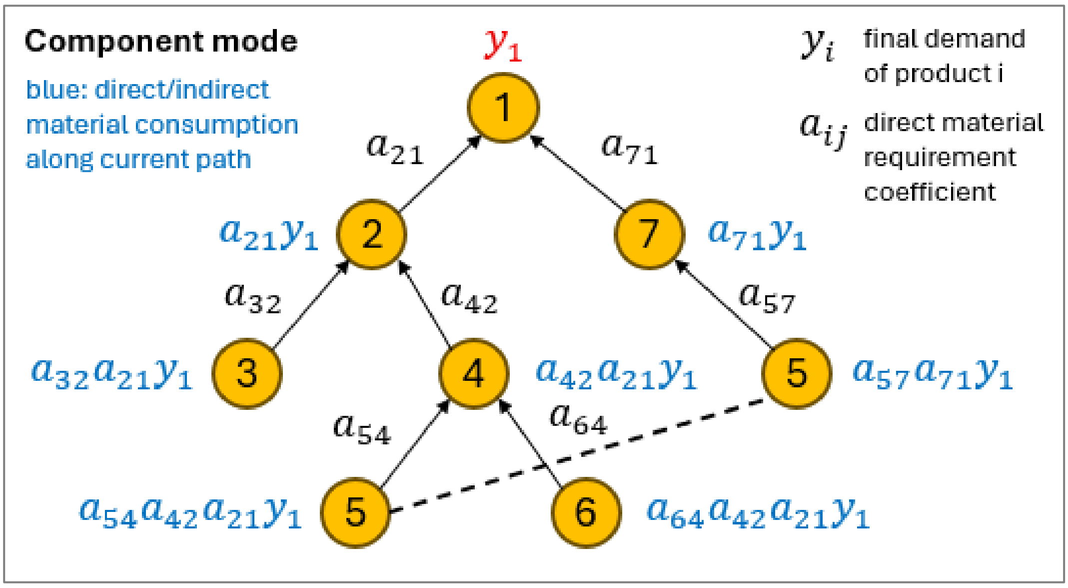
- **Component mode:** searching for paths of direct and indirect backward linkages
- **Find Linking Chains mode:** searching for paths showing the direct and indirect use of a given material for a certain product
- **Usage mode:** searching for paths of direct and indirect forward linkages

2.1.1 Component mode

In this mode, the graph is traversed backwards (upstream) to identify direct and indirect materials used for the production of a certain product.

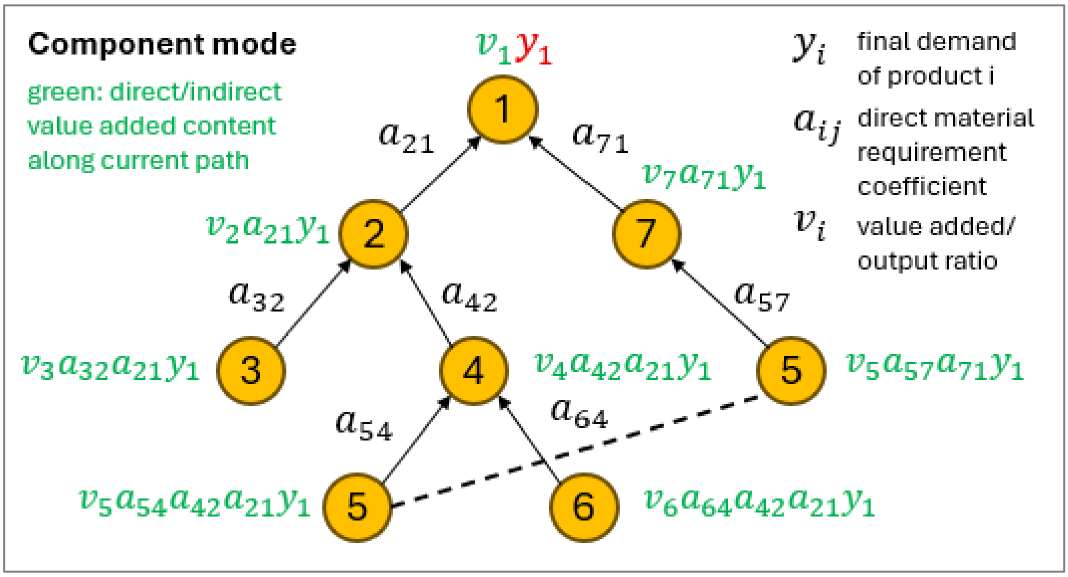
For each node in the graph, **material consumption** is calculated as the product of all direct material requirement coefficients along the chain leading to the selected product on top.

Figure 5 *- Component mode: Calculating material consumption*



For the calculation of the value added content of the nodes, the terms of material consumption for each node are multiplied with the value added/ output ratio of the material represented by the respective node.

Figure 6 *- Component mode: Calculating Value Added content*



The calculation can be traced in detail in the following **hierarchical table**.

Columns (b), (c) and (e) show the decomposition of the multipliers (i.e. the total material requirement coefficients), material consumption in absolute terms and value added content respectively.

The different nesting depths, indicated by the level of indentation of the descriptions, correspond to the respective stages of the series expansion of the Leontief inverse.

Table 1 *- Component mode: Hierarchical table with calculation*

Components of "Product 1"

		Material coefficient		Material consumption*	VA ratio	Value added content
		direct	total			
		a	*b = b(-1) * a*	*c = b * y1*	*d*	*e = d * c*
1	Product 1			y_1	v_1	$v_1 y_1$
2	. Product 2	a_{21}	a_{21}	$a_{21} y_1$	v_2	$v_2 a_{21} y_1$
3	. . Product 3	a_{32}	$a_{32} a_{21}$	$a_{32} a_{21} y_1$	v_3	$v_3 a_{32} a_{21} y_1$
4	. . Product 4	a_{42}	$a_{42} a_{21}$	$a_{42} a_{21} y_1$	v_4	$v_4 a_{42} a_{21} y_1$
5	. . . Product 5	a_{54}	$a_{54} a_{42} a_{21}$	$a_{54} a_{42} a_{21} y_1$	v_5	$v_5 a_{54} a_{42} a_{21} y_1$
6	. . . Product 6	a_{64}	$a_{64} a_{42} a_{21}$	$a_{64} a_{42} a_{21} y_1$	v_6	$v_6 a_{64} a_{42} a_{21} y_1$
7	. Product 7	a_{71}	a_{71}	$a_{71} y_1$	v_7	$v_7 a_{71} y_1$
8 → 5	. . Product 5	a_{57}	$a_{57} a_{71}$	$a_{57} a_{71} y_1$	v_5	$v_5 a_{57} a_{71} y_1$

* line 1: Final Demand

Line 6, for example, for the calculation of the total or cumulated material requirement coefficient for the path

"Product 6" → "Product 4" → "Product 2" → "Product 1"

shows the formula "$a_{64}a_{42}a_{21}$" as the product of 3 direct material requirement coefficients along the selected path. This term is related to the element A^3 of the series expansion of the Leontief inverse.

As can be seen from these illustrations, a **distinctive feature of the Visual Network Decomposition approach is the link between visualization and the underlying Input-Output calculations**.

2.1.2 Find Linking Chains mode

This mode searches for network paths that link a selected material to a given product. This is the most important mode for the decomposition of a certain multiplier in the Leontief inverse and derived matrices.

The algorithm in this mode works like in the components mode, but "remembers" only those paths, which end with the selected material - in this case "Product 5".

Figure 7 *- Find Linking Chains mode: Calculating material consumption*

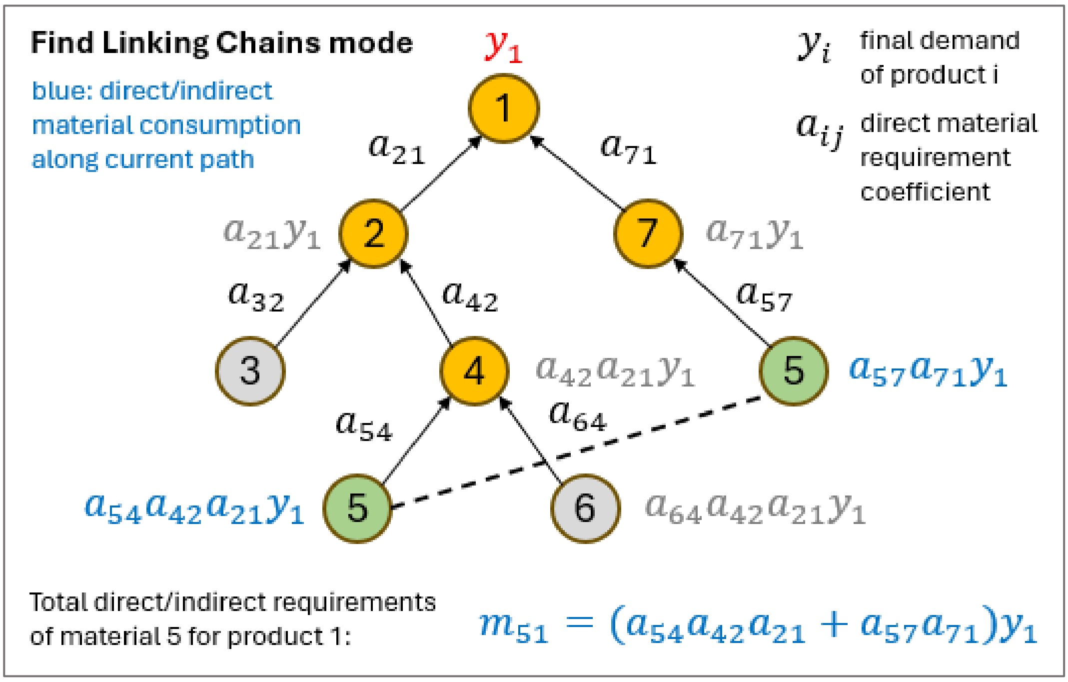


Figure 8 *- Find Linking Chains mode: Calculating Value Added content*

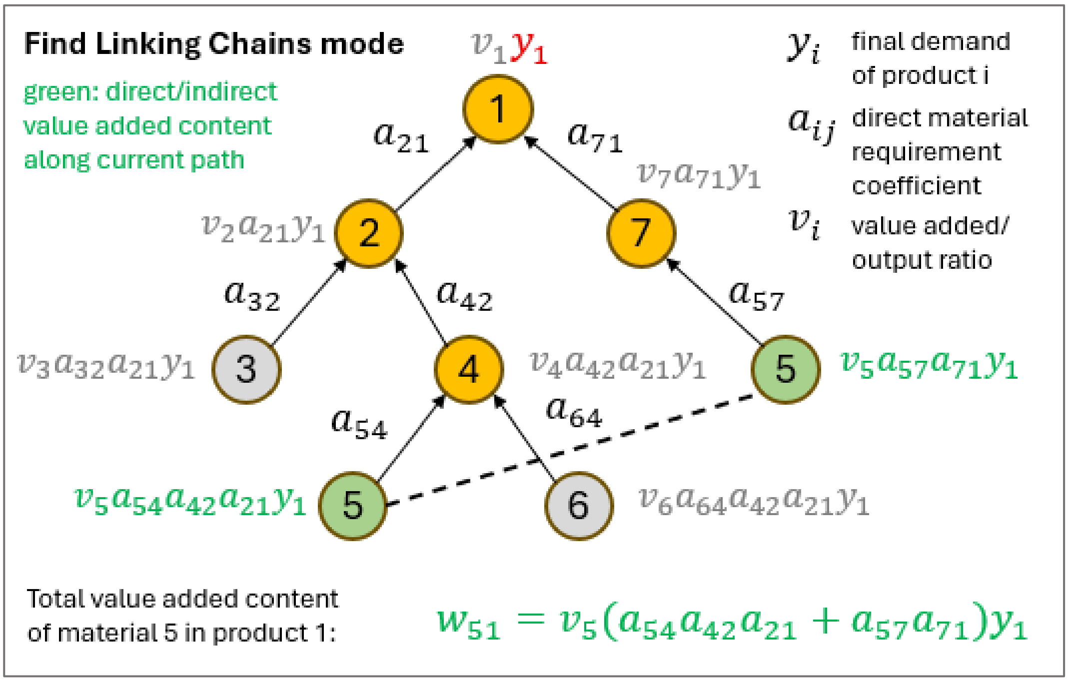


Table 2 *- Find Linking Chains mode: Hierarchical table with calculations*

Find Linking Chains of "Product 5" for "Product 1"

		Material coefficient		Material consumption*	VA ratio	Value added content
		direct	total			
		a	*b = b(-1) * a*	*c = b * y1*	*d*	*e = d * c*
Total in report			$a_{54}a_{42}a_{21}$ $+ a_{57}a_{71}$	$a_{54}a_{42}a_{21}y_1$ $+ a_{57}a_{71}y_1$		$v_5a_{54}a_{42}a_{21}y_1$ $+ v_5a_{57}a_{71}y_1$
1	Product 1			y_1	v_1	v_1y_1
2	. Product 2	a_{21}	a_{21}	$a_{21}y_1$	v_2	$v_2a_{21}y_1$
3	. . Product 4	a_{42}	$a_{42}a_{21}$	$a_{42}a_{21}y_1$	v_4	$v_4a_{42}a_{21}y_1$
4	. . . Product 5	a_{54}	$a_{54}a_{42}a_{21}$	$a_{54}a_{42}a_{21}y_1$	v_5	$v_5a_{54}a_{42}a_{21}y_1$
5	. Product 7	a_{71}	a_{71}	$a_{71}y_1$	v_7	$v_7a_{71}y_1$
6→4	. . Product 5	a_{57}	$a_{57}a_{71}$	$a_{57}a_{71}y_1$	v_5	$v_5a_{57}a_{71}y_1$

* line 1: Final Demand

2.1.3 Usage mode

In this mode, all paths of direct and indirect usage of a given material are identified. It can be used, e.g., for material footprint analysis.

Please consider, that, unlike in the cases of "Component mode" and "Find linking chains mode", the products in this chart are not the same as in the first two modes. We have chosen the same node identifiers, just with the reverse direction of the arrows, for the sake of simplicity only.

Figure 9 *- Usage mode: Calculating material consumption*

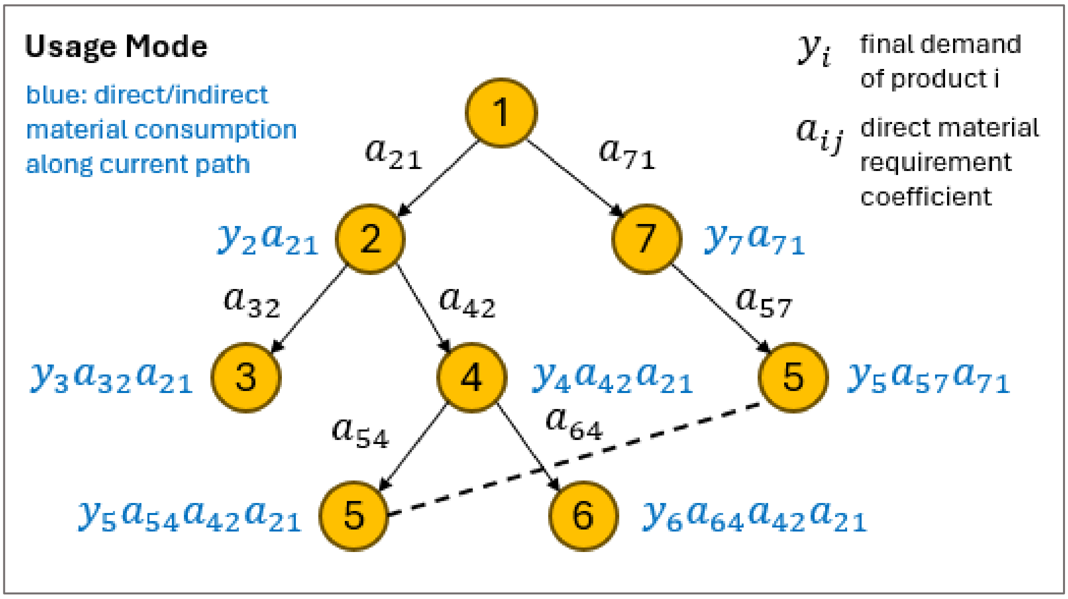
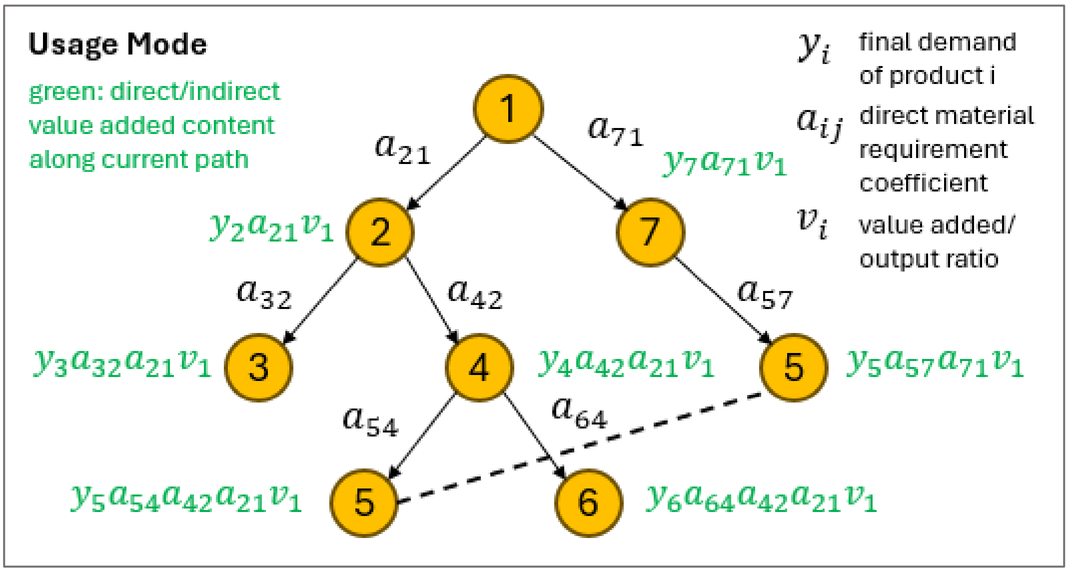
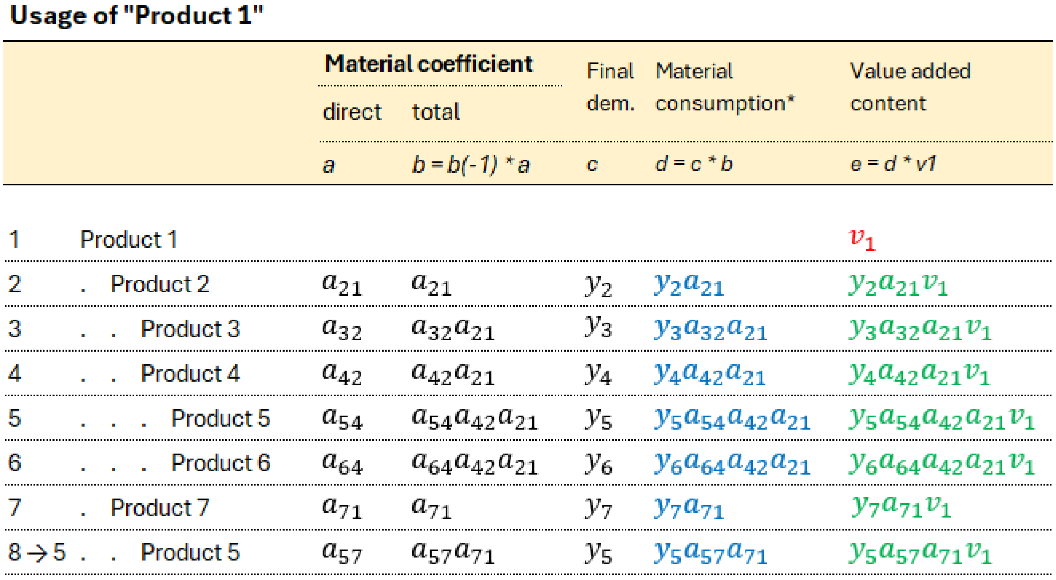


Figure 10 *- Usage mode: Calculating Value Added content*



In this mode, the calculation can again be traced looking at a hierarchical table.

Table 3 *- Usage mode: Hierarchical table with calculations*



Usage of "Product 1"

		Material coefficient		Final dem.	Material consumption*	Value added content
		direct	total			
		a	*b = b(-1) * a*	*c*	*d = c * b*	*e = d * v1*
1	Product 1					v_1
2	. Product 2	a_{21}	a_{21}	y_2	$y_2 a_{21}$	$y_2 a_{21} v_1$
3	. . Product 3	a_{32}	$a_{32} a_{21}$	y_3	$y_3 a_{32} a_{21}$	$y_3 a_{32} a_{21} v_1$
4	. . Product 4	a_{42}	$a_{42} a_{21}$	y_4	$y_4 a_{42} a_{21}$	$y_4 a_{42} a_{21} v_1$
5	. . . Product 5	a_{54}	$a_{54} a_{42} a_{21}$	y_5	$y_5 a_{54} a_{42} a_{21}$	$y_5 a_{54} a_{42} a_{21} v_1$
6	. . . Product 6	a_{64}	$a_{64} a_{42} a_{21}$	y_6	$y_6 a_{64} a_{42} a_{21}$	$y_6 a_{64} a_{42} a_{21} v_1$
7	. Product 7	a_{71}	a_{71}	y_7	$y_7 a_{71}$	$y_7 a_{71} v_1$
8→5	. . Product 5	a_{57}	$a_{57} a_{71}$	y_5	$y_5 a_{57} a_{71}$	$y_5 a_{57} a_{71} v_1$

2.2 Instant inversion of self-consumption

Already during the development of "Product Chain Analysis" in the 1980s an effect became evident, that significantly impacted the readability and clarity of flowcharts and hierarchical tables.

The elements of the diagonal of the matrix of intermediate consumption represent the consumption of products as intermediate materials for the same products. The share of this "self-consumption" depends, first of all, on the level of aggregation.

For the US Input-Output tables for 2017, the share of the total diagonal elements in total intermediate consumption amounts to

15.27%	for the aggregate table with 73 summary groups
7.51%	for the detailed table with 402 commodities (products)

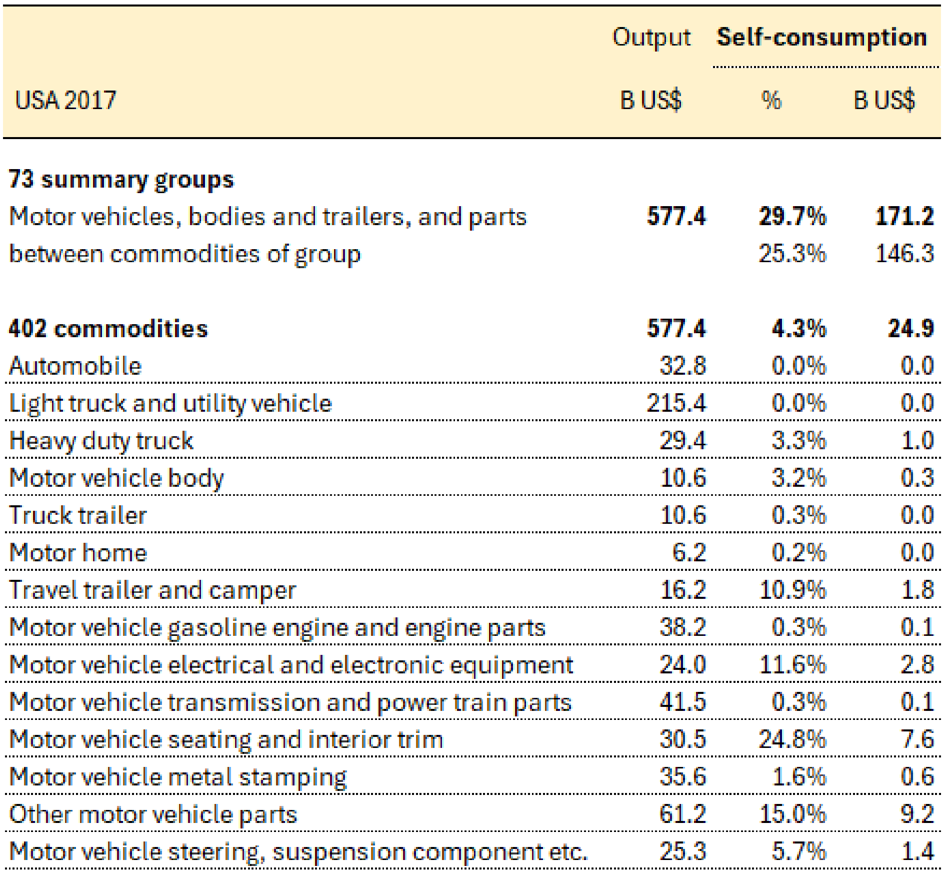
(Source: US BEA, 2026, own calculations)

As an example, the summary group

"3361 MV - Motor vehicles, bodies and trailers, and parts"

in the detailed tables is split into 14 commodities:

Table 4 *- Self-consumption of "USA Motor vehicles and parts"*



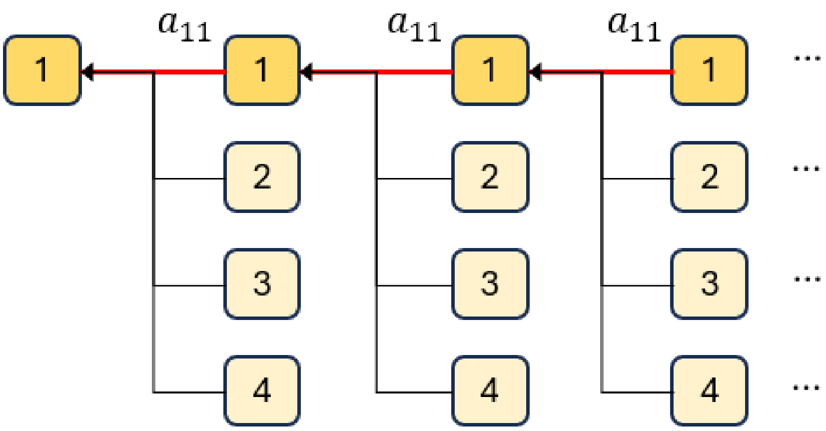
	Output	**Self-consumption**	
USA 2017	B US$	%	B US$
73 summary groups			
Motor vehicles, bodies and trailers, and parts	**577.4**	**29.7%**	**171.2**
between commodities of group		25.3%	146.3
402 commodities	**577.4**	**4.3%**	**24.9**
Automobile	32.8	0.0%	0.0
Light truck and utility vehicle	215.4	0.0%	0.0
Heavy duty truck	29.4	3.3%	1.0
Motor vehicle body	10.6	3.2%	0.3
Truck trailer	10.6	0.3%	0.0
Motor home	6.2	0.2%	0.0
Travel trailer and camper	16.2	10.9%	1.8
Motor vehicle gasoline engine and engine parts	38.2	0.3%	0.1
Motor vehicle electrical and electronic equipment	24.0	11.6%	2.8
Motor vehicle transmission and power train parts	41.5	0.3%	0.1
Motor vehicle seating and interior trim	30.5	24.8%	7.6
Motor vehicle metal stamping	35.6	1.6%	0.6
Other motor vehicle parts	61.2	15.0%	9.2
Motor vehicle steering, suspension component etc.	25.3	5.7%	1.4

Source: own calculations based on U.S. Bureau of Economic Analysis (2026)

Of the self-consumption of the aggregated group (domestic and import), 146 B US$ or 85% are related to intermediate consumption of commodities within the group. The average self-consumption of commodities on detail level is just 4.3% of gross output.

During the traversal of the Input-Output network, cases with relevant self-consumption lead to an "echo effect" of multiple repetitions of the same input structures:

Figure 11 *- Echo effect of self-consumption*



As a pragmatic solution, the diagonal with the self-consumption coefficients can be separated from the A matrix of direct material requirement coefficients and subjected to "instant inversion". For further processing, the A matrix will be used with a zero diagonal.

First the A matrix will be split into the "self-consumption diagonal" and a matrix with a zero diagonal:

$$A = As + Az \tag{1}$$

$$as_{ij} = \begin{cases} a_{ii}, & i = j \\ 0, & i \neq j \end{cases} \tag{2}$$

$$az_{ij} = \begin{cases} 0, & i = j \\ a_{ij}, & i \neq j \end{cases} \tag{3}$$

We substitute matrix A in the standard Input-Output equation and transform:

$$x = Ax + y = (As + Az)x + y \tag{4}$$

$$(I - As)x = Az \cdot x + y \tag{5}$$

$$x = (I - As)^{-1}[Az \cdot x + y] \tag{6}$$

Defining the matrix of instant inversion of the self-consumption diagonal:

$$Ast = (I - As)^{-1} \tag{7}$$

$$ast_{ij} = \begin{cases} \dfrac{1}{(1 - a_{ii})}, i = j \\ 0, i \neq j \end{cases} \tag{8}$$

Substituting and factoring out in equation before

$$[I - Ast \cdot Az]\, x = Ast \cdot y \tag{9}$$

After multiplying with the inverse of the term in square brackets we get:

$$\boxed{x = [I - Ast \cdot Az]^{-1} Ast \cdot y} \tag{10}$$

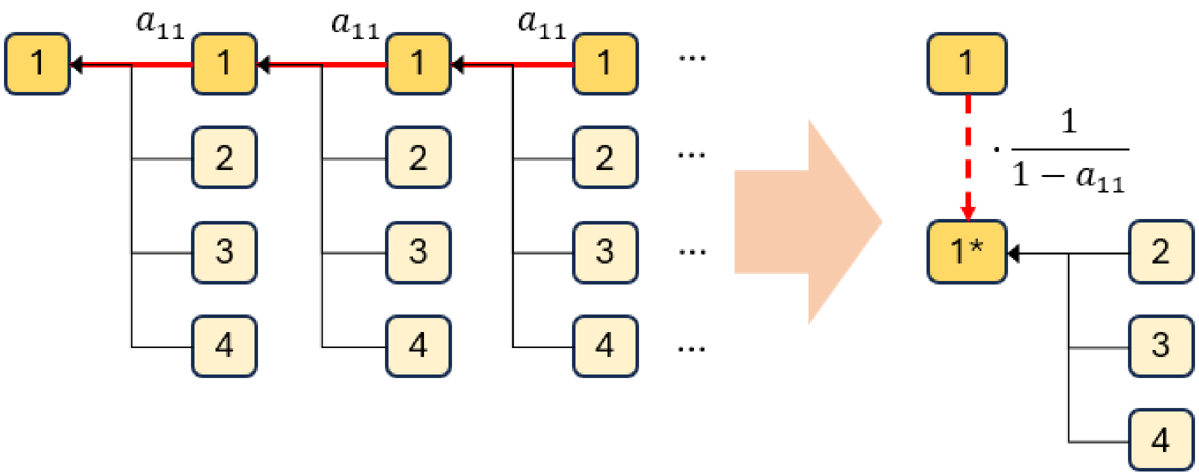
This equation means that for the traversal of the Input-Output graph **we can proceed with matrix Az with a zero diagonal, as soon as we have applied instant inversion factors from matrix Ast to final demand and any calculated material requirements before.**

The technique of "instant inversion of self-consumption" brings several benefits:

- removes redundant replication of whole substructures
- improves convergence speed of series expansion
- increases coverage with fewer chains to be selected

This diagram illustrates, how the flowchart with the echo effect caused by self consumption is reduced by applying the total effect of self consumption (i.e. the sum of the infinite series) at once:

Figure 12 *- Effect of instant inversion of self-consumption*



2.3 Link to the Leontief inverse – the coverage of decomposition

2.3.1 The VA contribution matrix: starting point and coverage criteria

The Visual Network Decomposition approach uses the Leontief inverse in two ways:

- as the starting point of analysis via interactive drill-down
- as the criteria to measure the coverage of the paths of direct and indirect backward and forward linkages selected by the algorithm

What presentation of the Leontief inverse best fits the analysis objective? The different options correspond to the three calculated columns in the hierarchical tables presented in the previous section:

A	Multipliers, total material requirement coefficients:	$L = (I - A)^{-1}$	(11)
B	Total material consumption for final demand in absolute terms:	$L\hat{y} = (I - A)^{-1}\hat{y}$	(12)
C	Value Added contribution matrix (or VA decomposition matrix):	$W = \hat{v}(I - A)^{-1}\hat{y}$	(13)

where:

A matrix of direct material requirement coefficients

$\hat{y}$ diagonal matrix of final demand

$\hat{v}$ diagonal matrix of value-added/output ratios

Options A and B, though equal or closest to the Leontief inverse by definition, imply double-counting, e.g., when considering totals of columns or rows.

In comparison, the Value Added contribution matrix (option C) has a number of clear advantages:

- Each cell represents the value added contribution of the material shown in the row to final demand of the product displayed in the column
- Column totals are equal to final demand of the respective products
- Elements in a column describe the sources of the value of final demand, summarizing backward linkages of this product
- Row totals are equal to value added of the material assigned to the row
- Elements of rows describe the final use of a material in terms of value added, summarizing the forward linkages of this material
- There is no double-counting in the matrix
- The total of the matrix in principle equals Gross Domestic Product (GDP)

Valuable information can be derived from the series expansion of matrix W:

$$(I - A)^{-1} = \sum_{k=0}^{\infty} A^k \tag{14}$$

$$W = \hat{v}(I - A)^{-1}\hat{y} = \hat{v}\sum_{k=0}^{\infty} A^k \,\hat{y} \tag{15}$$

$$W^{(k)} = \hat{v}A^k\hat{y} \tag{16}$$

$$W = \sum_{k=0}^{\infty} W^{(k)} \tag{17}$$

There are some differences to keep in mind, when looking at Multiregional Input-Output frameworks (MRIO) and national Input-Output tables:

- Whereas especially in global MRIOs trade is usually endogenous due to the inclusion of artificial countries reflecting the "Rest of the World", in national tables with a standard distinction of domestic and import materials consumption, the value of final demand besides the value added of domestic production includes also total import requirements, represented in a second matrix

- For MRIOs, it makes a difference, whether the value added for exports, which are used directly for final demand, are assigned to the producing or the using country. In the second case, the diagonal of the final demand vector is replaced by a block matrix with diagonal blocks to ensure this reallocation (see 3.1).

In Visual Network Decomposition, these differences are reflected in two different setups: one for national tables and one for MRIOs, with the options of "Final Demand produced" and "Final Demand used" for the latter.

This approach can easily be adopted from Value Added contents to, e.g., the **decomposition of air emission footprints**. In this case, in equation (15) the value added coefficient vector diagonal $\hat{v}$ is replaced by the air emission coefficients vector diagonal $\hat{c}$. This will give the **total emission footprint contribution matrix** C:

$$C = \hat{c}(I - A)^{-1}\hat{y} = \hat{c}\sum_{k=0}^{\infty} A^k \, \hat{y} \tag{18}$$

In the same way, any other external stressors directly linked to output and free of double-counting can be handled, like, e.g., labor requirements by gender or skill level.

2.3.2 How many paths are required for sufficient decomposition coverage?

In the first chapter, the use of energy for the US beer production was used as an example for Visual Network Decomposition in the "Find Linking Chains" mode.

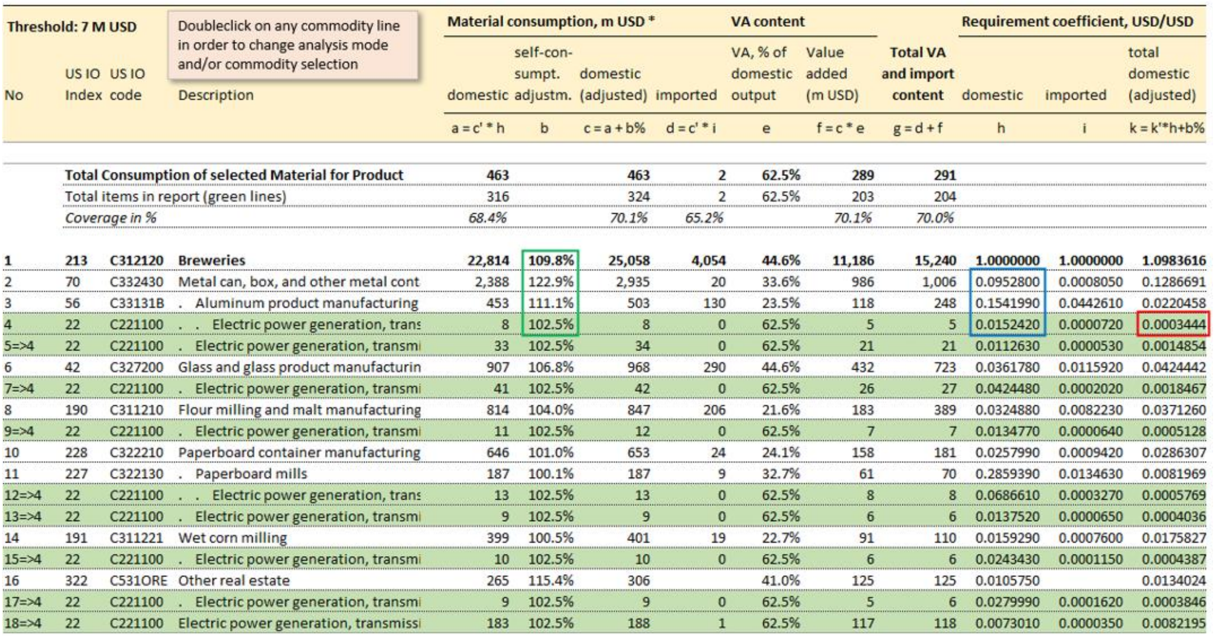
Table 5 shows the hierarchical table that was introduced in segment 2.1, **Table 2**, for this real-world example. This is a screenshot of the application "**IOPathFinder**", that has been developed by the author for all use cases of visual network decomposition described in the present paper.

The calculation logic is described in the bottom line of the table heading. Please pay attention to the "self-consumption adjustment" columns (b) and (c). They show the implementation of instant self consumption inversion introduced in section 2.2.

Lines 1-4 represent the first linking chain on top of **Figure 2** on page 8. The coefficient of total domestic requirement of energy for beer via this chain (red box in column "k") is the product of three coefficients of direct material requirements (blue box in column "h") along the chain and four coefficients of instantly inverted self-consumption (green box in column "b").

The product of the three direct material requirement coefficients is an element of the fourth member of the series expansion A^3, or more precisely, matrix Az^3 from equation (10).

Table 5 *- Hierarchical table: Electric energy for US Beer production in 2017*



Threshold: 7 M USD			Doubleclick on any commodity line in order to change analysis mode and/or commodity selection	**Material consumption, m USD ***				**VA content**			**Requirement coefficient, USD/USD**		
No	US IO Index	US IO code	Description	domestic	self-con-sumpt. adjustm.	domestic (adjusted)	imported	VA, % of domestic output	Value added (m USD)	**Total VA and import content**	domestic	imported	total domestic (adjusted)
				a = c' * h	b	c = a + b%	d = c' * i	e	f = c * e	g = d + f	h	i	k = k'*h+b%
			Total Consumption of selected Material for Product	**463**		**463**	**2**	**62.5%**	**289**	**291**			
			Total items in report (green lines)	316		324	2	62.5%	203	204			
			Coverage in %	*68.4%*		*70.1%*	*65.2%*		*70.1%*	*70.0%*			
1	**213**	**C312120**	**Breweries**	**22,814**	**109.8%**	**25,058**	**4,054**	**44.6%**	**11,186**	**15,240**	**1.0000000**	**1.0000000**	**1.0983616**
2	70	C332430	Metal can, box, and other metal cont	2,388	122.9%	2,935	20	33.6%	986	1,006	0.0952800	0.0008050	0.1286691
3	56	C33131B	. Aluminum product manufacturing	453	111.1%	503	130	23.5%	118	248	0.1541990	0.0442610	0.0220458
4	22	C221100	. . Electric power generation, trans	8	102.5%	8	0	62.5%	5	5	0.0152420	0.0000720	0.0003444
5=>4	22	C221100	. Electric power generation, transmi	33	102.5%	34	0	62.5%	21	21	0.0112630	0.0000530	0.0014854
6	42	C327200	Glass and glass product manufacturin	907	106.8%	968	290	44.6%	432	723	0.0361780	0.0115920	0.0424442
7=>4	22	C221100	. Electric power generation, transmi	41	102.5%	42	0	62.5%	26	27	0.0424480	0.0002020	0.0018467
8	190	C311210	Flour milling and malt manufacturing	814	104.0%	847	206	21.6%	183	389	0.0324880	0.0082230	0.0371260
9=>4	22	C221100	. Electric power generation, transmi	11	102.5%	12	0	62.5%	7	7	0.0134770	0.0000640	0.0005128
10	228	C322210	Paperboard container manufacturing	646	101.0%	653	24	24.1%	158	181	0.0257990	0.0009420	0.0286307
11	227	C322130	. Paperboard mills	187	100.1%	187	9	32.7%	61	70	0.2859390	0.0134630	0.0081969
12=>4	22	C221100	. . Electric power generation, trans	13	102.5%	13	0	62.5%	8	8	0.0686610	0.0003270	0.0005769
13=>4	22	C221100	. Electric power generation, transmi	9	102.5%	9	0	62.5%	6	6	0.0137520	0.0000650	0.0004036
14	191	C311221	Wet corn milling	399	100.5%	401	19	22.7%	91	110	0.0159290	0.0007600	0.0175827
15=>4	22	C221100	. Electric power generation, transmi	10	102.5%	10	0	62.5%	6	6	0.0243430	0.0001150	0.0004387
16	322	C531ORE	Other real estate	265	115.4%	306		41.0%	125	125	0.0105750		0.0134024
17=>4	22	C221100	. Electric power generation, transmi	9	102.5%	9	0	62.5%	5	6	0.0279990	0.0001620	0.0003846
18=>4	22	C221100	Electric power generation, transmissi	183	102.5%	188	1	62.5%	117	118	0.0073010	0.0000350	0.0082195

Source: own calculations based on U.S. Bureau of Economic Analysis (2026)

The self-consumption adjustment coefficients are elements of matrix Ast in equation (10) - the first applied to final demand (to the right of the equation), the other three to direct material requirements (in the bracket term to the left).

In **Table 5**, the 9 linking chains marked in green represent 70.1% of the total value added contribution according to matrix obtained from formula (13).

The application offers a diagram that shows the coverage by "stage", i.e. by length of linking chains - see **Figure 13**.

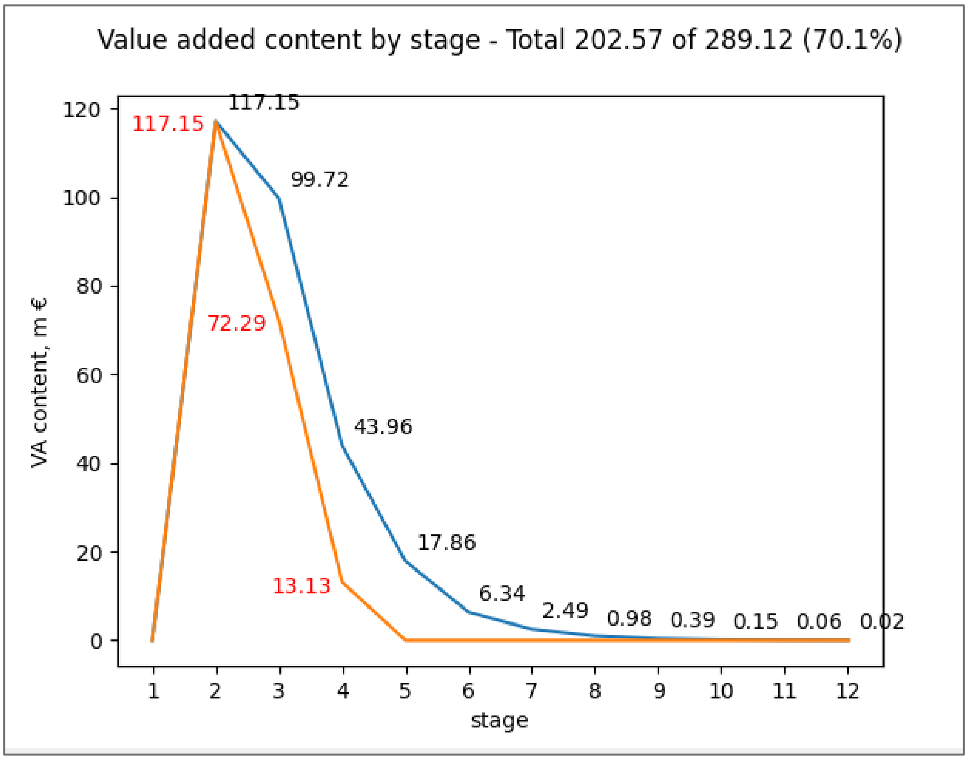
The blue line with the black numbers is derived from the series expansion of matrix W (see equation (17)), while the orange line with the red numbers is the summary of the green rows in **Table 5**.

For example: 13.13 = 5 + 8 from rows 4 and 12 in the table (level 4 with 2 indents).

The last line in the table shows the direct consumption of electric energy for the production of beer. In the flowchart, the corresponding chain is the one at the very bottom. In the stage diagram, this path is shown as "stage 2". The coverage in this case is 100% (117 M US$), the only relevant path is included in the decomposition.

Stage 1 is referring to the production of beer itself. In the "Find Linking Chains" mode, it is zero as soon as the material is not equal to the product.

Figure 13 *- Value added content by stage (length of chain)*



Source: own calculations based on U.S. Bureau of Economic Analysis (2026)

This chart **links the "Depth First Search" approach** of the Visual Network Decomposition with the IOPathFinder application **to the "Breadth First Search" perspective** of the series expansion of the Leontief inverse.

The image points to the question of the **length of the chains** of backward and forward linkages, which is also linked the concept of downstreamness and upstreamness.

How many linking chains (or paths) are required to obtain a representative decomposition of the total values derived from the Leontief inverse? Is it even possible get a full 100% breakdown? What can be considered a "representative" decomposition?

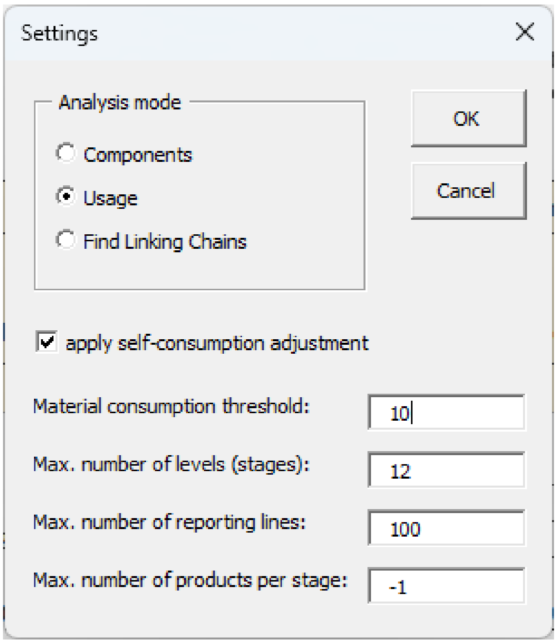
In the following, we will consider some intuitive properties of digraphs without referring to exact proofs according to the theory of graphs.

If for a given product the **digraph** derived from the matrix of direct material requirements from the perspective of backward linkages **does not include any direct or indirect cycles**, this graph is considered a **tree graph**. Direct cycles correspond to self-consumption.

For a tree graph, the "depth first search (DFS)" algorithm always gives a finite number of paths, which still can be very large. The opposite is true for digraphs with cycles, which lead to an infinite number of paths. However, production networks without cycles are rare exceptions on an aggregated level, such as in Input-Output analysis. But even in a corporate environment with materials represented on the lowest level of stock-keeping units (SKU), cycles can appear in cases of reuse, recycling or as a method to account for yield losses in the production process.

To cope with the large number of paths, the IOPathFinder application uses a number of parameters to set limitations for the search.

Figure 14 *- Control parameters for the search for linkages*



Analysis mode
- ○ Components
- ● Usage
- ○ Find Linking Chains

☑ apply self-consumption adjustment

Material consumption threshold: 10

Max. number of levels (stages): 12

Max. number of reporting lines: 100

Max. number of products per stage: -1

Material consumption threshold: stops search as soon as material consumption, falls below this threshold in monetary terms. As direct material requirement coefficients $a_{ij} < 1$, the resulting absolute material consumption gets smaller with each stage of the linkages.

Maximum number of levels: maximum length of backward or forward chains

Maximum number of products per stage: allows to show longer chains even with smaller thresholds, not showing all backward resp. forward linkages on reach stage ("-1" means "ignore parameter, show all".)

Figure 15 illustrates how the coverage of the value added contribution and the number of selected chains increase with lower thresholds for our example of electric energy consumption for the US beer production in 2017.

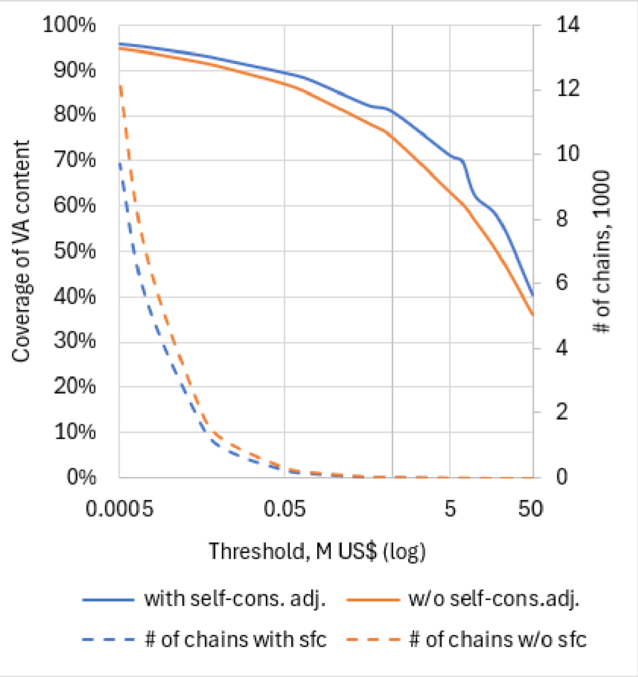
The fourth line with a threshold of 7 M US$ corresponds to the settings of our US example as shown in **Figure 2** and **Table 5**.

The threshold is displayed on the x-axis. Please note the log scale used to match the wide range of values. The primary y-axis to the left shows the percentage coverage: the solid blue line with instant inversion of self-consumption, the solid orange line - the case with self-consumption handled as "normal coefficients".

The dashed lines indicate the number of chains selected with the respective threshold for both cases. They refer to the secondary y-axis to the right (in thousands). Despite the large range of values, the linear scale was deliberately retained. **Above a certain level of coverage, further increases are only achieved at the cost of a large number of additional chains**.

Figure 15 *- Coverage of VA content depending on the threshold*

	self-cons. adj.			**no self-cons. adj.**		
Thresh-old, M US$	num-ber of chains	max lev-el	VA cov-erage %	num-ber of chains	max lev-el	VA cov-erage %
50	1	1	40.5%	1	1	36.0%
20	3	2	56.9%	3	2	48.8%
10	5	3	62.3%	6	3	56.7%
7	9	3	70.1%	8	3	60.4%
5	10	3	71.2%	10	3	62.9%
1	27	4	80.9%	32	4	75.1%
0.5	36	4	82.4%	52	4	78.3%
0.1	154	5	87.9%	192	5	84.6%
0.05	255	5	89.4%	336	6	86.8%
0.01	852	6	92.2%	1,119	7	90.3%
0.005	1,591	6	93.3%	1,985	7	91.6%
0.001	5,641	7	95.2%	7,082	8	93.9%
0.0005	9,677	7	95.8%	12,229	9	94.7%



Source: own calculations based on U.S. Bureau of Economic Analysis (2026)

The comparison of the lines with and without the **self-consumption technique** applied demonstrates the efficiency of the algorithm. **With the same thresholds, a higher coverage is gained**. For smaller thresholds, much less and also shorter chains are selected with instant inversion of self-consumption. On the other hand, the coverage advantage of the algorithm tends to shrink for very small thresholds.

2.4 Visual Network Decomposition: the standard analysis path

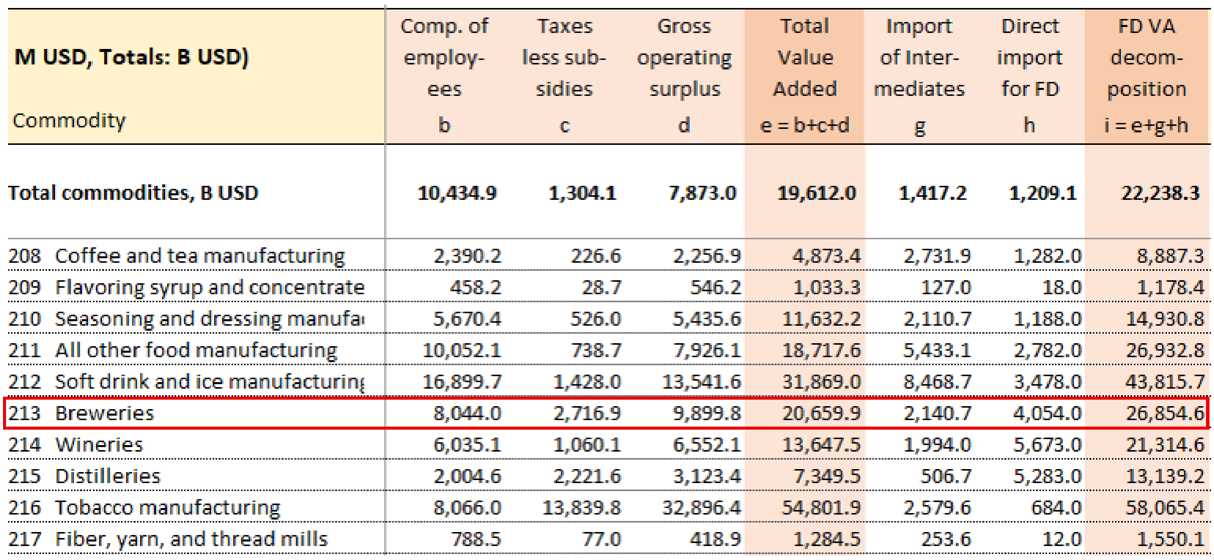
In the standard case, the Visual Network Decomposition

A. starts with the Value Added contribution matrix on an aggregated level.
B. In the following step, a selected item on the top level will be broken down to the next layer of detail, which provides summary figures already on the product level.
C. From here, the actual visual network decomposition process can be invoked

In the interactive "IOPathFinder" application, the drill-down to the next level of analysis is started by double-clicking on the respective cell in a report.

For the US Input-Output table, analysis starts with this report as entry level "A":

***Table 6** - Decomposition of Final Demand by VA and import contribution*



M USD, Totals: B USD) Commodity	Comp. of employ-ees b	Taxes less sub-sidies c	Gross operating surplus d	Total Value Added e = b+c+d	Import of Inter-mediates g	Direct import for FD h	FD VA decom-position i = e+g+h
Total commodities, B USD	**10,434.9**	**1,304.1**	**7,873.0**	**19,612.0**	**1,417.2**	**1,209.1**	**22,238.3**
208 Coffee and tea manufacturing	2,390.2	226.6	2,256.9	4,873.4	2,731.9	1,282.0	8,887.3
209 Flavoring syrup and concentrate	458.2	28.7	546.2	1,033.3	127.0	18.0	1,178.4
210 Seasoning and dressing manufa	5,670.4	526.0	5,435.6	11,632.2	2,110.7	1,188.0	14,930.8
211 All other food manufacturing	10,052.1	738.7	7,926.1	18,717.6	5,433.1	2,782.0	26,932.8
212 Soft drink and ice manufacturin	16,899.7	1,428.0	13,541.6	31,869.0	8,468.7	3,478.0	43,815.7
213 Breweries	8,044.0	2,716.9	9,899.8	20,659.9	2,140.7	4,054.0	26,854.6
214 Wineries	6,035.1	1,060.1	6,552.1	13,647.5	1,994.0	5,673.0	21,314.6
215 Distilleries	2,004.6	2,221.6	3,123.4	7,349.5	506.7	5,283.0	13,139.2
216 Tobacco manufacturing	8,066.0	13,839.8	32,896.4	54,801.9	2,579.6	684.0	58,065.4
217 Fiber, yarn, and thread mills	788.5	77.0	418.9	1,284.5	253.6	12.0	1,550.1

Source: own calculations based on U.S. Bureau of Economic Analysis (2026)

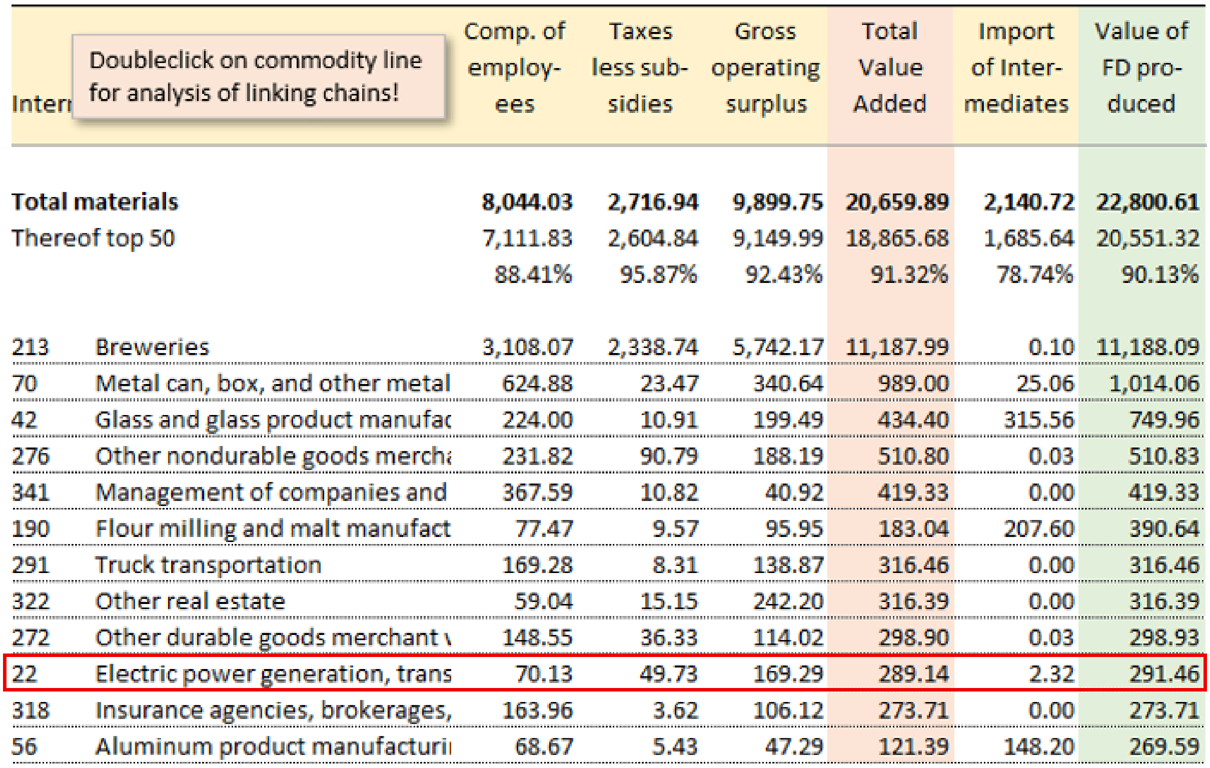
For each commodity, the value of Final Demand is broken down by the elements of total value added contribution and total intermediate and direct import contents.

These items are calculated using formulas (48), (49) and (50) in Appendix C. Formula (48) is applied for each component value added to obtain the first three columns.

We have scrolled down already to see the data for "breweries". When double-clicking on a data line, e.g., for "Breweries", the information will be broken down by contributing material. This is level "B" in the list at the beginning of this chapter.

The report can be sorted by any column, in this case by "Value of FD produced":

***Table 7** - Decomposition of Final Demand of "Breweries" by contributing material*



Doubleclick on commodity line for analysis of linking chains!

Interi		Comp. of employ-ees	Taxes less sub-sidies	Gross operating surplus	Total Value Added	Import of Inter-mediates	Value of FD pro-duced
Total materials		**8,044.03**	**2,716.94**	**9,899.75**	**20,659.89**	**2,140.72**	**22,800.61**
Thereof top 50		7,111.83	2,604.84	9,149.99	18,865.68	1,685.64	20,551.32
		88.41%	95.87%	92.43%	91.32%	78.74%	90.13%
213	Breweries	3,108.07	2,338.74	5,742.17	11,187.99	0.10	11,188.09
70	Metal can, box, and other metal	624.88	23.47	340.64	989.00	25.06	1,014.06
42	Glass and glass product manufac	224.00	10.91	199.49	434.40	315.56	749.96
276	Other nondurable goods mercha	231.82	90.79	188.19	510.80	0.03	510.83
341	Management of companies and	367.59	10.82	40.92	419.33	0.00	419.33
190	Flour milling and malt manufact	77.47	9.57	95.95	183.04	207.60	390.64
291	Truck transportation	169.28	8.31	138.87	316.46	0.00	316.46
322	Other real estate	59.04	15.15	242.20	316.39	0.00	316.39
272	Other durable goods merchant	148.55	36.33	114.02	298.90	0.03	298.93
22	Electric power generation, trans	70.13	49.73	169.29	289.14	2.32	291.46
318	Insurance agencies, brokerages,	163.96	3.62	106.12	273.71	0.00	273.71
56	Aluminum product manufacturi	68.67	5.43	47.29	121.39	148.20	269.59

Source: own calculations based on U.S. Bureau of Economic Analysis (2026)

With a double-click on the line for "Electric power generation etc.", the reports shown before in **Figure 2** (flowchart) and **Table 5** (hierarchical table) will be displayed. All reports are fully interactive. Any boxes in the flowcharts or lines in the reports can be selected to start the analysis for another product or to switch between the three analysis modes.

The next two flowcharts illustrate the other two analysis modes introduced earlier in segment 2.1.

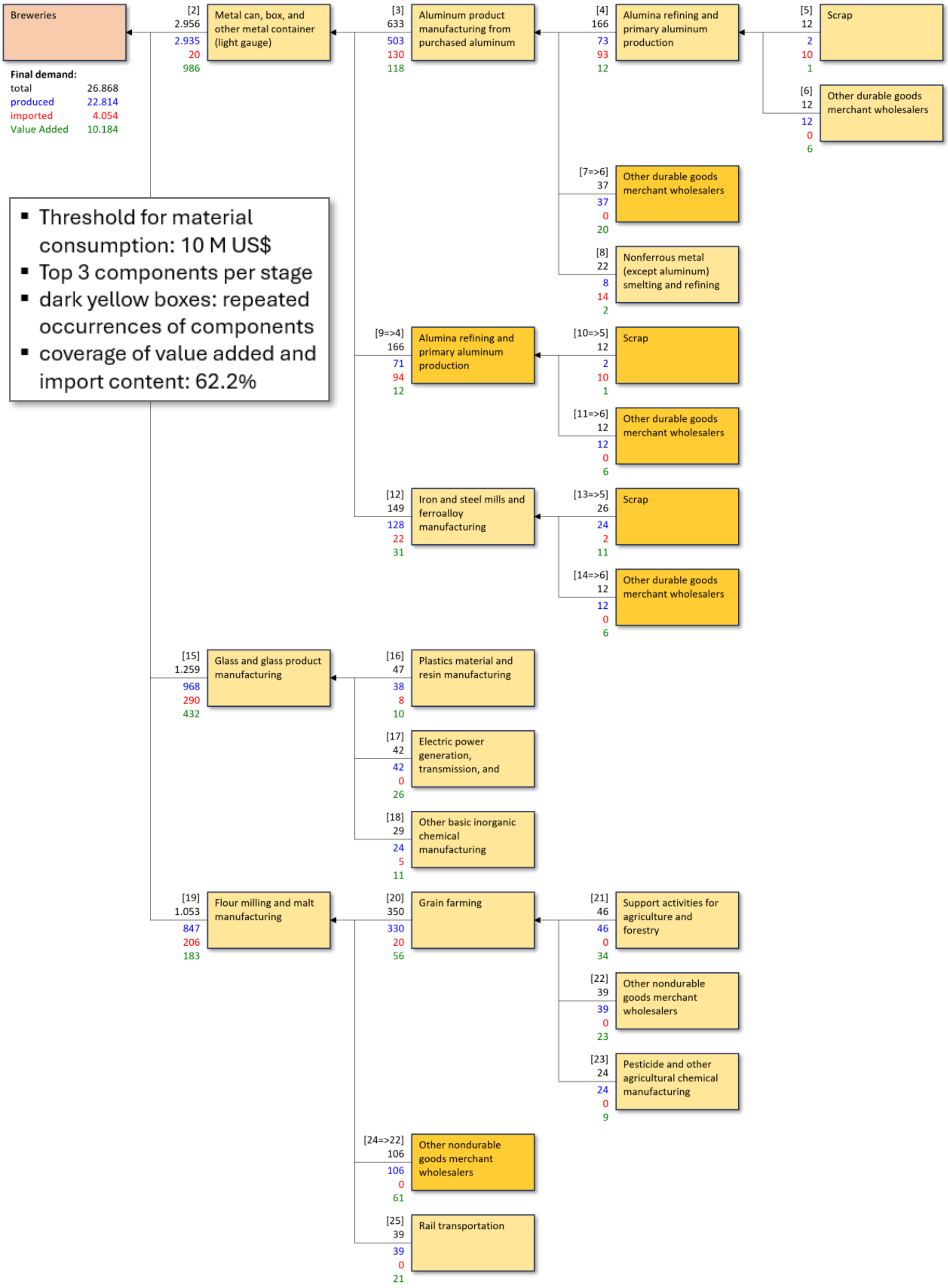
Figure 16 is an example of the "Components" mode. Just like the reports for the "Find Linking Chains" mode presented above, it reveals the direct and indirect components of brewery products, but this time not restricted to the consumption of electric energy at the beginning of the different chains.

With a threshold for material consumption of 10 M US$, 16 chains are selected with a relatively high coverage of value added and import contents of 62.2%,

Finally, **Figure 17** illustrates the "Usage" mode for "Aluminum products from purchased aluminum". Besides the usage for brewery products, other usages are included such as canning of fruit and vegetables or architectural metal products for office, commercial and residential structures.

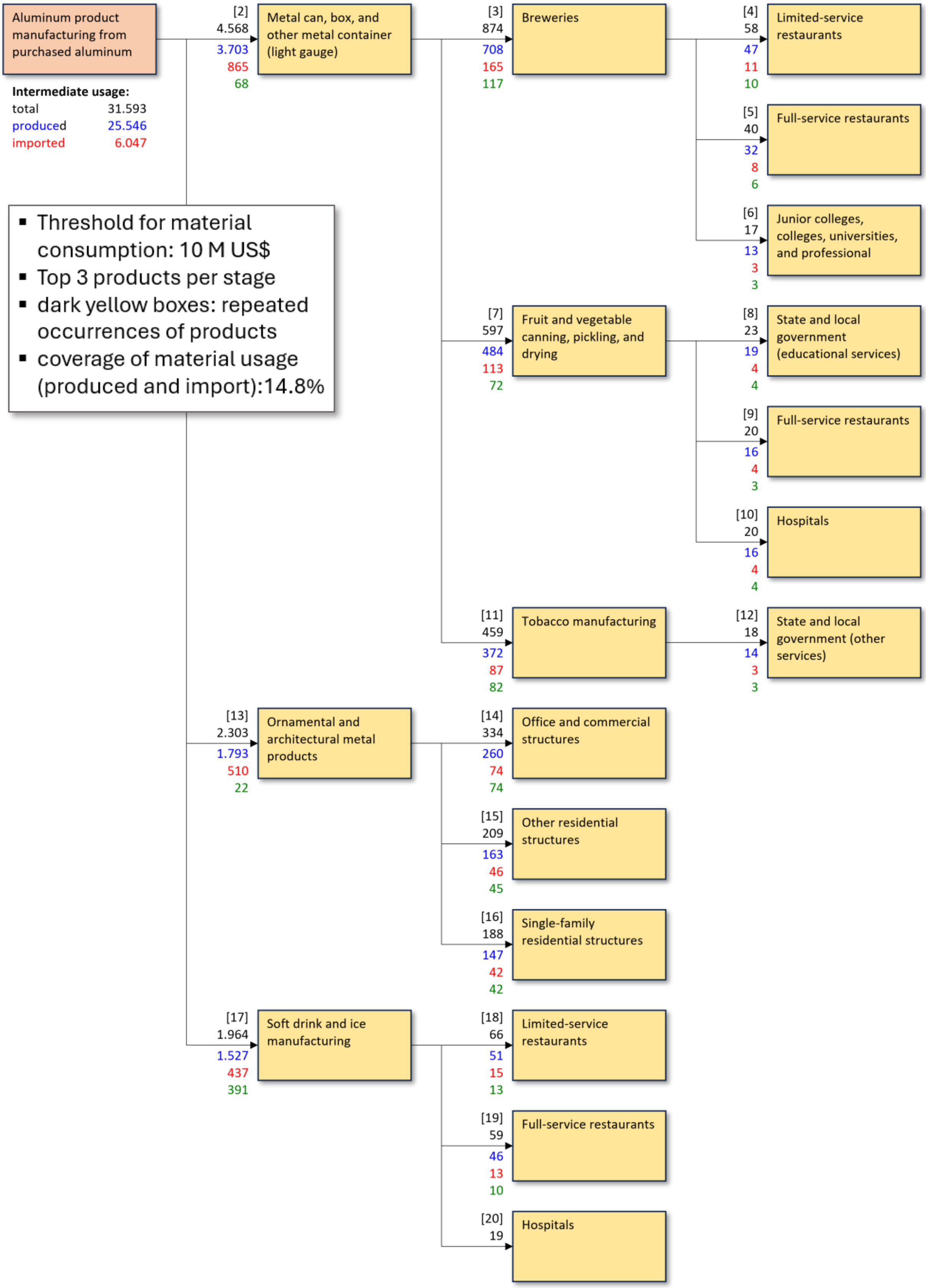
The low coverage of the 13 selected chains of forward linkages (14.8%) can be explained by the wide range of applications of aluminum products in most industries.

Figure 16 *- Direct and indirect components of US Brewery products 2017*



Source: own calculations based on U.S. Bureau of Economic Analysis (2026)

Figure 17 *- Direct and indirect usage of US Aluminum products 2017*



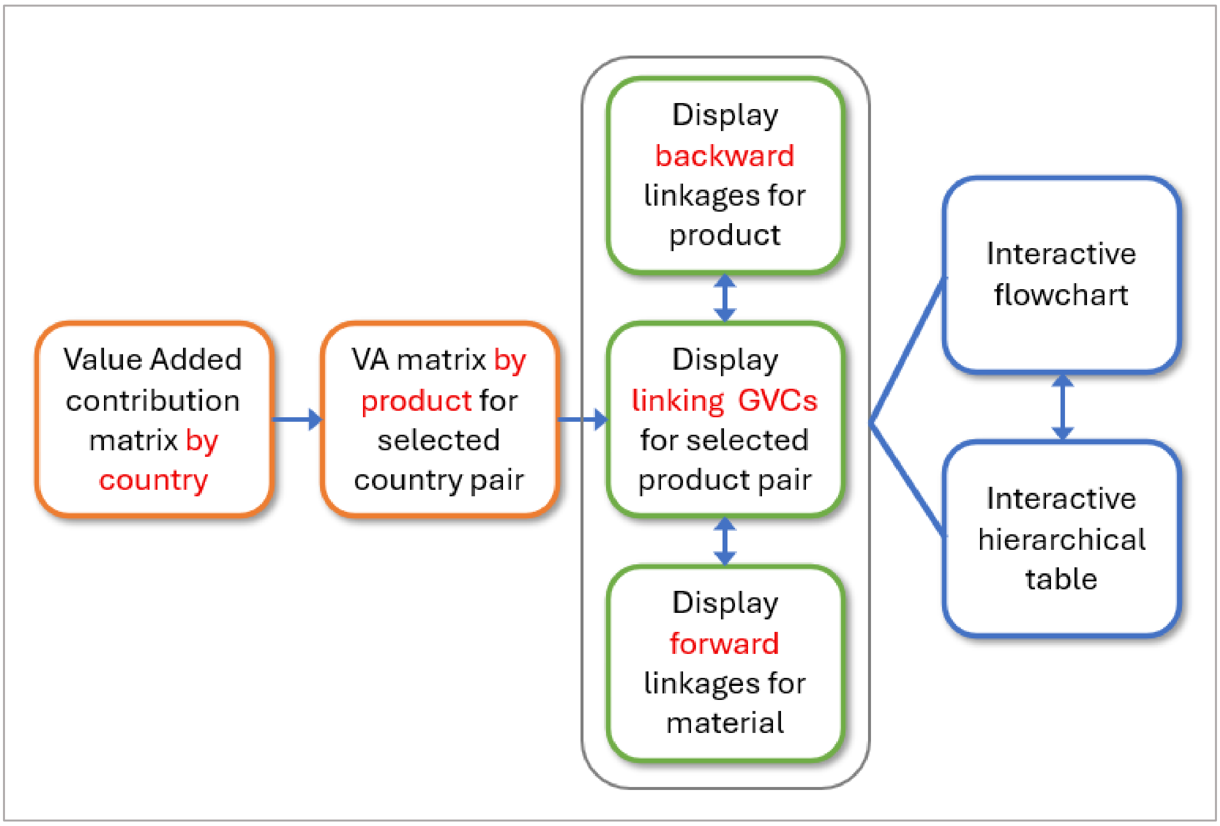
Source: own calculations based on U.S. Bureau of Economic Analysis (2026)

In the case of Multiregional Input-Output tables (MRIO) like WIOD, FIGARO or OECD-ICIO, the levels of the standard analysis path are different:

A. The analysis starts with the Value Added contribution matrix aggregated by country. This can be either a table or a world map. Besides a geographical map, which might be difficult to read due to the clustering of countries depending on the regional focus, a schematic world map may also be used.
B. After selecting a pair of countries, the value added contribution of the first to the second will be broken down by product
C. From here, the actual visual network decomposition process can be invoked

Figure 18 illustrates this sequence of analysis, including the options to switch back and forth between the different reports and analysis modes.

***Figure 18** - Standard analysis path for MRIOs*



The reports for MRIOs will be discussed in detail in the next chapter.

3 Use cases – Input-Output Analysis and extensions

The technique of visual network decomposition offers a unique perspective on the world of Input-Output tables. In this chapter, it will be demonstrated how this approach can be used to gain deeper insights into several analytical applications and special extensions of Input-Output analysis.

So far, the detailed Input-Output table of the U.S. Bureau of Economic Analysis for 2017 has been used an example.

The natural next step was to take a look at **Multiregional Input-Output models (MRIO)**. A major milestone in this area was the development of the **World Input-Output Database** (WIOD), which is still often used as a key reference (Timmer et al., 2015). This was the first MRIO database for which the visual network decomposition with its main "IOPathFinder" application approach was applied.

In 2021, Eurostat started to publish official **FIGARO** MRIO tables on an annual basis - a joint development of the JRC and Eurostat (Rémond-Tiedrez et al., 2019). Next, with **FIGARO-E3**, the JRC developed a much more detailed version of FIGARO with special focus on air emissions, energy and labor contents (Cazcarro et al., 2025).

Segments 3.1 and 3.2 will deal with the visual network decomposition for FIGARO-E3 to explore value added contents (including TiVA) and carbon footprints respectively.

Segments 3.3 and 3.4 illustrate how visual network decomposition can be used to attribute the different linking paths of the Input-Output network to components of a decomposition of the value added content matrix that was introduced in section 2.3.1.

The investigation follows the approach developed in Wang et al. (2017, 2021), using FIGARO-E3 (3.3) and - for the distinction of foreign-owned and domestic-owned enterprises (3.4) - the OECD AMNE database (Cadestin, C. et al., 2018).

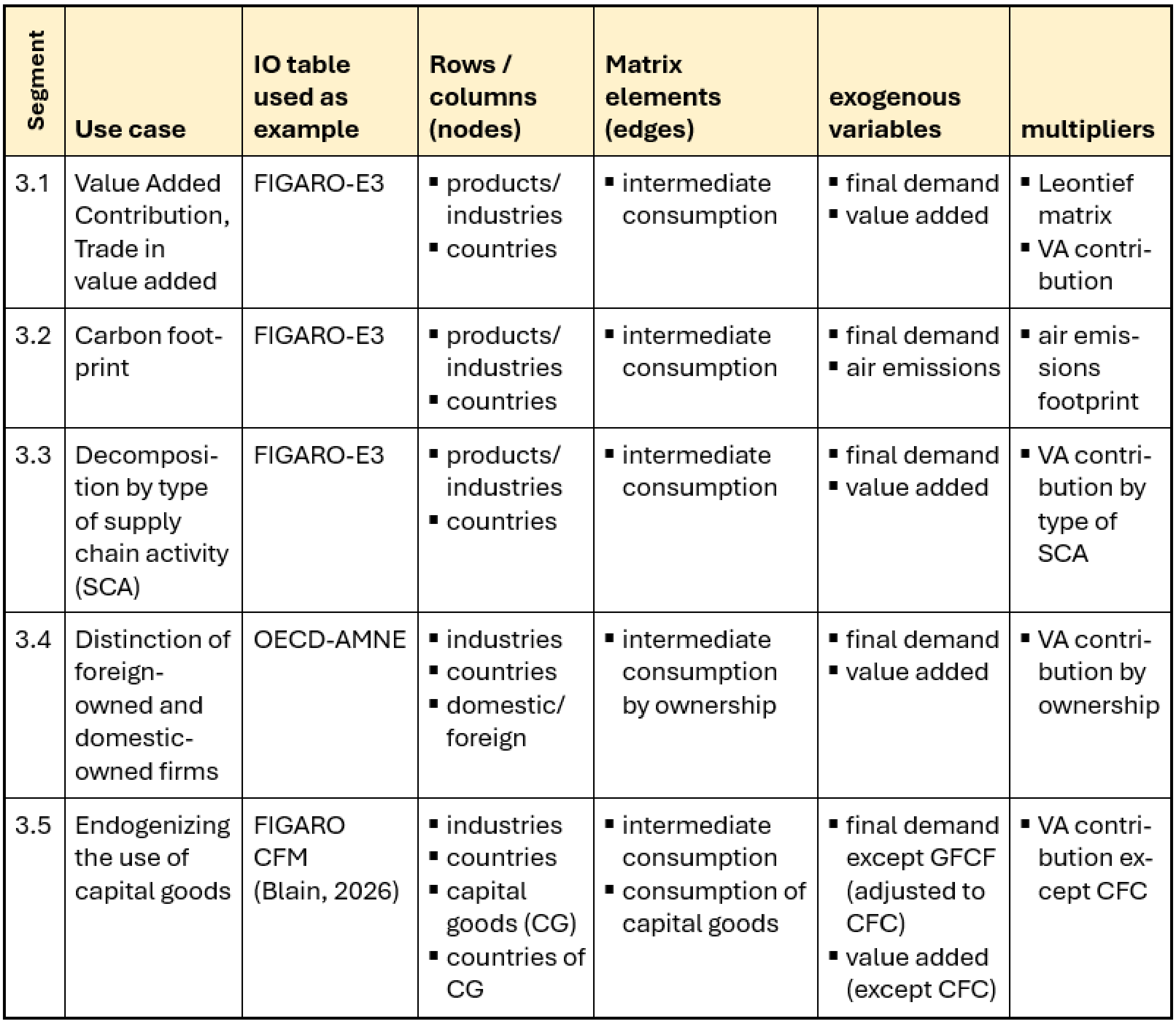
A standard approach to assign value added contents and energy footprints of capital goods to the products, for which these fixed assets are used, is based on capital flow matrices (see, e.g., Ye M. et al., 2024). Visual network decomposition can be extended to show this use of capital goods, beyond just intermediate material consumption. This type of "capital endogenization" will be demonstrated in segment 3.5.

Table 8 provides an overview of the setup of data structures for the different cases. In most cases, original data sources can be used directly or with some basic rearrangement.

Certain properties of the rows and columns, such as countries or ownership are reflected by including them in the descriptions and keys of the objects

For use cases 3.3, 3.4 and 3.5, the algorithm has been extended to detect certain characteristics of supply chains and to display them besides the nodes in the flowchart.

***Table 8** - Overview of use cases with main characteristics*



Segment	Use case	IO table used as example	Rows / columns (nodes)	Matrix elements (edges)	exogenous variables	multipliers
3.1	Value Added Contribution, Trade in value added	FIGARO-E3	▪ products/ industries ▪ countries	▪ intermediate consumption	▪ final demand ▪ value added	▪ Leontief matrix ▪ VA contribution
3.2	Carbon footprint	FIGARO-E3	▪ products/ industries ▪ countries	▪ intermediate consumption	▪ final demand ▪ air emissions	▪ air emissions footprint
3.3	Decomposition by type of supply chain activity (SCA)	FIGARO-E3	▪ products/ industries ▪ countries	▪ intermediate consumption	▪ final demand ▪ value added	▪ VA contribution by type of SCA
3.4	Distinction of foreign-owned and domestic-owned firms	OECD-AMNE	▪ industries ▪ countries ▪ domestic/ foreign	▪ intermediate consumption by ownership	▪ final demand ▪ value added	▪ VA contribution by ownership
3.5	Endogenizing the use of capital goods	FIGARO CFM (Blain, 2026)	▪ industries ▪ countries ▪ capital goods (CG) ▪ countries of CG	▪ intermediate consumption ▪ consumption of capital goods	▪ final demand except GFCF (adjusted to CFC) ▪ value added (except CFC)	▪ VA contribution except CFC

For the first three use cases, the FIGARO-E3 MRIO for 2015 is used as an example for demonstration (Cazcarro et al., 2025, data source: European Commission, Joint Research Centre, 2025). This database is an extension of the Eurostat FIGARO Input-Output tables, with 213 instead of 64 products for the same 45 countries plus "Rest of the World". It includes data on air emissions, labor by gender and skill level as well as on primary supply and net use of energy.

FIGARO-E3, just as FIGARO, is published both in a "product by product" and an "industry by industry" version. We will strictly **focus on the "product by product" tables**.

The implementation of the concept will now be described in detail for each of the use cases.

3.1 Value Added contents – Trade in Value Added

3.1.1 Decomposition of the Value Added Contribution Matrix

The visual network decomposition of the elements of the value added contribution matrix is the standard case. It follows the MRIO analysis path as described in **Figure 18**.

The data on intermediate consumption, final demand by component and value added by component from the original files provided by the JRC can be used directly.

There is one modification to the calculation of the value added contribution matrix as described in equation (13). The expression $\hat{y}$ implies that final demand is represented by a single vector, i.e. with one value per country and product.

However, in a standard MRIO setup, final demand is given as a matrix Y_{cc} with one column per country and component of final demand and one row per country and product. Elements of this matrix show for each producing country and each product, which amounts of this product are used in which country for the different final demand components.

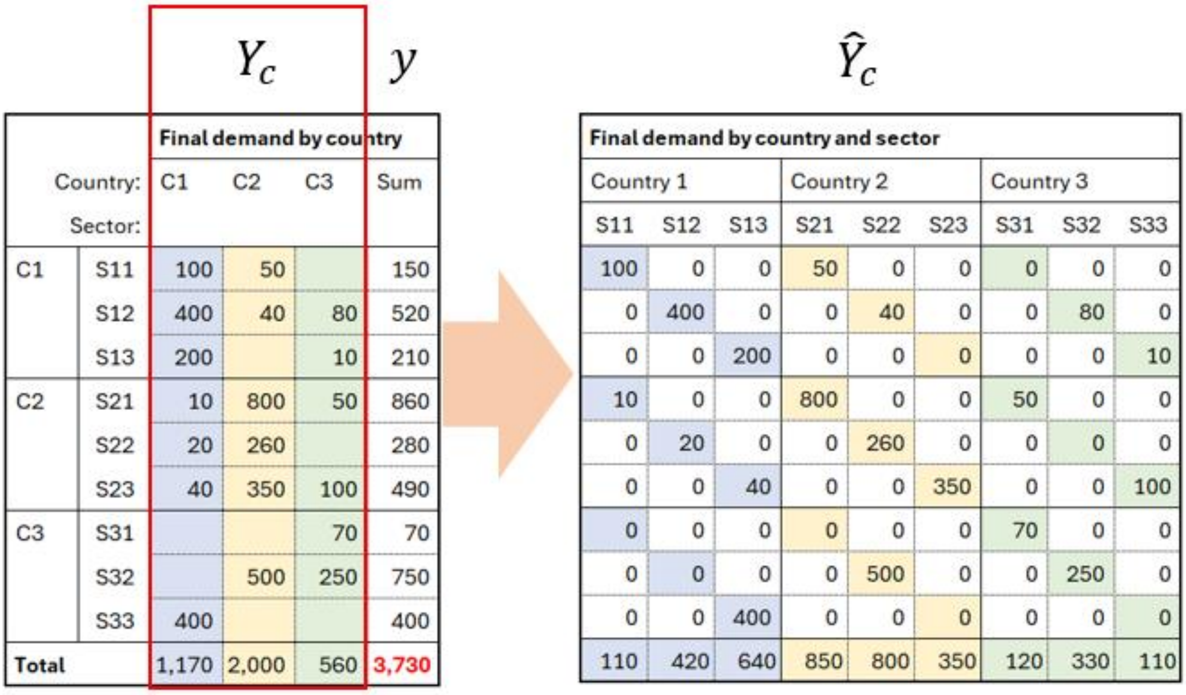
If now using the vector of row totals of this matrix $y = Y_{cc}e$ with e being a vector of ones according to the number of countries times the number of final demand components, the resulting value added contribution matrix would assign the export for direct final use in other countries to the exporting country instead of the using country.

In this way, e.g., the value added embodied in exports of motor vehicles from Spain for direct final use in France would be shown in Spain rather than in France as the using country.

The solution is a blockwise diagonalization of final demand by using country. First, matrix Y_{cc} is transformed into a matrix Y_c by summarizing all final demand components for each using country. Y_c has one column per country. In a second step, each block of Y_c for a pair of producing and using countries is diagonalized. The resulting matrix $\hat{Y}_c$ is a block matrix with diagonal blocks, a square matrix with a number of rows and columns both equal to the number of countries time the number of products.

For the Sample WIOT presented in Appendix B, this transformation would look like this:

Figure 19 *- Final Demand by country with diagonal blocks for the Sample WIOT*



		Final demand by country			
	Country:	C1	C2	C3	Sum
	Sector:				
C1	S11	100	50		150
	S12	400	40	80	520
	S13	200		10	210
C2	S21	10	800	50	860
	S22	20	260		280
	S23	40	350	100	490
C3	S31			70	70
	S32		500	250	750
	S33	400			400
Total		1,170	2,000	560	3,730

Final demand by country and sector								
Country 1			Country 2			Country 3		
S11	S12	S13	S21	S22	S23	S31	S32	S33
100	0	0	50	0	0	0	0	0
0	400	0	0	40	0	0	80	0
0	0	200	0	0	0	0	0	10
10	0	0	800	0	0	50	0	0
0	20	0	0	260	0	0	0	0
0	0	40	0	0	350	0	0	100
0	0	0	0	0	0	70	0	0
0	0	0	0	500	0	0	250	0
0	0	400	0	0	0	0	0	0
110	420	640	850	800	350	120	330	110

Equation (13) will be adjusted as follows:

$$W_u = \hat{v}(I - A)^{-1}\hat{Y}_c \tag{19}$$

Where W_u denotes the value added contribution matrix for final demand (used).

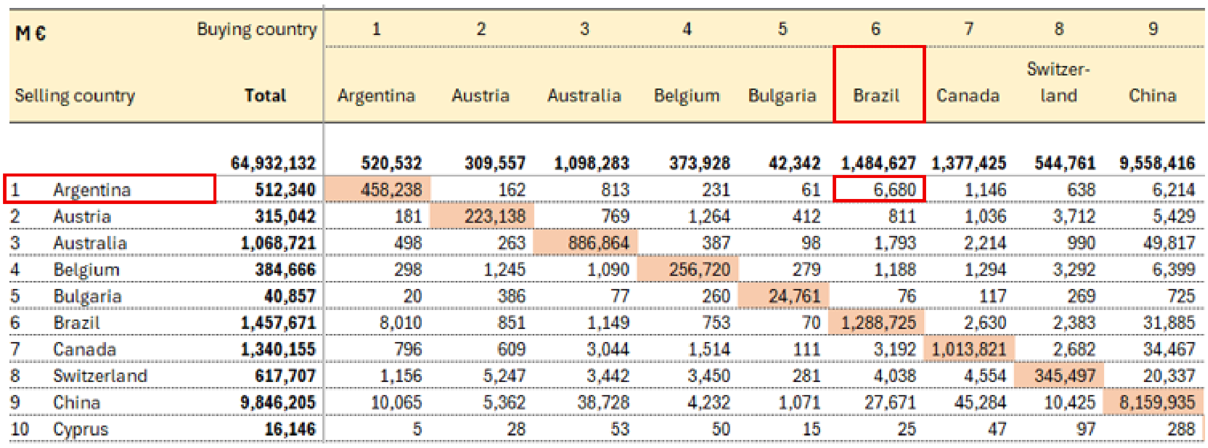
According to **Figure 18**, the analysis starts with matrix W_u, aggregated by country.

Table 9 shows the matrix, sorted by the sequence of countries in the original MRIO data table. For example, the value added contribution of Argentina for Brazil amounts to 6.7 billions of €.

In **Table 10**, columns are sorted by descending column totals (final demand) and descending row totals (value added).

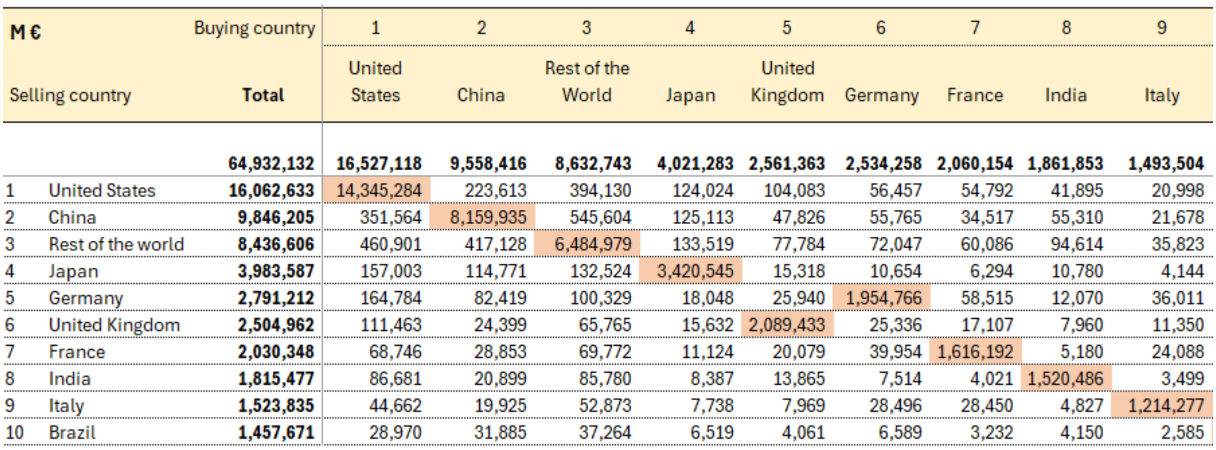
To start an in-depth analysis of Germany's value added contribution to Spain's final demand, in **Table 11** columns and rows are sorted respectively.

__Table 9__ - Value added contribution matrix (VACM) by country, no sorting



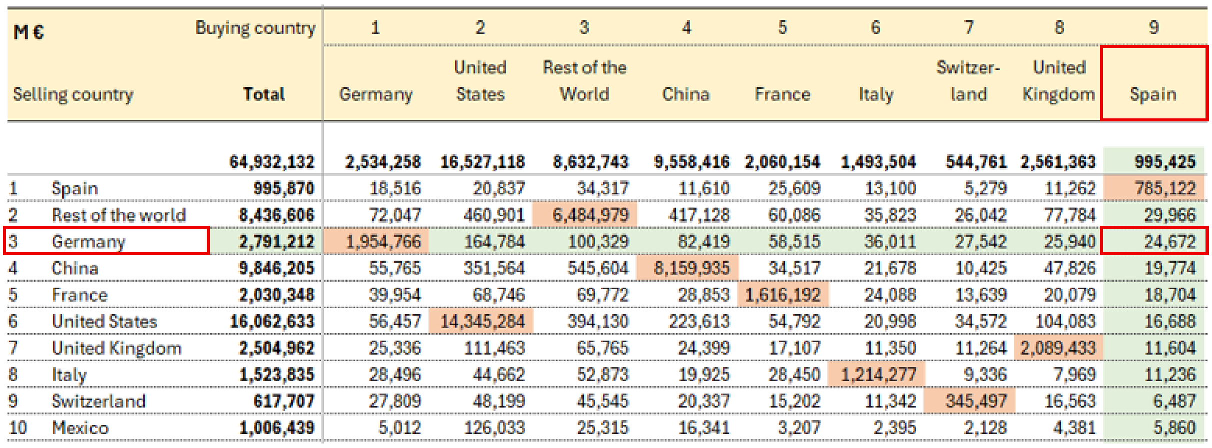
M €		Buying country	1	2	3	4	5	6	7	8	9
Selling country		Total	Argentina	Austria	Australia	Belgium	Bulgaria	Brazil	Canada	Switzer-land	China
		64,932,132	**520,532**	**309,557**	**1,098,283**	**373,928**	**42,342**	**1,484,627**	**1,377,425**	**544,761**	**9,558,416**
1	Argentina	**512,340**	458,238	162	813	231	61	6,680	1,146	638	6,214
2	Austria	**315,042**	181	223,138	769	1,264	412	811	1,036	3,712	5,429
3	Australia	**1,068,721**	498	263	886,864	387	98	1,793	2,214	990	49,817
4	Belgium	**384,666**	298	1,245	1,090	256,720	279	1,188	1,294	3,292	6,399
5	Bulgaria	**40,857**	20	386	77	260	24,761	76	117	269	725
6	Brazil	**1,457,671**	8,010	851	1,149	753	70	1,288,725	2,630	2,383	31,885
7	Canada	**1,340,155**	796	609	3,044	1,514	111	3,192	1,013,821	2,682	34,467
8	Switzerland	**617,707**	1,156	5,247	3,442	3,450	281	4,038	4,554	345,497	20,337
9	China	**9,846,205**	10,065	5,362	38,728	4,232	1,071	27,671	45,284	10,425	8,159,935
10	Cyprus	**16,146**	5	28	53	50	15	25	47	97	288

__Table 10__ - VACM by country, descending VA (rows) and FD (columns)



M €		Buying country	1	2	3	4	5	6	7	8	9
Selling country		Total	United States	China	Rest of the World	Japan	United Kingdom	Germany	France	India	Italy
		64,932,132	**16,527,118**	**9,558,416**	**8,632,743**	**4,021,283**	**2,561,363**	**2,534,258**	**2,060,154**	**1,861,853**	**1,493,504**
1	United States	**16,062,633**	14,345,284	223,613	394,130	124,024	104,083	56,457	54,792	41,895	20,998
2	China	**9,846,205**	351,564	8,159,935	545,604	125,113	47,826	55,765	34,517	55,310	21,678
3	Rest of the world	**8,436,606**	460,901	417,128	6,484,979	133,519	77,784	72,047	60,086	94,614	35,823
4	Japan	**3,983,587**	157,003	114,771	132,524	3,420,545	15,318	10,654	6,294	10,780	4,144
5	Germany	**2,791,212**	164,784	82,419	100,329	18,048	25,940	1,954,766	58,515	12,070	36,011
6	United Kingdom	**2,504,962**	111,463	24,399	65,765	15,632	2,089,433	25,336	17,107	7,960	11,350
7	France	**2,030,348**	68,746	28,853	69,772	11,124	20,079	39,954	1,616,192	5,180	24,088
8	India	**1,815,477**	86,681	20,899	85,780	8,387	13,865	7,514	4,021	1,520,486	3,499
9	Italy	**1,523,835**	44,662	19,925	52,873	7,738	7,969	28,496	28,450	4,827	1,214,277
10	Brazil	**1,457,671**	28,970	31,885	37,264	6,519	4,061	6,589	3,232	4,150	2,585

__Table 11__ - VACM by country, sorted by Germany (columns) and Spain (rows)



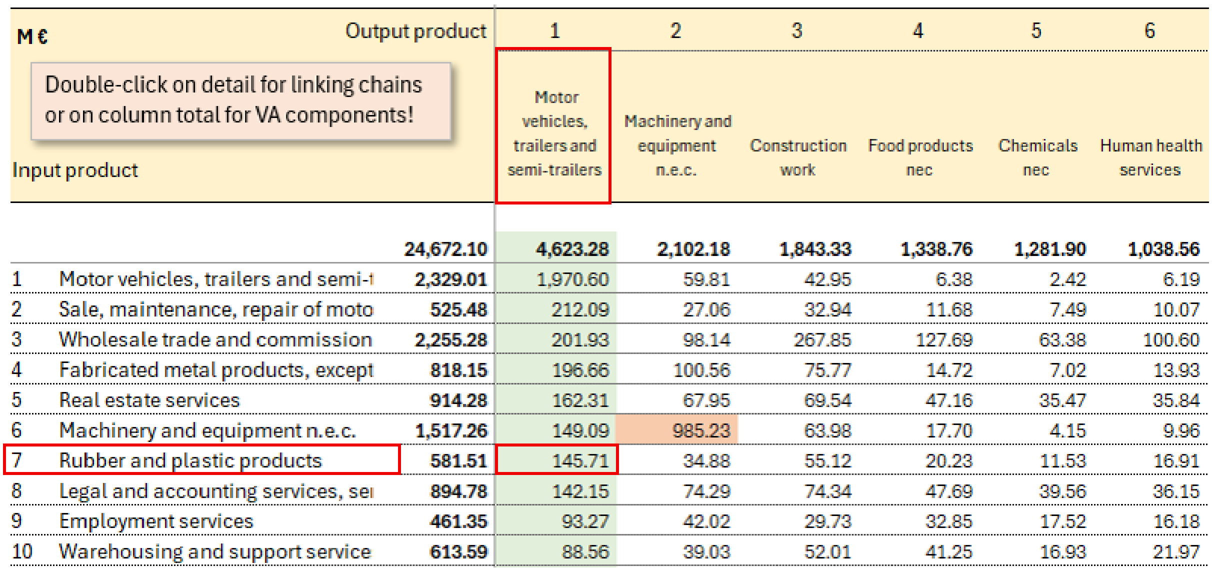
M €		Buying country	1	2	3	4	5	6	7	8	9
Selling country		Total	Germany	United States	Rest of the World	China	France	Italy	Switzer-land	United Kingdom	Spain
		64,932,132	**2,534,258**	**16,527,118**	**8,632,743**	**9,558,416**	**2,060,154**	**1,493,504**	**544,761**	**2,561,363**	**995,425**
1	Spain	**995,870**	18,516	20,837	34,317	11,610	25,609	13,100	5,279	11,262	785,122
2	Rest of the world	**8,436,606**	72,047	460,901	6,484,979	417,128	60,086	35,823	26,042	77,784	29,966
3	Germany	**2,791,212**	1,954,766	164,784	100,329	82,419	58,515	36,011	27,542	25,940	24,672
4	China	**9,846,205**	55,765	351,564	545,604	8,159,935	34,517	21,678	10,425	47,826	19,774
5	France	**2,030,348**	39,954	68,746	69,772	28,853	1,616,192	24,088	13,639	20,079	18,704
6	United States	**16,062,633**	56,457	14,345,284	394,130	223,613	54,792	20,998	34,572	104,083	16,688
7	United Kingdom	**2,504,962**	25,336	111,463	65,765	24,399	17,107	11,350	11,264	2,089,433	11,604
8	Italy	**1,523,835**	28,496	44,662	52,873	19,925	28,450	1,214,277	9,336	7,969	11,236
9	Switzerland	**617,707**	27,809	48,199	45,545	20,337	15,202	11,342	345,497	16,563	6,487
10	Mexico	**1,006,439**	5,012	126,033	25,315	16,341	3,207	2,395	2,128	4,381	5,860

Source: own calculations based on FIGARO-E3 2015 (EC, Joint Research Center, 2025)

The value added contribution of Germany for Spain is 24.7 billions of €. A double click on this value opens the breakdown by product as presented in **Table 12**.

Please note, that on the top-left appears the value we have just clicked on. Because "Motor Vehicles" absorb 4.6 billions of € or 18.7% of the total value added contribution, the rows with the contribution materials have been sorted by this column.

Table 12 *- Value added contribution of Germany to Spain by product*



M €	Output product		1	2	3	4	5	6
Input product	Double-click on detail for linking chains or on column total for VA components!		Motor vehicles, trailers and semi-trailers	Machinery and equipment n.e.c.	Construction work	Food products nec	Chemicals nec	Human health services
		24,672.10	**4,623.28**	**2,102.18**	**1,843.33**	**1,338.76**	**1,281.90**	**1,038.56**
1	Motor vehicles, trailers and semi-t	**2,329.01**	1,970.60	59.81	42.95	6.38	2.42	6.19
2	Sale, maintenance, repair of moto	**525.48**	212.09	27.06	32.94	11.68	7.49	10.07
3	Wholesale trade and commission	**2,255.28**	201.93	98.14	267.85	127.69	63.38	100.60
4	Fabricated metal products, except	**818.15**	196.66	100.56	75.77	14.72	7.02	13.93
5	Real estate services	**914.28**	162.31	67.95	69.54	47.16	35.47	35.84
6	Machinery and equipment n.e.c.	**1,517.26**	149.09	985.23	63.98	17.70	4.15	9.96
7	Rubber and plastic products	**581.51**	145.71	34.88	55.12	20.23	11.53	16.91
8	Legal and accounting services, se	**894.78**	142.15	74.29	74.34	47.69	39.56	36.15
9	Employment services	**461.35**	93.27	42.02	29.73	32.85	17.52	16.18
10	Warehousing and support service	**613.59**	88.56	39.03	52.01	41.25	16.93	21.97

Source: own calculations based on FIGARO-E3 2015 (EC, Joint Research Center, 2025)

Next we want to understand how "Rubber and plastic products" from Germany contribute to the value added absorbed by "Motor vehicles" in Spain. A double-click on the value in the red box invokes the respective "Find linking chains" reports - as a flowchart (**Figure 21**) and a nested (hierarchical) table (**Table 13**).

Figure 20 *- Find Linking Chains mode for FIGARO-E3: Flowchart legend*

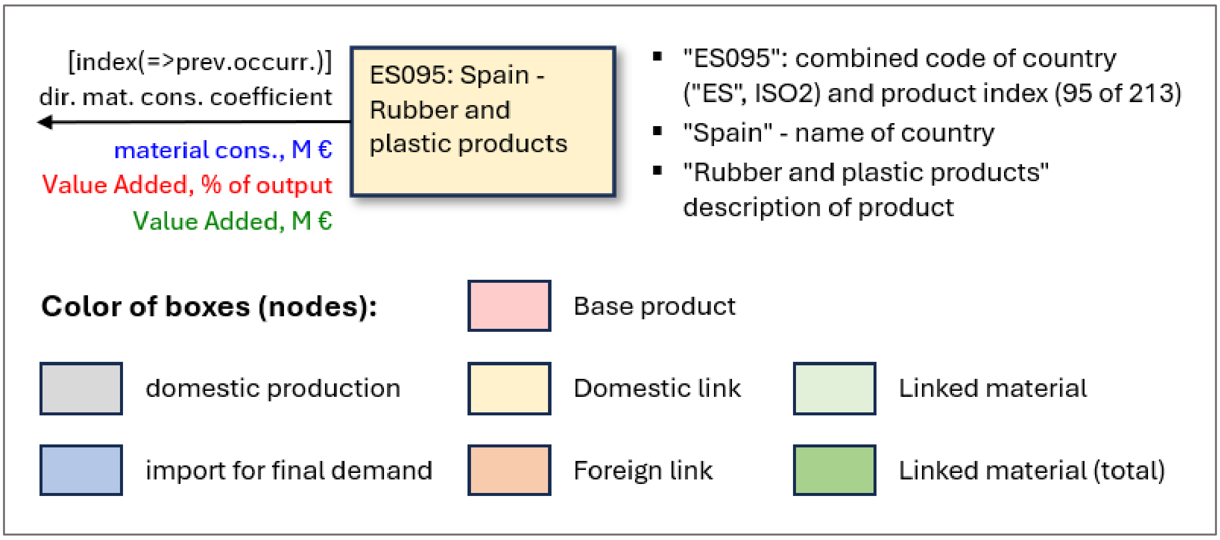
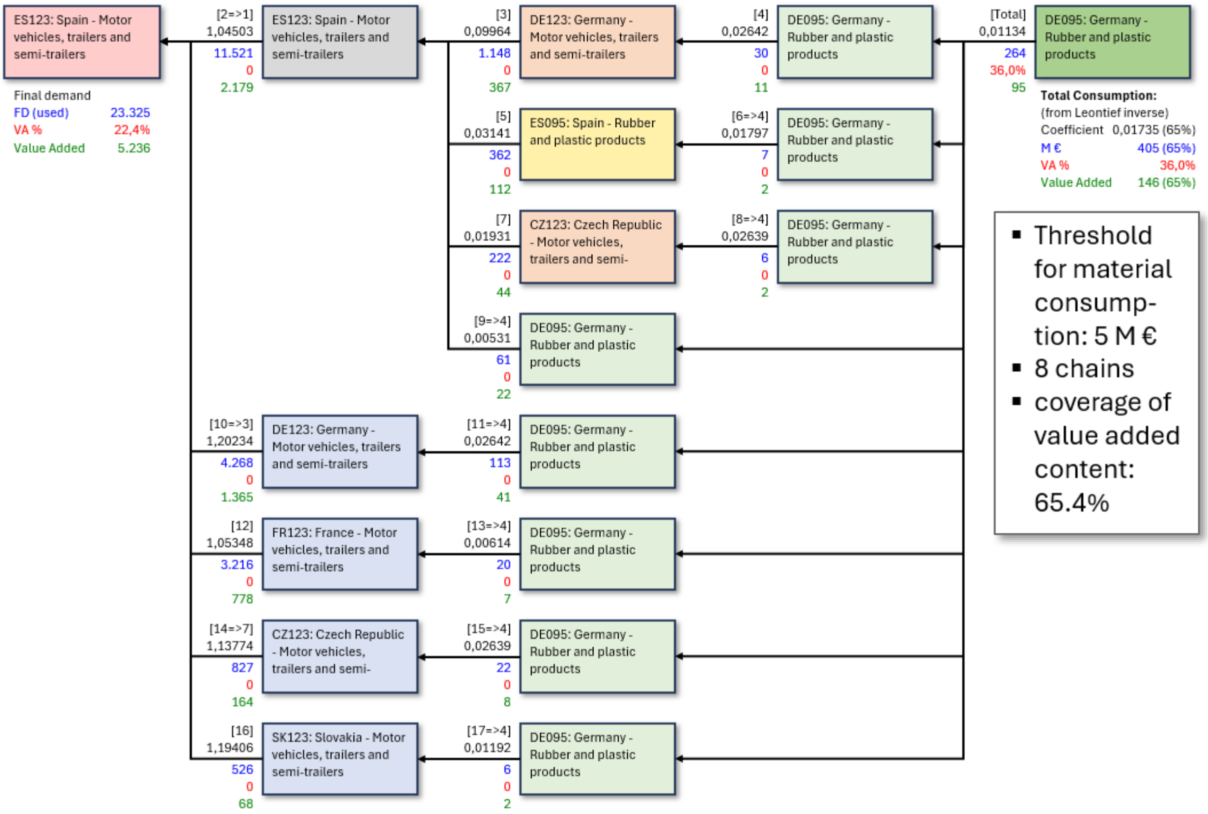
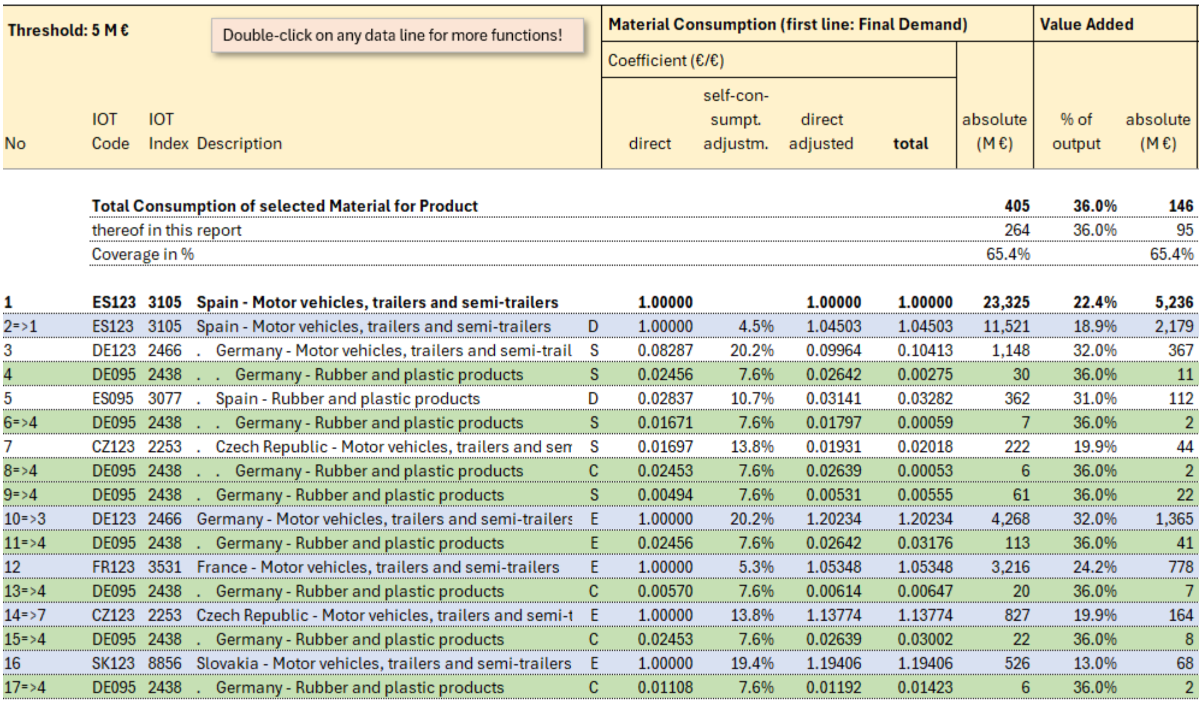


Figure 21 *- Linking chains: German "Rubber & plastic" for Spanish "Motor vehicles"*



- Threshold for material consump-tion: 5 M €
- 8 chains
- coverage of value added content: 65.4%

Table 13 *- Nested table: German "Rubber & plastic" for Spanish "Motor vehicles"*



Threshold: 5 M €

Double-click on any data line for more functions!

					Material Consumption (first line: Final Demand)					**Value Added**	
					Coefficient (€/€)						
No	IOT Code	IOT Index	Description		direct	self-con-sumpt. adjustm.	direct adjusted	**total**	absolute (M €)	% of output	absolute (M €)
	Total Consumption of selected Material for Product								**405**	**36.0%**	**146**
	thereof in this report								264	36.0%	95
	Coverage in %								65.4%		65.4%
1	**ES123**	**3105**	**Spain - Motor vehicles, trailers and semi-trailers**		**1.00000**		**1.00000**	**1.00000**	**23,325**	**22.4%**	**5,236**
2=>1	ES123	3105	Spain - Motor vehicles, trailers and semi-trailers	D	1.00000	4.5%	1.04503	1.04503	11,521	18.9%	2,179
3	DE123	2466	. Germany - Motor vehicles, trailers and semi-trail	S	0.08287	20.2%	0.09964	0.10413	1,148	32.0%	367
4	DE095	2438	. . Germany - Rubber and plastic products	S	0.02456	7.6%	0.02642	0.00275	30	36.0%	11
5	ES095	3077	. Spain - Rubber and plastic products	D	0.02837	10.7%	0.03141	0.03282	362	31.0%	112
6=>4	DE095	2438	. . Germany - Rubber and plastic products	S	0.01671	7.6%	0.01797	0.00059	7	36.0%	2
7	CZ123	2253	. Czech Republic - Motor vehicles, trailers and sem	S	0.01697	13.8%	0.01931	0.02018	222	19.9%	44
8=>4	DE095	2438	. . Germany - Rubber and plastic products	C	0.02453	7.6%	0.02639	0.00053	6	36.0%	2
9=>4	DE095	2438	. Germany - Rubber and plastic products	S	0.00494	7.6%	0.00531	0.00555	61	36.0%	22
10=>3	DE123	2466	Germany - Motor vehicles, trailers and semi-trailers	E	1.00000	20.2%	1.20234	1.20234	4,268	32.0%	1,365
11=>4	DE095	2438	. Germany - Rubber and plastic products	E	0.02456	7.6%	0.02642	0.03176	113	36.0%	41
12	FR123	3531	France - Motor vehicles, trailers and semi-trailers	E	1.00000	5.3%	1.05348	1.05348	3,216	24.2%	778
13=>4	DE095	2438	. Germany - Rubber and plastic products	C	0.00570	7.6%	0.00614	0.00647	20	36.0%	7
14=>7	CZ123	2253	Czech Republic - Motor vehicles, trailers and semi-t	E	1.00000	13.8%	1.13774	1.13774	827	19.9%	164
15=>4	DE095	2438	. Germany - Rubber and plastic products	C	0.02453	7.6%	0.02639	0.03002	22	36.0%	8
16	SK123	8856	Slovakia - Motor vehicles, trailers and semi-trailers	E	1.00000	19.4%	1.19406	1.19406	526	13.0%	68
17=>4	DE095	2438	. Germany - Rubber and plastic products	C	0.01108	7.6%	0.01192	0.01423	6	36.0%	2

Source: own calculations based on FIGARO-E3 2015 (EC, Joint Research Center, 2025)

How to read the flowchart?

With a threshold of 5 M € for material consumption, the visual network decomposition algorithm of the IOPathFinder application detected 8 linking chains for the selected product and material. These 8 chains cover already 65.4% of the total value added contribution derived from the Leontief inverse.

The first set of four chains linked to the grey box reflects the domestic production for final demand of "Motor vehicles" in Spain and backward linkages of intermediate consumption with a connection to "Rubber and plastic products" from Germany.

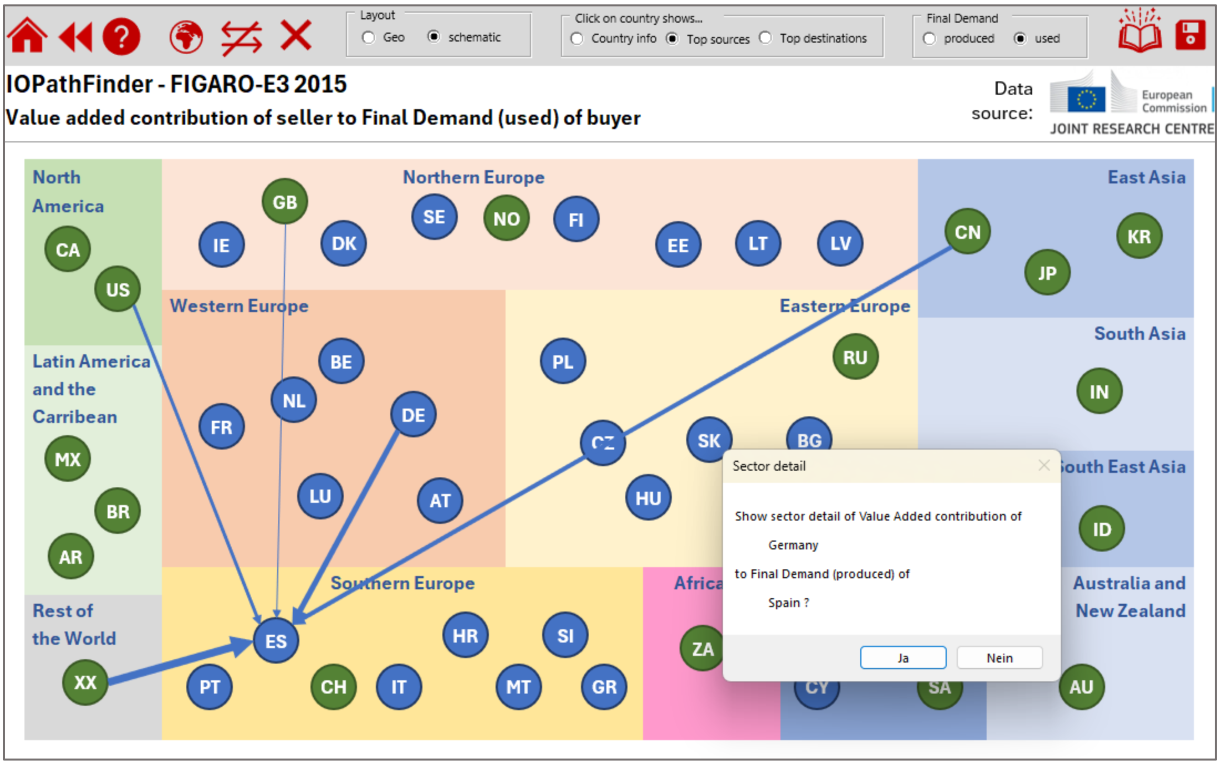
The third chain, besides Spain and Germany, even involves Czechia as a third country:

- Rubber and plastic products (DE095) are imported from Germany to Czechia
- ... to be used as intermediates for the production of motor vehicles (including parts and assemblies) in Czechia (CZ123)
- ... which then are imported by Spain to be used in the production of Motor vehicles (ES123) for domestic final demand

The highest single value added contribution (41 M €) comes from the fifth chain. It is caused by the use of German rubber and plastic products (DE095) for the production of motor vehicles in Germany (DE123), that are be exported to Spain to be directly used for final demand with no further processing (indicated by the blue box).

As mentioned earlier (see 2.4), instead of the value added contribution matrix aggregated by country, a world map can be used as an alternative starting point for the analysis.

Figure 22 *- Schematic world map with top 5 value added source countries of Spain*



Layout ○ Geo ● schematic

Click on country shows... ○ Country info ● Top sources ○ Top destinations

Final Demand ○ produced ● used

IOPathFinder - FIGARO-E3 2015

Value added contribution of seller to Final Demand (used) of buyer

Data source: European Commission JOINT RESEARCH CENTRE

Sector detail

Show sector detail of Value Added contribution of

Germany

to Final Demand (produced) of

Spain ?

Ja Nein

Besides a geographic map, there is a schematic map for easier navigation, as the EU-27 countries are difficult to distinguish on a physical map with Mercator projection.

Figure 22 shows this schematic map with arrows depicting the top 5 source countries in terms of value added contribution to final demand in Spain.

Clicking on the arrow that links Germany to Spain will produce the report by product presented earlier in **Table 12**.

It must be emphasized, that the **arrows in this map do not represent the value added content of bilateral trade between the two countries, but total value added contribution, part of which might be routed via third countries**. The example presented in **Figure 21** and **Table 13** shows several of these paths involving third countries. Besides the above-mentioned chain via Czechia, the last three paths with imports for final demand from France, again Czechia, and Slovakia are of the same category.

3.1.2 The link to "Trade in Value Added" (TiVA)

"Trade in Value Added" or "TiVA" is mentioned both in the title of this paper and of the current segment. How is TiVA linked to the value added contribution matrix? Can the IOPathFinder application be used for the visual network decomposition of TiVA?

TiVA indicators were developed to measure international trade not just in gross values of trade flows, but also in terms of direct and indirect value added embodied in these flows. These indicators provide a more appropriate measure of the real contribution of the individual countries to global value chains.

In order to determine the value added content of exports, in equation (19) the expression for the diagonal matrix of total final demand $\hat{Y}_c$ can be replaced by an analogous expression for total exports:

$$TiVA_u = \hat{v}(I - A)^{-1}\hat{E}_c \tag{20}$$

The elements e_{rsi} of matrix E_c (subscript "c" for "by using country") include exports for both intermediate consumption (first term) and final consumption (second term) in the importing country:

$$\forall\, r \neq s, \forall\, i\colon\; e_{rsi}^{(c)} = e_{rsi}^{(int)} + e_{rsi}^{(fin)} = \sum_{j}^{n} a_{rsij} x_{sj} + y_{rsi} \tag{21}$$

where:

r, s	indices of supplying and using country
i, j	indices of input and output product with n as the number of products
a_{rsij}	direct material requirement coefficients

x_{sj} output by product and using country

y_{rsi} final demand by product, supplying and using country

By substitution in (20) we obtain

$$TiVA_u = \hat{v}(I - A)^{-1}\hat{E}_c^{(int)} + \hat{v}(I - A)^{-1}\hat{E}_c^{(fin)} \tag{22}$$

Subscripts "u" and "c" and the "hat" accent above the export matrices indicate that these are block matrices with diagonal blocks, just like the matrix $\hat{Y}_c$ introduced before in **Figure 19**. A major difference is that the block matrices on the diagonal are zero matrices, as they stand for domestic intermediate and final consumption, which is excluded here.

It is worth noting that this does not mean that the blocks on the diagonal of the TiVA matrix are also zero matrices. They are populated by exports used abroad as intermediates for the production of goods and services, possibly over several stages, which then are imported again to the current country.

The crux of the TiVA equation (22) is that it involves double-counting. Value added embodied in exports for intermediate consumption eventually will end up as value added embodied in final demand. Hence, for the MRIO as a whole, the first term is part of the second term. But even when looking at individual countries or bilateral trade, there are always cases of value added embodied in intermediate exports, which return from abroad on some stage to be used for domestic final demand, where they, however, have already been accounted for.

The issue is inherent already in the TiVA definition, as it mixes the final demand perspective with that of gross values like output and material consumption. Usually the Leontief inverse would not be applied to such gross values.

The TiVA double-counting issue has since been intensively discussed by several authors - see, e.g., Koopman et al. (2014) and Arto et al. (2019). The identification and elimination of different double-counting components involves complex decomposition procedures.

Zhi Wang, one of the coauthors of Koopman et al. (2014), has continued this work and developed a decomposition approach, which will be discussed later in segments 3.3 and 3.4. Like the visual network decomposition technique, this concept focusses on the Value Added contribution matrix, which does not imply double-counting.

These can also be applied for the decomposition of TiVA, in particular to identify and to eliminate double-counting. This is one point on the agenda for future improvements (see 4 - Conclusions and prospects).

3.2 Footprint of CO_2 and other air emissions

For many years, environmental extensions and applications have been a strong and ever growing part of Input-Output analysis. Global environmental models based on MRIOs in particular have proven to provide transparent insights into worldwide interrelationships of climate impacts. For good reason, many research reports related to Input-Output analysis are published in renowned ecological journals. Scientists with a profound Input-Output background are recognized as partners by key stakeholders.

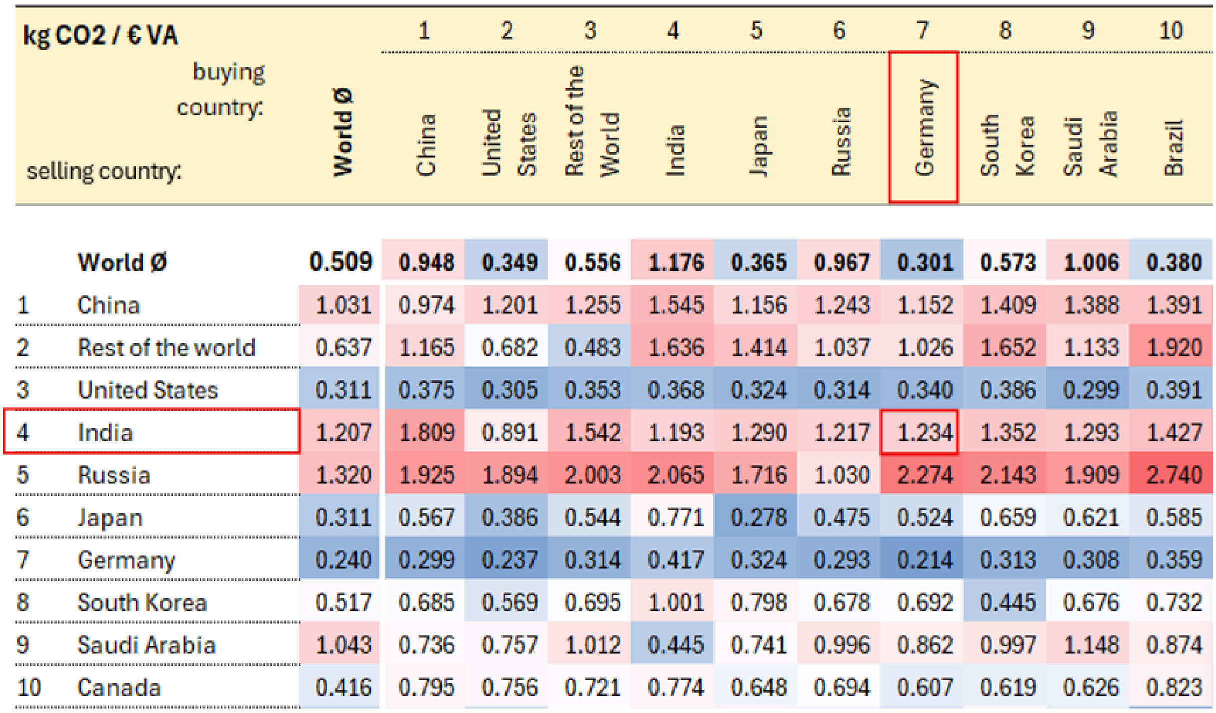
In section 2.3.1 it was already mentioned, that the visual network decomposition approach can be applied to track the footprint of CO_2 and other air emissions along supply chains. In the following this will be demonstrated, using the air emission data provided together with the FIGARO-E3 tables.

For a proper allocation of emission footprint to countries of final use, in formula (18) for the total emission footprint contribution matrix final demand has to be switched to the format with diagonal blocks, just like in equation (19):

$$C_u = \hat{c}(I - A)^{-1}\hat{Y}_c = \hat{c}\sum_{k=0}^{\infty} A^k\,\hat{Y}_c \tag{23}$$

FIGARO-E3 offers air emission data for four types of green house gases (GHG), both in original values and CO_2 equivalents. The following explanations will focus on CO_2 emissions.

Table 14 *- CO_2 intensity of VA contribution 2015 by country, sorted by emission in kg*



kg CO2 / € VA	buying country: selling country:	**World Ø**	1 China	2 United States	3 Rest of the World	4 India	5 Japan	6 Russia	7 Germany	8 South Korea	9 Saudi Arabia	10 Brazil
	World Ø	**0.509**	**0.948**	**0.349**	**0.556**	**1.176**	**0.365**	**0.967**	**0.301**	**0.573**	**1.006**	**0.380**
1	China	1.031	0.974	1.201	1.255	1.545	1.156	1.243	1.152	1.409	1.388	1.391
2	Rest of the world	0.637	1.165	0.682	0.483	1.636	1.414	1.037	1.026	1.652	1.133	1.920
3	United States	0.311	0.375	0.305	0.353	0.368	0.324	0.314	0.340	0.386	0.299	0.391
4	India	1.207	1.809	0.891	1.542	1.193	1.290	1.217	1.234	1.352	1.293	1.427
5	Russia	1.320	1.925	1.894	2.003	2.065	1.716	1.030	2.274	2.143	1.909	2.740
6	Japan	0.311	0.567	0.386	0.544	0.771	0.278	0.475	0.524	0.659	0.621	0.585
7	Germany	0.240	0.299	0.237	0.314	0.417	0.324	0.293	0.214	0.313	0.308	0.359
8	South Korea	0.517	0.685	0.569	0.695	1.001	0.798	0.678	0.692	0.445	0.676	0.732
9	Saudi Arabia	1.043	0.736	0.757	1.012	0.445	0.741	0.996	0.862	0.997	1.148	0.874
10	Canada	0.416	0.795	0.756	0.721	0.774	0.648	0.694	0.607	0.619	0.626	0.823

Source: own calculations based on FIGARO-E3 2015 (EC, Joint Research Center, 2025)

As a starting point, the application displays a heatmap by country of **total CO_2 footprint per value added contribution in kg per €**. This indicator is aligned with "carbon intensity of GDP" and combines the results of equations (23) and (19).

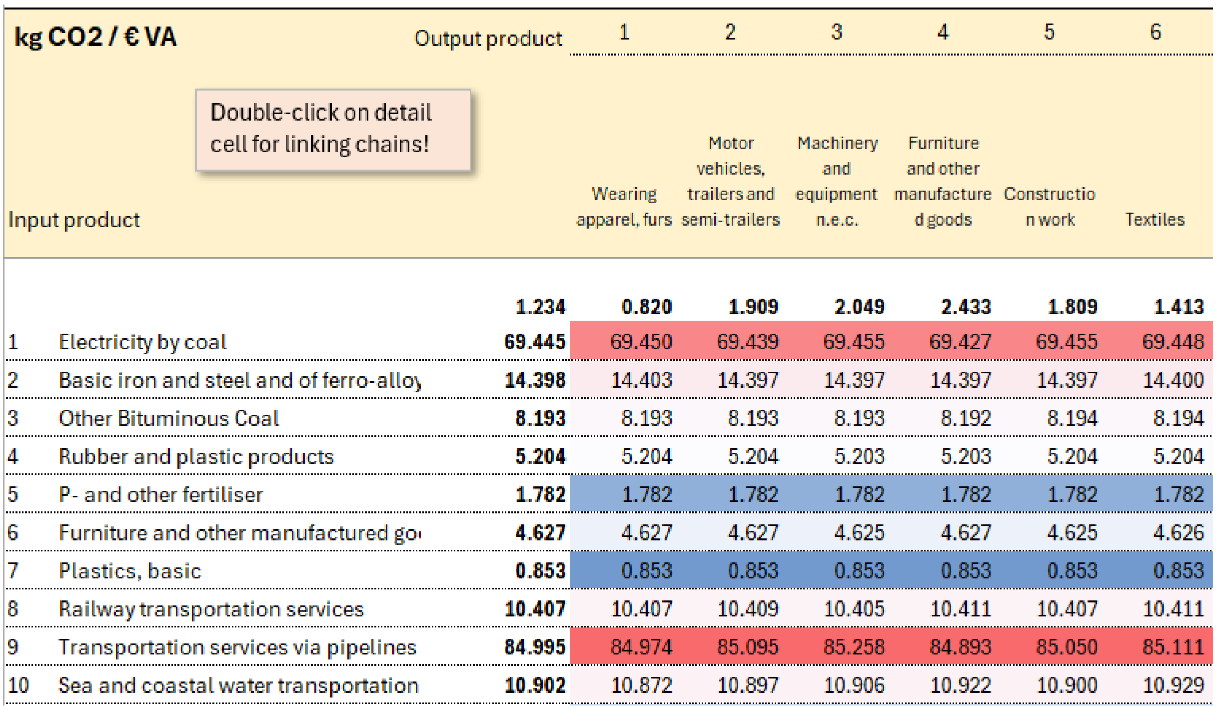
In the next step, the total CO_2 contribution from India to Germany will be examined. A double-click on the crossing of the respective row for India and column for Germany opens the report by product - see **Table 15**.

On the top left, the average value from the selected cell **Table 14** is displayed. The values in the rows should be equal across columns, because both CO_2 emission and value added in relation to output depend only on the material. Small deviations are caused by rounding errors due to the amount of detail involved in the calculation.

In terms of absolute emission, used as a sorting criteria, the indirect CO_2 contribution of "Electricity by coal" in India to the final use of "Wearing apparel, furs" is first in ranking. Coal-based power generation shows a high CO_2 intensity of almost 70 kg/€.

As can be seen in **Table 16**, in terms of the absolute footprint, 5.6% of total CO_2 contributions from India to Germany can be attributed to this relation. When including "Textiles" (to the right of the table), the share is even 8.9%.

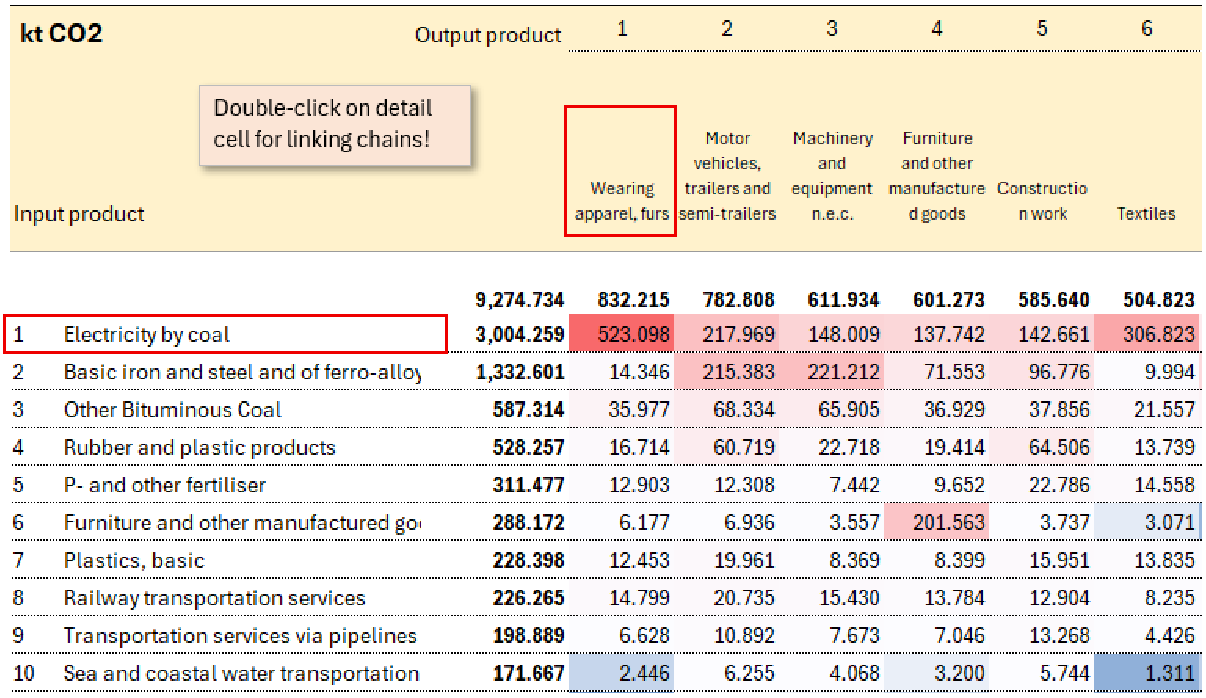
Table 15 *- CO_2/VA by product, India to Germany, sorted by emission in kg*



Double-click on detail cell for linking chains!

kg CO2 / € VA		Output product	1	2	3	4	5	6
Input product			Wearing apparel, furs	Motor vehicles, trailers and semi-trailers	Machinery and equipment n.e.c.	Furniture and other manufactured goods	Construction work	Textiles
		1.234	**0.820**	**1.909**	**2.049**	**2.433**	**1.809**	**1.413**
1	Electricity by coal	**69.445**	69.450	69.439	69.455	69.427	69.455	69.448
2	Basic iron and steel and of ferro-alloy	**14.398**	14.403	14.397	14.397	14.397	14.397	14.400
3	Other Bituminous Coal	**8.193**	8.193	8.193	8.193	8.192	8.194	8.194
4	Rubber and plastic products	**5.204**	5.204	5.204	5.203	5.203	5.204	5.204
5	P- and other fertiliser	**1.782**	1.782	1.782	1.782	1.782	1.782	1.782
6	Furniture and other manufactured go	**4.627**	4.627	4.627	4.625	4.627	4.625	4.626
7	Plastics, basic	**0.853**	0.853	0.853	0.853	0.853	0.853	0.853
8	Railway transportation services	**10.407**	10.407	10.409	10.405	10.411	10.407	10.411
9	Transportation services via pipelines	**84.995**	84.974	85.095	85.258	84.893	85.050	85.111
10	Sea and coastal water transportation	**10.902**	10.872	10.897	10.906	10.922	10.900	10.929

Table 16 *- CO$_2$ emission by product, India to Germany, sorted by emission in kg*



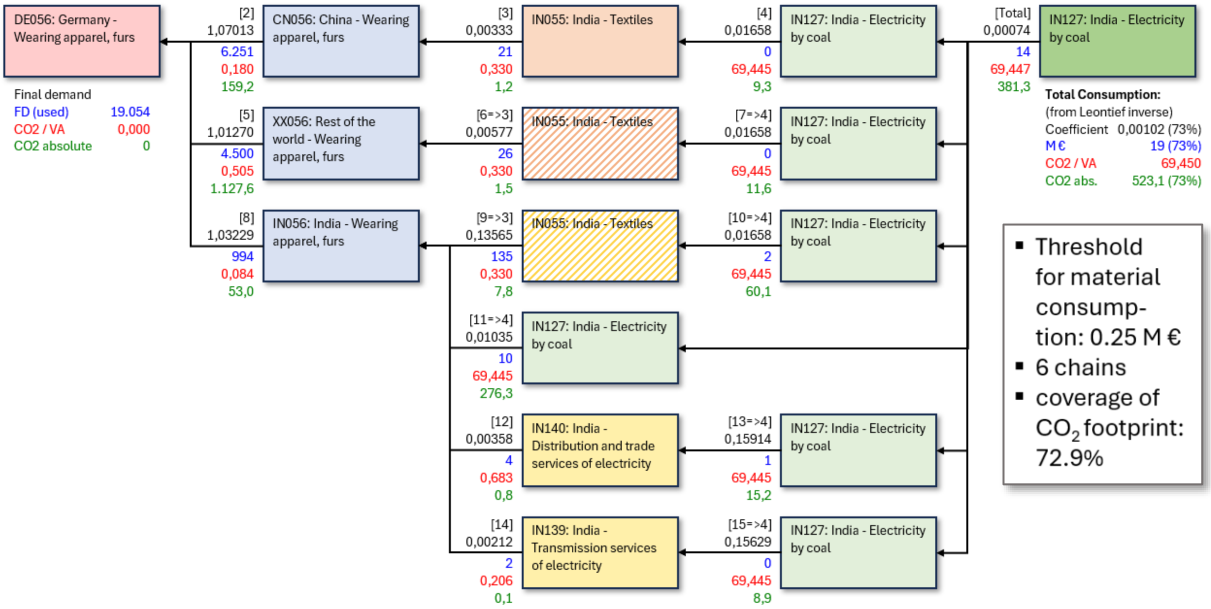
Double-click on detail cell for linking chains!

kt CO2		Output product	1	2	3	4	5	6
Input product			Wearing apparel, furs	Motor vehicles, trailers and semi-trailers	Machinery and equipment n.e.c.	Furniture and other manufactured goods	Construction work	Textiles
		9,274.734	**832.215**	**782.808**	**611.934**	**601.273**	**585.640**	**504.823**
1	Electricity by coal	**3,004.259**	523.098	217.969	148.009	137.742	142.661	306.823
2	Basic iron and steel and of ferro-alloy	**1,332.601**	14.346	215.383	221.212	71.553	96.776	9.994
3	Other Bituminous Coal	**587.314**	35.977	68.334	65.905	36.929	37.856	21.557
4	Rubber and plastic products	**528.257**	16.714	60.719	22.718	19.414	64.506	13.739
5	P- and other fertiliser	**311.477**	12.903	12.308	7.442	9.652	22.786	14.558
6	Furniture and other manufactured go	**288.172**	6.177	6.936	3.557	201.563	3.737	3.071
7	Plastics, basic	**228.398**	12.453	19.961	8.369	8.399	15.951	13.835
8	Railway transportation services	**226.265**	14.799	20.735	15.430	13.784	12.904	8.235
9	Transportation services via pipelines	**198.889**	6.628	10.892	7.673	7.046	13.268	4.426
10	Sea and coastal water transportation	**171.667**	2.446	6.255	4.068	3.200	5.744	1.311

Source: own calculations based on FIGARO-E3 2015 (EC, Joint Research Center, 2025)

In the next step, a double-click on the value for "Electricity by coal" for "Wearing apparel, furs" will start the "Find Linking Chains" analysis.

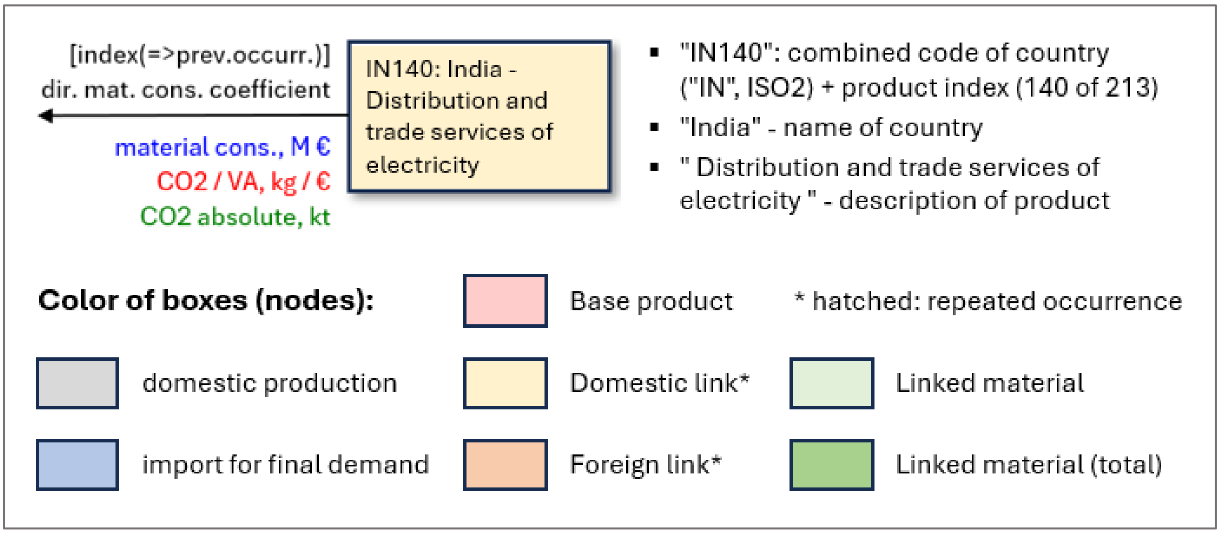
Figure 23 *- CO$_2$ footprint of India electricity by coal for German Wearing apparel*



- Threshold for material consumption: 0.25 M €
- 6 chains
- coverage of CO$_2$ footprint: 72.9%

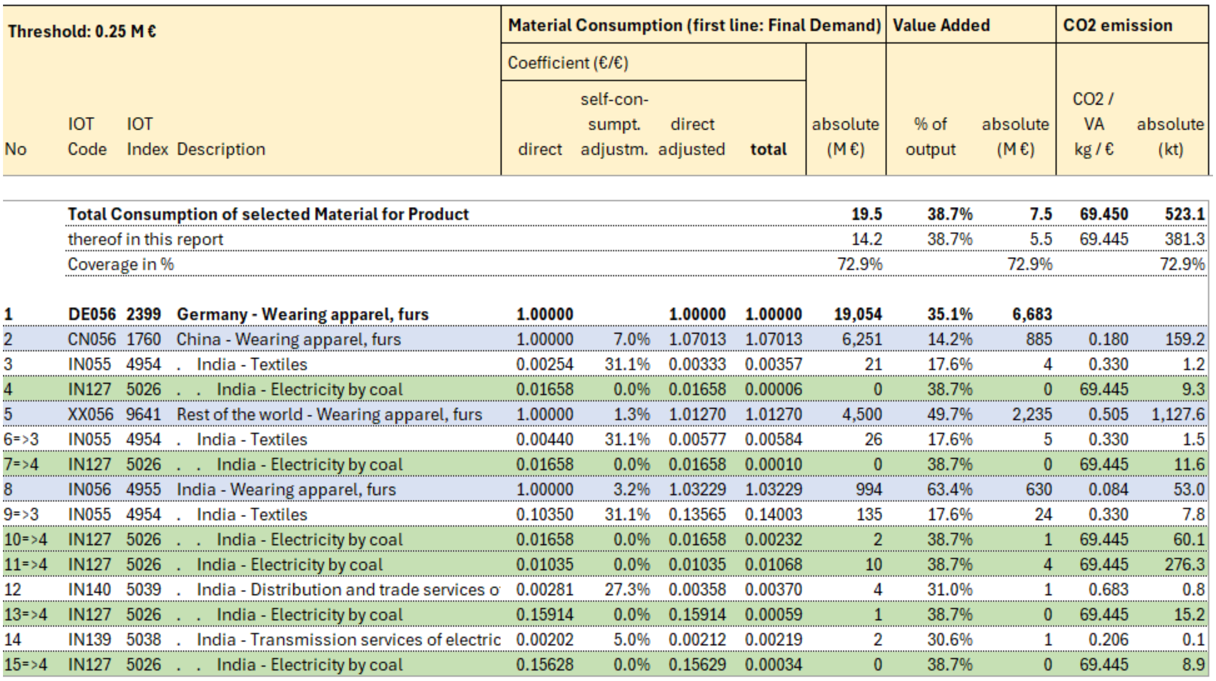
Source: own calculations based on FIGARO-E3 2015 (EC, Joint Research Center, 2025)

***Figure 24** - CO$_2$ footprint: Flowchart legend*



Instead of value added contribution, CO$_2$ contents are displayed. In the hierarchical table, special columns are added that show the underlying values and allow to trace the calculation:

***Table 17** - CO$_2$ footprint of India electricity by coal for German Wearing apparel*



Threshold: 0.25 M €				Material Consumption (first line: Final Demand)					Value Added		CO2 emission	
				Coefficient (€/€)								
No	IOT Code	IOT Index	Description	direct	self-con-sumpt. adjustm.	direct adjusted	**total**	absolute (M €)	% of output	absolute (M €)	CO2 / VA kg / €	absolute (kt)
	Total Consumption of selected Material for Product							**19.5**	**38.7%**	**7.5**	**69.450**	**523.1**
	thereof in this report							14.2	38.7%	5.5	69.445	381.3
	Coverage in %							72.9%		72.9%		72.9%
1	**DE056**	**2399**	**Germany - Wearing apparel, furs**	**1.00000**		**1.00000**	**1.00000**	**19,054**	**35.1%**	**6,683**		
2	CN056	1760	China - Wearing apparel, furs	1.00000	7.0%	1.07013	1.07013	6,251	14.2%	885	0.180	159.2
3	IN055	4954	. India - Textiles	0.00254	31.1%	0.00333	0.00357	21	17.6%	4	0.330	1.2
4	IN127	5026	. . India - Electricity by coal	0.01658	0.0%	0.01658	0.00006	0	38.7%	0	69.445	9.3
5	XX056	9641	Rest of the world - Wearing apparel, furs	1.00000	1.3%	1.01270	1.01270	4,500	49.7%	2,235	0.505	1,127.6
6=>3	IN055	4954	. India - Textiles	0.00440	31.1%	0.00577	0.00584	26	17.6%	5	0.330	1.5
7=>4	IN127	5026	. . India - Electricity by coal	0.01658	0.0%	0.01658	0.00010	0	38.7%	0	69.445	11.6
8	IN056	4955	India - Wearing apparel, furs	1.00000	3.2%	1.03229	1.03229	994	63.4%	630	0.084	53.0
9=>3	IN055	4954	. India - Textiles	0.10350	31.1%	0.13565	0.14003	135	17.6%	24	0.330	7.8
10=>4	IN127	5026	. . India - Electricity by coal	0.01658	0.0%	0.01658	0.00232	2	38.7%	1	69.445	60.1
11=>4	IN127	5026	. India - Electricity by coal	0.01035	0.0%	0.01035	0.01068	10	38.7%	4	69.445	276.3
12	IN140	5039	. India - Distribution and trade services o	0.00281	27.3%	0.00358	0.00370	4	31.0%	1	0.683	0.8
13=>4	IN127	5026	. . India - Electricity by coal	0.15914	0.0%	0.15914	0.00059	1	38.7%	0	69.445	15.2
14	IN139	5038	. India - Transmission services of electric	0.00202	5.0%	0.00212	0.00219	2	30.6%	1	0.206	0.1
15=>4	IN127	5026	. . India - Electricity by coal	0.15628	0.0%	0.15629	0.00034	0	38.7%	0	69.445	8.9

Source: own calculations based on FIGARO-E3 2015 (EC, Joint Research Center, 2025)

This approach can be applied to other "stressors" directly linked to output and free of double-counting, such as additional emissions, labor requirements by gender and skill level or primary supply of energy.

3.3 Decomposition by type of supply chain activity

For a deeper understanding of the nature and complexity of global value chains (GVC), Wang et al. (2017) have proposed a concept of decomposition of the value added contribution matrix (VACM) in a MRIO by the "type of supply chain activity". In a later article (Wang et al., 2107), the approach was applied to the OECD AMNE database with a distinction of foreign-owned and domestic-owned firms.

In the following, it will be shown, how visual network decomposition can provide even more transparency for such a concept by identifying single GVCs and assigning them to the elements of the decomposition.

In this segment, the analysis will start with the case without ownership distinction using the FIGARO-E3 database.

In their articles (Wang et al., 2017, 2021) the authors split the value added contribution matrix into four types of supply chain activity. We retain the notation of the original source, which slightly differs from variable names used in the current paper.

$$\boxed{\hat{V}B\hat{Y} = \hat{V}L\hat{Y}^L + \hat{V}L\hat{Y}^E + \hat{V}LA^E L\hat{Y}^L + \hat{V}LA^E\left(B\hat{Y} - L\hat{Y}^L\right)} \tag{24}$$

Source: Wang et al. (2021), p. 13, equation (1) [3]

where:

A^L	diagonal submatrix of A (intermediate consumption in the same country)
A^E	off-diagonal submatrix of A (intermediate consumption in other countries
$\hat{Y}^L$	diagonal submatrix of $\hat{Y}$ [4] - domestic final demand
$\hat{Y}^E$	off-diagonal submatrix of $\hat{Y}$ - export for final demand in other countries
L	local Leontief inverse, $L = (I - A^L)^{-1}$
B	global Leontief inverse, $B = (I - A^L - A^E)^{-1}$

The four components of equation (24) are explained in **Table 18** (numbers in first column and text in quotation marks from Wang et al., 2017).

---

[3] In Wang et al. (2017), instead of superscripts "L" for "local" and "E" for "export", superscripts "D" and "F" were used. In the 2021 article, they were replaced to allow the use of "D" and "F" for the additional distinction of domestic-owned and foreign-owned firms.

[4] $\hat{Y}$ is equivalent to matrix $\hat{Y}_c$ according to the notation shown in **Figure 19**

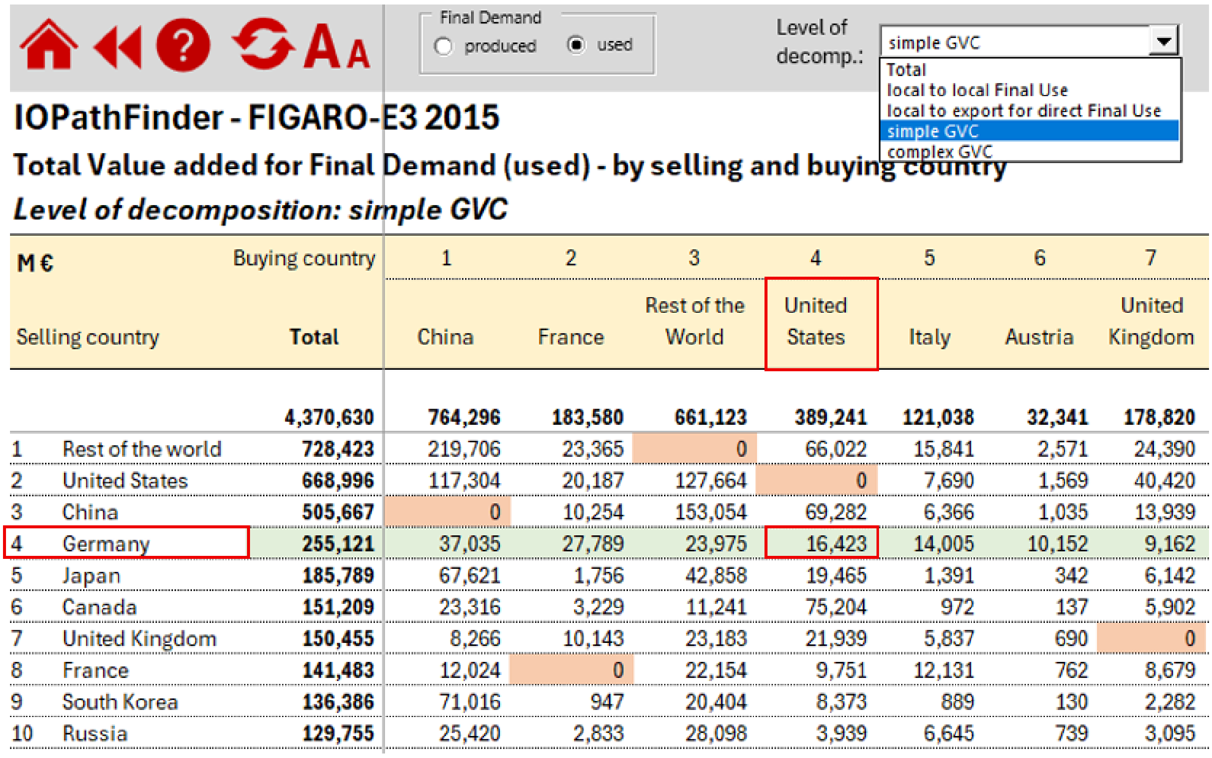
Table 18 *- Types of supply chain activity*

No.	ID	Formula	Contents	border cross.
1	D	$\hat{V}L\hat{Y}^{L}$	"Value added that is domestically produced and consumed" (pure domestic VC)	0
2	E	$\hat{V}L\hat{Y}^{E}$	"Value-added that is embodied in final product exports" ("classic trade")	1
3a	S	$\hat{V}LA^{E}L\hat{Y}^{L}$	VA embodied in "simple cross country production sharing activities" ("simple GVC")	1
3b	C	$\hat{V}LA^{E}\left(B\hat{Y}-L\hat{Y}^{L}\right)$	VA embodied in "complex cross country production sharing activities" ("complex GVC")	>= 2

The letters in the "ID" column will be used to identify the different supply chain types in flowcharts and hierarchical tables. The last column specifies the number of border crossings (see Wang et al., 2017, Figure 1b on page 9).

Like in the previously described MRIO use cases, the starting point for analysis is again the value added contribution matrix aggregated by country. Now in this report the type of supply chain activity can be selected.

Table 19 *- Value added contribution by country for simple GVC*



Final Demand ○ produced ● used

Level of decomp.: simple GVC

IOPathFinder - FIGARO-E3 2015

Total Value added for Final Demand (used) - by selling and buying country

Level of decomposition: simple GVC

M €		Buying country	1	2	3	4	5	6	7
Selling country		**Total**	China	France	Rest of the World	United States	Italy	Austria	United Kingdom
		4,370,630	**764,296**	**183,580**	**661,123**	**389,241**	**121,038**	**32,341**	**178,820**
1	Rest of the world	**728,423**	219,706	23,365	0	66,022	15,841	2,571	24,390
2	United States	**668,996**	117,304	20,187	127,664	0	7,690	1,569	40,420
3	China	**505,667**	0	10,254	153,054	69,282	6,366	1,035	13,939
4	Germany	**255,121**	37,035	27,789	23,975	16,423	14,005	10,152	9,162
5	Japan	**185,789**	67,621	1,756	42,858	19,465	1,391	342	6,142
6	Canada	**151,209**	23,316	3,229	11,241	75,204	972	137	5,902
7	United Kingdom	**150,455**	8,266	10,143	23,183	21,939	5,837	690	0
8	France	**141,483**	12,024	0	22,154	9,751	12,131	762	8,679
9	South Korea	**136,386**	71,016	947	20,404	8,373	889	130	2,282
10	Russia	**129,755**	25,420	2,833	28,098	3,939	6,645	739	3,095

Source: own calculations, Wang et al. (2017), FIGARO-E3 (EC, Joint Research Centre, 2025)

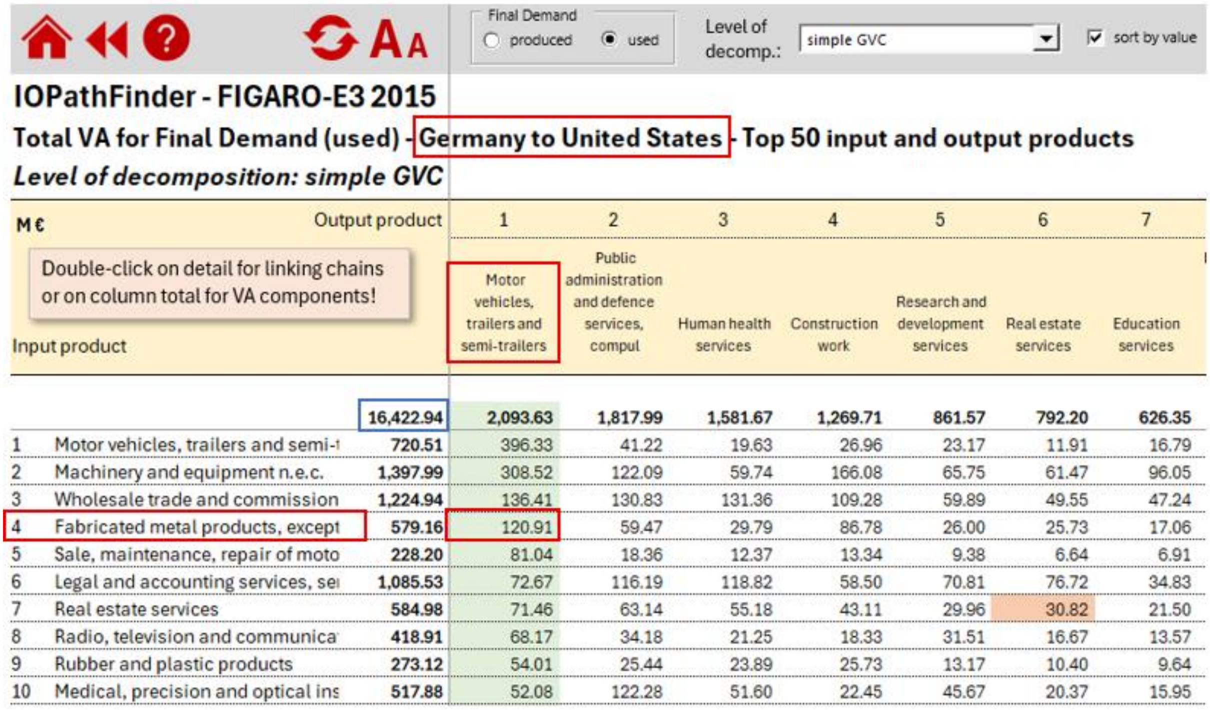
Diagonal elements are marked in brown. In the case of simple GVCs, they are zero, as borders are crossed only once, so there is no value added contribution of countries to themselves. For the same reason, diagonal elements are zero for case 2, final product exports.

For pure domestic use (case 1), only diagonal elements of the VACM by country can be greater than zero, because there are no border crossings.

In case 4 (complex GVC), diagonal elements can have non-zero values again, since with two and more border crossings, GVCs can return to the country they started from.

For a deeper analysis of the VA contributions of Germany to other countries via simple GVCs, the columns have been sorted by the VA contribution from Germany. A double-click on the value added contribution of Germany for the US will invoke the report with the product detail for this relation, now limited to simple GVCs.

Table 20 *- VA contribution by product for simple GVC, Germany to US*



IOPathFinder - FIGARO-E3 2015

Total VA for Final Demand (used) - Germany to United States - Top 50 input and output products

Level of decomposition: simple GVC

Double-click on detail for linking chains or on column total for VA components!

M€	Input product \ Output product		1 Motor vehicles, trailers and semi-trailers	2 Public administration and defence services, compul	3 Human health services	4 Construction work	5 Research and development services	6 Real estate services	7 Education services
		16,422.94	**2,093.63**	**1,817.99**	**1,581.67**	**1,269.71**	**861.57**	**792.20**	**626.35**
1	Motor vehicles, trailers and semi-t	**720.51**	396.33	41.22	19.63	26.96	23.17	11.91	16.79
2	Machinery and equipment n.e.c.	**1,397.99**	308.52	122.09	59.74	166.08	65.75	61.47	96.05
3	Wholesale trade and commission	**1,224.94**	136.41	130.83	131.36	109.28	59.89	49.55	47.24
4	Fabricated metal products, except	**579.16**	120.91	59.47	29.79	86.78	26.00	25.73	17.06
5	Sale, maintenance, repair of moto	**228.20**	81.04	18.36	12.37	13.34	9.38	6.64	6.91
6	Legal and accounting services, se	**1,085.53**	72.67	116.19	118.82	58.50	70.81	76.72	34.83
7	Real estate services	**584.98**	71.46	63.14	55.18	43.11	29.96	30.82	21.50
8	Radio, television and communica	**418.91**	68.17	34.18	21.25	18.33	31.51	16.67	13.57
9	Rubber and plastic products	**273.12**	54.01	25.44	23.89	25.73	13.17	10.40	9.64
10	Medical, precision and optical ins	**517.88**	52.08	122.28	51.60	22.45	45.67	20.37	15.95

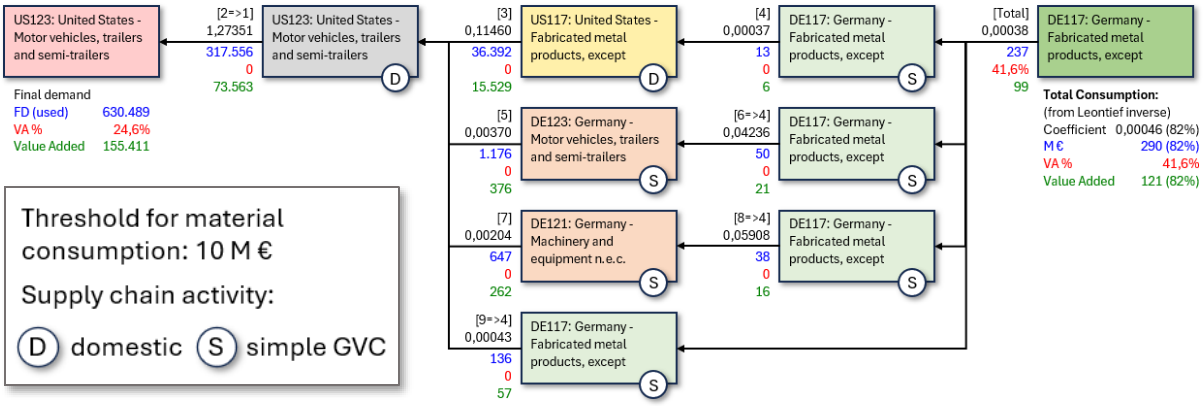
Source: own calculations, Wang et al. (2017), FIGARO-E3 (EC, Joint Research Centre, 2025)

The rows have been sorted by their relevance for the final use of "Motor vehicles etc." in the US.

From here, the "Find Linking Chains" mode can be started to reveal the product chain detail of the value added contribution of "Fabricated metal products" from Germany for the production of cars in the US.

For an explanation of the flowchart elements, please refer to the legend in **Figure 20**.

__Figure 25__ - Simple GVC - DE Fabricated metal products for US Motor vehicles



Source: own calculations, Wang et al. (2017), FIGARO-E3 (EC, Joint Research Centre, 2025)

Four selected chains cover already 82% of the total value added contribution according to the formula in line "3a" in **Table 18**.

The supply chain types are displayed as markers in the circles on the bottom right of each of the boxes. In order to obtain these markers, the recursive traversal algorithm of the IOPathFinder application was extended by counting the border crossings with a distinction between final demand from production or import.

Table 21 lists the rules how supply chain activity types are derived from the initial source of final demand and the number of border crossings.

__Table 21__ - Derivation of supply chain activity type

Starting point: final demand from...	box color	No. of border crossings	Supply chain activity type
domestic production (D)	grey	0	D - domestic for final demand
		1	S - simple GVC
		>= 2	C - complex GVC
import = export by other country (E)	blue	1	E - export for final demand
		>= 2	C - complex GVC

Special attention should be paid to the way, how the type of supply chain activity changes along the supply chain. In the first value chain on top of the flowchart, grey box 2 is still "domestic (D)", as is the consumption of US fabricated metal products for US cars. The next box reflects intermediate use of German fabricated metal products for US fabricated metal products. As this involves a border crossing, the type of supply chain activity switches to "simple GVC (S)".

In the hierarchical table, the type of supply chain activity is displayed in the column to the right of the product descriptions.

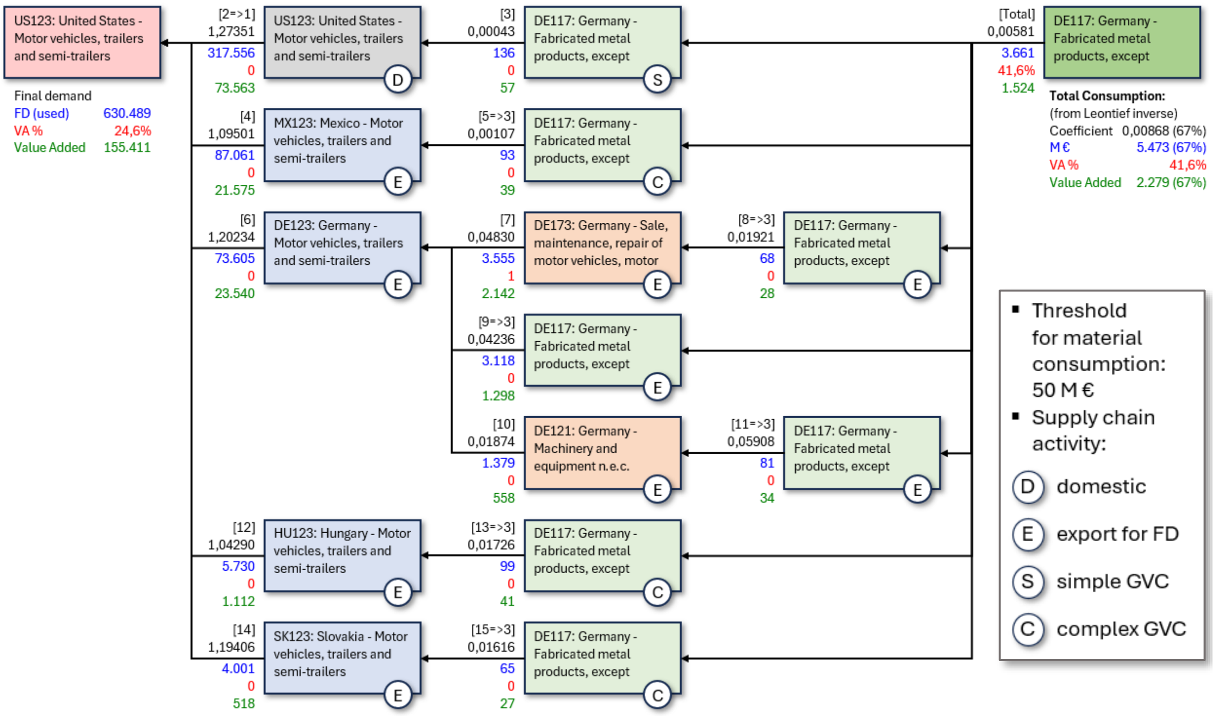
Table 22 *- Simple GVC - DE Fabricated metal products for US Motor vehicles*

simple GVC, Threshold: 10 M €					**Material Consumption (first line: Final Demand)**					**Value Added**	
					Coefficient (€/€)						
No	IOT Code	IOT Index	Description		direct	self-con-sumpt. adjustm.	direct adjusted	**total**	absolute (M €)	% of output	absolute (M €)
	Total Consumption of selected Material for Product								**290.4**	**41.6%**	**120.9**
	thereof in this report								237.4	41.6%	98.8
	Coverage in %								81.7%		81.7%
1	**US123**	**9282**	**United States - Motor vehicles, trailer**		**1.00000**		**1.00000**	**1.00000**	**630,489**	**24.6%**	**155,411**
2=>1	US123	9282	United States - Motor vehicles, trailers	D	1.00000	27.4%	1.27351	1.27351	317,556	23.2%	73,563
3	US117	9276	. United States - Fabricated metal pro	D	0.10353	10.7%	0.11460	0.14594	36,392	42.7%	15,529
4	DE117	2460	. . Germany - Fabricated metal prod	S	0.00031	18.3%	0.00037	0.00005	13	41.6%	6
5	DE123	2466	. Germany - Motor vehicles, trailers ai	S	0.00308	20.2%	0.00370	0.00472	1,176	32.0%	376
6=>4	DE117	2460	. . Germany - Fabricated metal prod	S	0.03581	18.3%	0.04236	0.00020	50	41.6%	21
7	DE121	2464	. Germany - Machinery and equipmer	S	0.00179	14.0%	0.00204	0.00259	647	40.4%	262
8=>4	DE117	2460	. . Germany - Fabricated metal prod	S	0.04994	18.3%	0.05908	0.00015	38	41.6%	16
9=>4	DE117	2460	. Germany - Fabricated metal product	S	0.00036	18.3%	0.00043	0.00055	136	41.6%	57

Source: own calculations, Wang et al. (2017), FIGARO-E3 (EC, Joint Research Centre, 2025)

To get the full picture across all types of supply chain activity, in **Figure 26** the selection is adjusted accordingly. The threshold for material consumption is increased from 10 M € to 50 M € to keep the flowchart at a reasonable size.

Figure 26 *- DE Fabricated metal products for US Motor vehicles - all types of activity*



Source: own calculations, Wang et al. (2017), FIGARO-E3 (EC, Joint Research Centre, 2025)

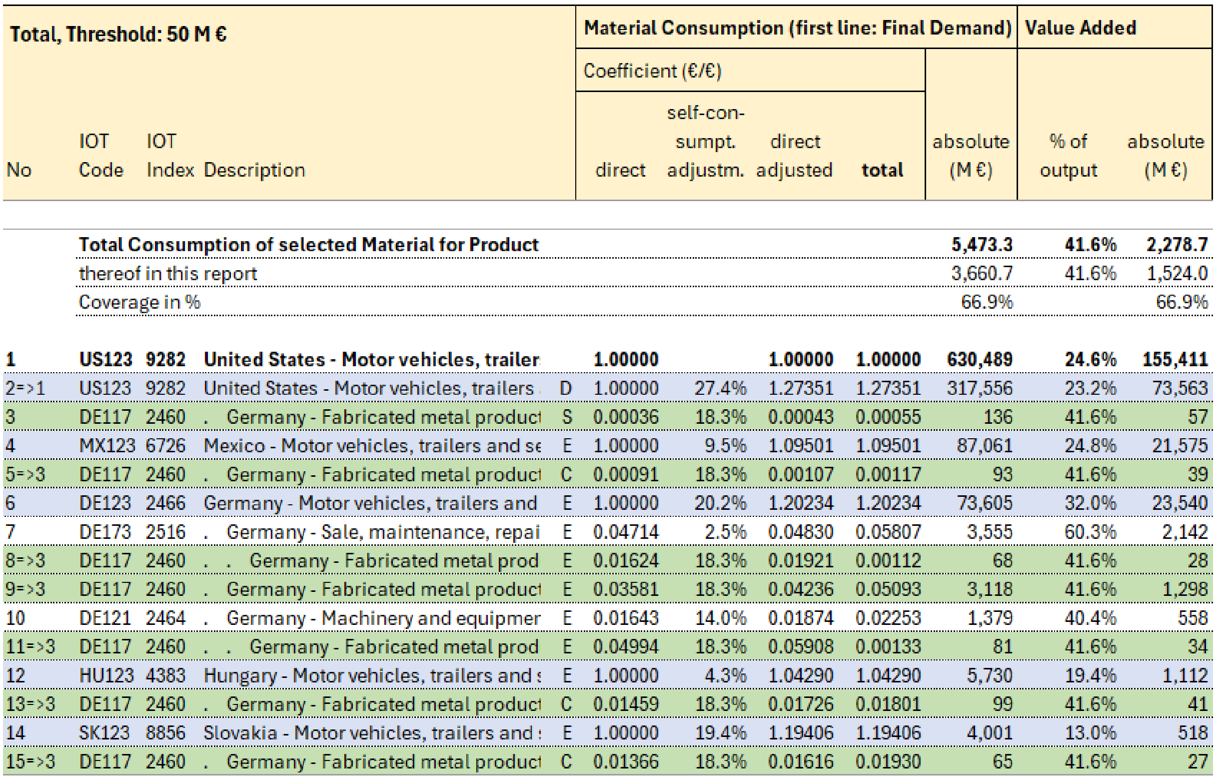
Due to the higher threshold, from all chains of type "S" in **Figure 25** only the last one with the highest value added contribution has remained in the picture.

For example, the supply chain activity type for the last chain at the bottom of the chart was derived as follows:

1. Export of "Motor vehicles" from Slovakia to the US for direct final demand (blue, "E"), first border crossing (counted from end of chain)
2. Intermediate use of "Fabricated metal products" imported from Germany, for the production of "Motor vehicles" in Slovakia, second border crossing
3. Target material "Fabricated metal products" from Germany found. As the chain starts with export for final demand and crosses two borders, the fourth line of **Table 21** applies. The type of supply chain activity is "C - complex GVC".

The respective hierarchical table is listed her again, with the supply chain activity type indicators shown to the right of the product descriptions.

Table 23 *- DE Fabricated metal products for US Motor vehicles - all types of activity*



Total, Threshold: 50 M €					Material Consumption (first line: Final Demand)					Value Added	
					Coefficient (€/€)						
No	IOT Code	IOT Index	Description		direct	self-con-sumpt. adjustm.	direct adjusted	**total**	absolute (M €)	% of output	absolute (M €)
	Total Consumption of selected Material for Product								**5,473.3**	**41.6%**	**2,278.7**
	thereof in this report								3,660.7	41.6%	1,524.0
	Coverage in %								66.9%		66.9%
1	**US123**	**9282**	**United States - Motor vehicles, trailer**		**1.00000**		**1.00000**	**1.00000**	**630,489**	**24.6%**	**155,411**
2=>1	US123	9282	United States - Motor vehicles, trailers	D	1.00000	27.4%	1.27351	1.27351	317,556	23.2%	73,563
3	DE117	2460	. Germany - Fabricated metal product	S	0.00036	18.3%	0.00043	0.00055	136	41.6%	57
4	MX123	6726	Mexico - Motor vehicles, trailers and se	E	1.00000	9.5%	1.09501	1.09501	87,061	24.8%	21,575
5=>3	DE117	2460	. Germany - Fabricated metal product	C	0.00091	18.3%	0.00107	0.00117	93	41.6%	39
6	DE123	2466	Germany - Motor vehicles, trailers and	E	1.00000	20.2%	1.20234	1.20234	73,605	32.0%	23,540
7	DE173	2516	. Germany - Sale, maintenance, repai	E	0.04714	2.5%	0.04830	0.05807	3,555	60.3%	2,142
8=>3	DE117	2460	. . Germany - Fabricated metal prod	E	0.01624	18.3%	0.01921	0.00112	68	41.6%	28
9=>3	DE117	2460	. Germany - Fabricated metal product	E	0.03581	18.3%	0.04236	0.05093	3,118	41.6%	1,298
10	DE121	2464	. Germany - Machinery and equipmer	E	0.01643	14.0%	0.01874	0.02253	1,379	40.4%	558
11=>3	DE117	2460	. . Germany - Fabricated metal prod	E	0.04994	18.3%	0.05908	0.00133	81	41.6%	34
12	HU123	4383	Hungary - Motor vehicles, trailers and s	E	1.00000	4.3%	1.04290	1.04290	5,730	19.4%	1,112
13=>3	DE117	2460	. Germany - Fabricated metal product	C	0.01459	18.3%	0.01726	0.01801	99	41.6%	41
14	SK123	8856	Slovakia - Motor vehicles, trailers and s	E	1.00000	19.4%	1.19406	1.19406	4,001	13.0%	518
15=>3	DE117	2460	. Germany - Fabricated metal product	C	0.01366	18.3%	0.01616	0.01930	65	41.6%	27

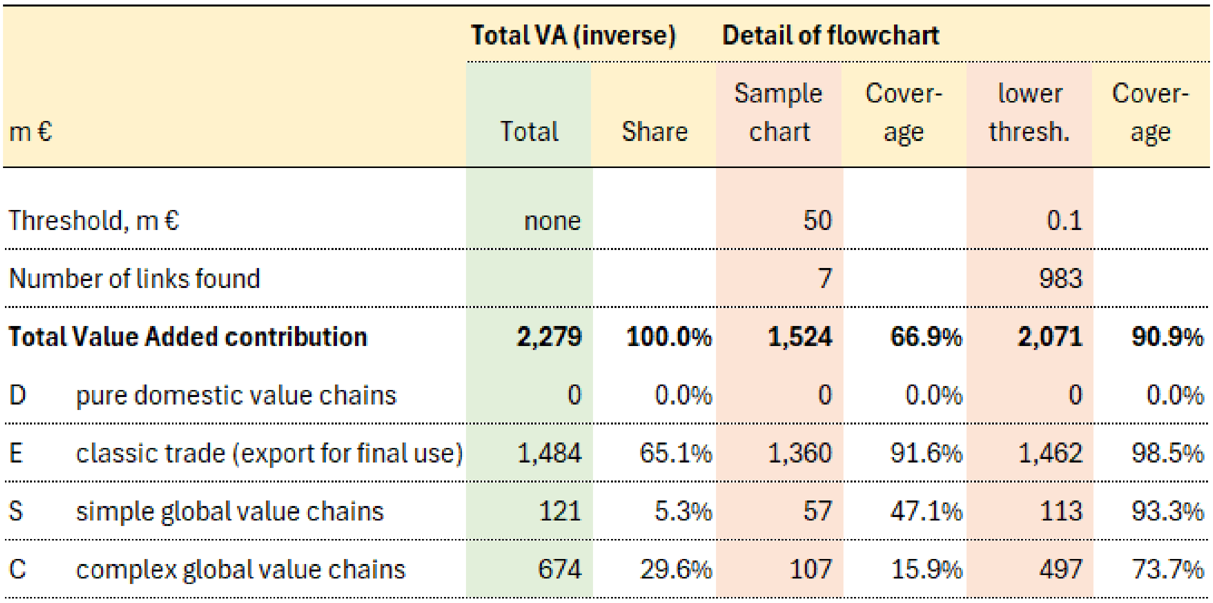
Source: own calculations, Wang et al. (2017), FIGARO-E3 (EC, Joint Research Centre, 2025)

How to check, whether the derivation of the supply chain activity types according to the rules in **Table 21** are in line with the matrix terms in **Table 18**?

When decreasing the threshold for material consumption, e.g., from 50 M € to 0.1 M €, the coverage of total values according to the formulas in increases from 66.9% to 90.9%.

With the lower threshold, instead of the 7 GVCs in the initial flowchart (**Figure 26**) and table (**Table 23**) now 983 GVCs are identified.

Table 24 *- Supply chain activity types: Total VA contribution vs. selected GVCs*



		Total VA (inverse)		Detail of flowchart			
m €		Total	Share	Sample chart	Cover-age	lower thresh.	Cover-age
Threshold, m €		none		50		0.1	
Number of links found				7		983	
Total Value Added contribution		**2,279**	**100.0%**	**1,524**	**66.9%**	**2,071**	**90.9%**
D	pure domestic value chains	0	0.0%	0	0.0%	0	0.0%
E	classic trade (export for final use)	1,484	65.1%	1,360	91.6%	1,462	98.5%
S	simple global value chains	121	5.3%	57	47.1%	113	93.3%
C	complex global value chains	674	29.6%	107	15.9%	497	73.7%

Source: own calculations, Wang et al. (2017), FIGARO-E3 (EC, Joint Research Centre, 2025)

With these 983 GVCs, the coverage has substantially increased not only on average, but also for the three relevant supply chain activity types.

There are no values in the category "D - pure domestic value chains", because the search for linking chains between "Fabricated metal products" from Germany as a material and "Motor vehicles" in the US as a finished product implies already crossing the border at least once.

This is a check, not a proof. On the other hand, if the coverage in only one of the three relevant categories had exceeded 100%, that would have indicated a misattribution of at least one global value chain.

3.4 Foreign-owned and domestic-owned enterprises

In 2018, the Organization of Economic Cooperation and Development (OECD) started the publication of the Analytical AMNE (Activities of Multinational Enterprises) database (Cadestin et al., 2018).

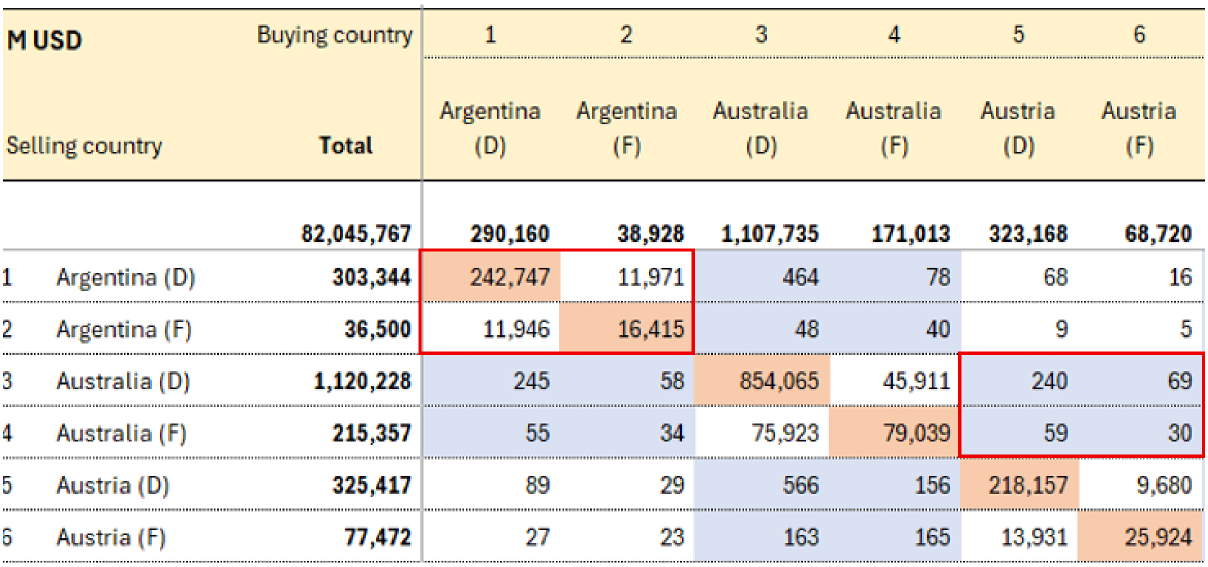
The AMNE database is consistent with the standard OECD ICIO tables with 41 industries and 76 countries plus an entry for the "Rest of the World". In addition, all information is subdivided into the shares of foreign-owned and domestic-owned firms[5].

In Wang et al. (2021), the decomposition by type of supply chain activity presented above, was extended to the OECD AMNE database. Visual network decomposition can follow this extension and provide additional insights based on the distinction between foreign-owned and domestic-owned firms. This will be illustrated based on the data for 2020 included in the 2024 edition of the OECD AMNE database (see OECD, 2025).

In the AMNE database, for each country there are two sets of rows and columns by product - one with the values for domestic-owned firms , one for foreign-owned firms. In the following, they will be distinguished by the letter "(D)" and "(F)" respectively.

In technical terms, further calculations follow the same steps like for a standard MRIO table (see 3.1), just as if there were twice as many countries. Consequently, in the value added contribution margin aggregated by country there is a square field with 4 cells for each pair of countries:

Table 25 *- Value added contribution matrix with distinction of ownership*



M USD		Buying country	1	2	3	4	5	6
Selling country		**Total**	Argentina (D)	Argentina (F)	Australia (D)	Australia (F)	Austria (D)	Austria (F)
		82,045,767	**290,160**	**38,928**	**1,107,735**	**171,013**	**323,168**	**68,720**
1	Argentina (D)	**303,344**	242,747	11,971	464	78	68	16
2	Argentina (F)	**36,500**	11,946	16,415	48	40	9	5
3	Australia (D)	**1,120,228**	245	58	854,065	45,911	240	69
4	Australia (F)	**215,357**	55	34	75,923	79,039	59	30
5	Austria (D)	**325,417**	89	29	566	156	218,157	9,680
6	Austria (F)	**77,472**	27	23	163	165	13,931	25,924

Source: OECD AMNE database, OECD (2025), own calculations

---

[5] It should be mentioned, that parent companies of multinational groups of companies (MNE) are considered "domestic MNE" and as such are subsumed under "domestic-owned". Information about the share of these domestic-owned MNE is provided by the OECD as additional information, but is not shown throughout the AMNE database.

In Wang et al. (2021) the authors have applied the decomposition approach laid out in the previous segment (3.3) to the OECD AMNE Database.

Formula (24) is extended according to the distinction by domestic and foreign ownership. This results in four times as many components - see Wang et al. (2021), p. 14, formula (5):

$$\hat{V}B\hat{Y} = \tag{25}$$

$$\hat{V}_D L\hat{Y}_D^L + \hat{V}_D L\hat{Y}_D^E + \hat{V}_D LA^E L\hat{Y}_D^L + \hat{V}_D LA^E\left(B\hat{Y}_D - L\hat{Y}_D^L\right) \qquad \text{(D → D)}$$

$$+\hat{V}_D L\hat{Y}_F^L + \hat{V}_D L\hat{Y}_F^E + \hat{V}_D LA^E L\hat{Y}_F^L + \hat{V}_D LA^E\left(B\hat{Y}_F - L\hat{Y}_F^L\right) \qquad \text{(D → F)}$$

$$+\hat{V}_F L\hat{Y}_D^L + \hat{V}_F L\hat{Y}_D^E + \hat{V}_F LA^E L\hat{Y}_D^L + \hat{V}_F LA^E\left(B\hat{Y}_D - L\hat{Y}_D^L\right) \qquad \text{(F → D)}$$

$$+\hat{V}_F L\hat{Y}_F^L + \hat{V}_F L\hat{Y}_F^E + \hat{V}_F LA^E L\hat{Y}_F^L + \hat{V}_F LA^E\left(B\hat{Y}_F - L\hat{Y}_F^L\right) \qquad \text{(F → F)}$$

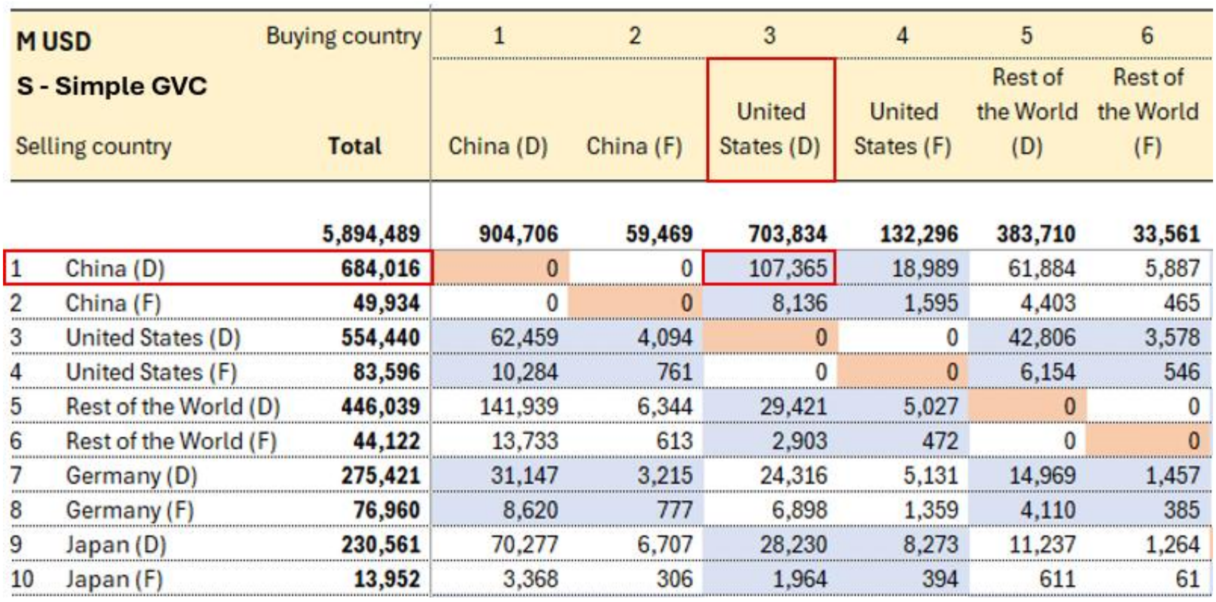
Diagonal matrices of value added coefficients and final demand are named

$\hat{V}_D, \hat{Y}_D, \hat{Y}_D^L, \hat{Y}_D^E$ for domestic-owned enterprises and

$\hat{V}_F, \hat{Y}_F, \hat{Y}_F^L, \hat{Y}_F^E$ for foreign-owned enterprises.

In the visual network decomposition, the different types of supply chain activity are implemented as separate matrices, while the distinction of ownership is reflected in additional rows and columns. As before, the analysis starts with the value added contribution matrix aggregated by country, this time for "S - Simple GVC":

Table 26 *- VACM by country and ownership, Simple GVC*



M USD **S - Simple GVC** Selling country		Buying country **Total**	1 China (D)	2 China (F)	3 United States (D)	4 United States (F)	5 Rest of the World (D)	6 Rest of the World (F)
		5,894,489	**904,706**	**59,469**	**703,834**	**132,296**	**383,710**	**33,561**
1	China (D)	**684,016**	0	0	107,365	18,989	61,884	5,887
2	China (F)	**49,934**	0	0	8,136	1,595	4,403	465
3	United States (D)	**554,440**	62,459	4,094	0	0	42,806	3,578
4	United States (F)	**83,596**	10,284	761	0	0	6,154	546
5	Rest of the World (D)	**446,039**	141,939	6,344	29,421	5,027	0	0
6	Rest of the World (F)	**44,122**	13,733	613	2,903	472	0	0
7	Germany (D)	**275,421**	31,147	3,215	24,316	5,131	14,969	1,457
8	Germany (F)	**76,960**	8,620	777	6,898	1,359	4,110	385
9	Japan (D)	**230,561**	70,277	6,707	28,230	8,273	11,237	1,264
10	Japan (F)	**13,952**	3,368	306	1,964	394	611	61

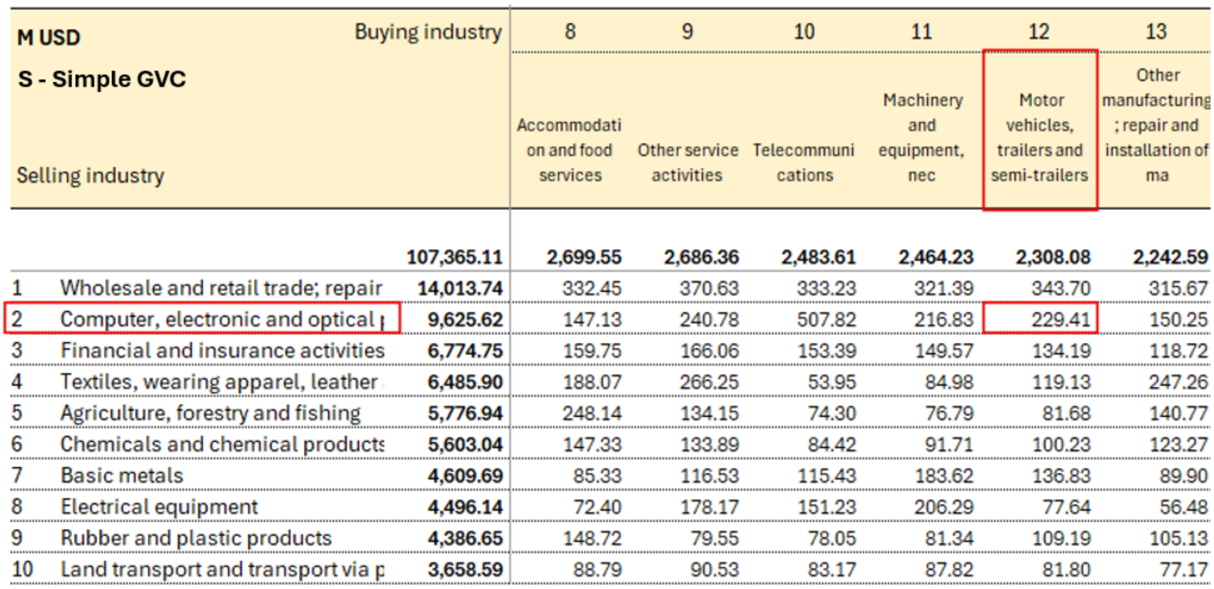
Source: own calculations based on OECD (2025), Wang et al. (2021)

In **Table 26**, rows and columns by the totals of both domestic-owned and foreign-owned enterprises in order to keep together the data for each country.

As an example, the value added contribution of Chinese domestic-owned enterprises to the final demand produced by US domestic-owned firms or imported to the US directly for final use will be investigated. As a reminder, it should be mentioned again, that Chinese and US parent companies of multinational enterprises are subsumed under domestic-owned firms in China and in the US respectively (see footnote 5 on page 53).

A double-click on the respective cell in the VACM by country leads to the report by industry:

Table 27 *- VA of CN domestic for US domestic firms by product, simple GVC*



M USD **S - Simple GVC** Selling industry		Buying industry	8	9	10	11	12	13
			Accommodation and food services	Other service activities	Telecommunications	Machinery and equipment, nec	Motor vehicles, trailers and semi-trailers	Other manufacturing; repair and installation of ma
		107,365.11	**2,699.55**	**2,686.36**	**2,483.61**	**2,464.23**	**2,308.08**	**2,242.59**
1	Wholesale and retail trade; repair	**14,013.74**	332.45	370.63	333.23	321.39	343.70	315.67
2	Computer, electronic and optical	**9,625.62**	147.13	240.78	507.82	216.83	229.41	150.25
3	Financial and insurance activities	**6,774.75**	159.75	166.06	153.39	149.57	134.19	118.72
4	Textiles, wearing apparel, leather	**6,485.90**	188.07	266.25	53.95	84.98	119.13	247.26
5	Agriculture, forestry and fishing	**5,776.94**	248.14	134.15	74.30	76.79	81.68	140.77
6	Chemicals and chemical products	**5,603.04**	147.33	133.89	84.42	91.71	100.23	123.27
7	Basic metals	**4,609.69**	85.33	116.53	115.43	183.62	136.83	89.90
8	Electrical equipment	**4,496.14**	72.40	178.17	151.23	206.29	77.64	56.48
9	Rubber and plastic products	**4,386.65**	148.72	79.55	78.05	81.34	109.19	105.13
10	Land transport and transport via p	**3,658.59**	88.79	90.53	83.17	87.82	81.80	77.17

Source: own calculations based on OECD (2025), Wang et al. (2021)

The sum on the top left of the table matches the value that was just selected in the VACM by country.

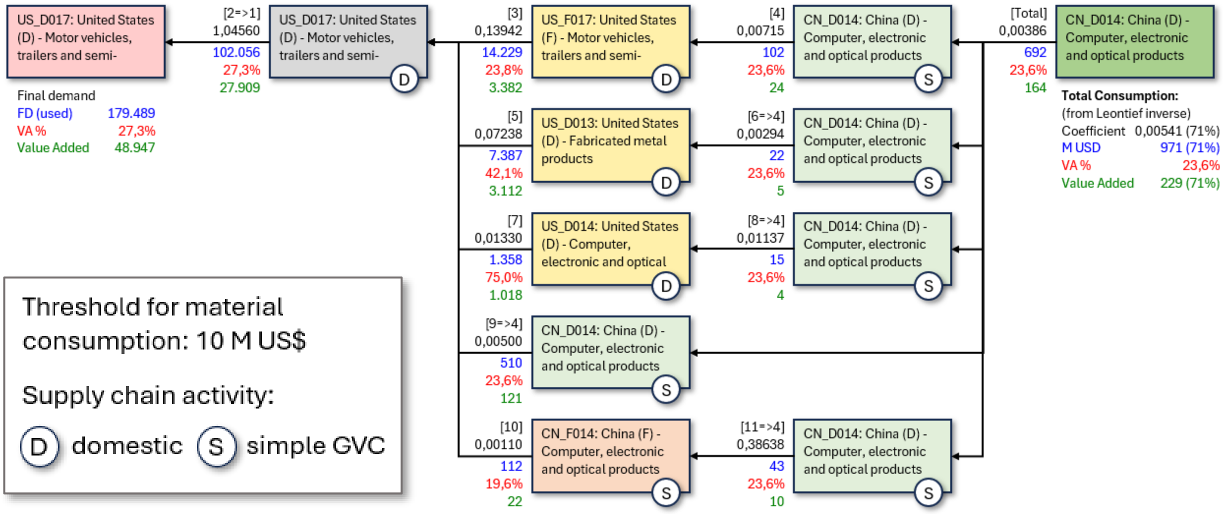
In the next step, special attention will be paid to the value added contribution of "Computer, electronic and optical products" from Chinese domestic-owned companies to the final demand of "Motor vehicles, trailers and semi-trailers" produced by US domestic-owned firms or imported for direct final use.

A double-click on the respective cell in the VACM by product starts the generation of the flowchart (see **Figure 27**). Chart elements are explained in

Figure 28. The assignment of the industries to domestic-owned or foreign-owned firms is represented by the letters "D" and "F" respectively, both in the code and after the country description.

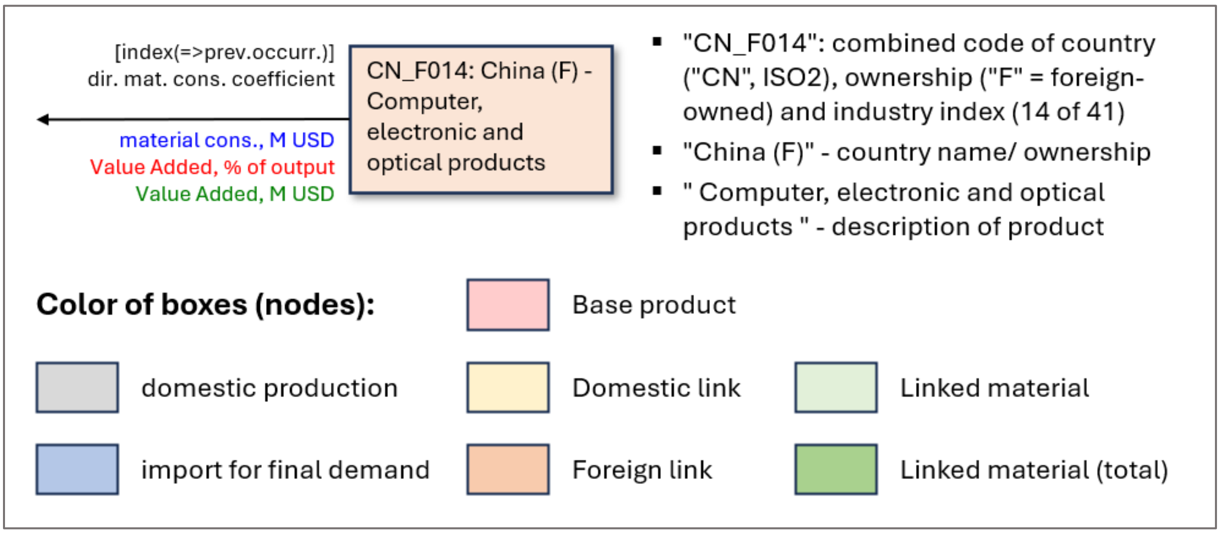
The five selected chains cover already 71% of the total value added contribution via "Simple GVC" according to the term $\hat{V}_D L A^E L \hat{Y}_D^L$ in equation (25), first line (D → D), third term.

Figure 27 *- CN (domestic) Electronic products for US (domestic) Cars, Simple GVC*



Source: own calculations based on OECD (2025), Wang et al. (2021)

Figure 28 *- Find Linking Chains mode for OECD-AMNE: Flowchart legend*

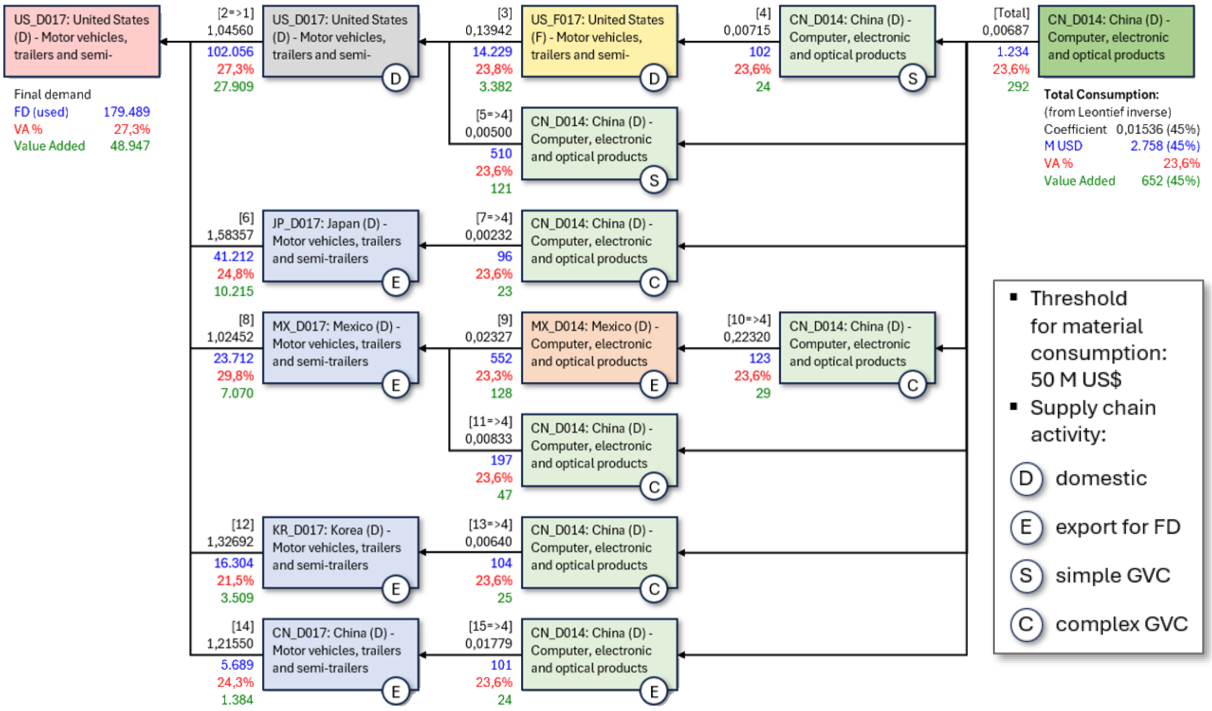


- "CN_F014": combined code of country ("CN", ISO2), ownership ("F" = foreign-owned) and industry index (14 of 41)
- "China (F)" - country name/ ownership
- " Computer, electronic and optical products " - description of product

Like in the previous segment (3.3), in the next step the flowchart is expanded to show all types of supply chain activity. The material consumption threshold is increased from 10 M US$ to 50 M US$ to keep the flowchart at a reasonable size (see **Figure 29**).

For both flowcharts - **Figure 27** and **Figure 29**, hierarchical tables can be used again to trace calculation in detail.

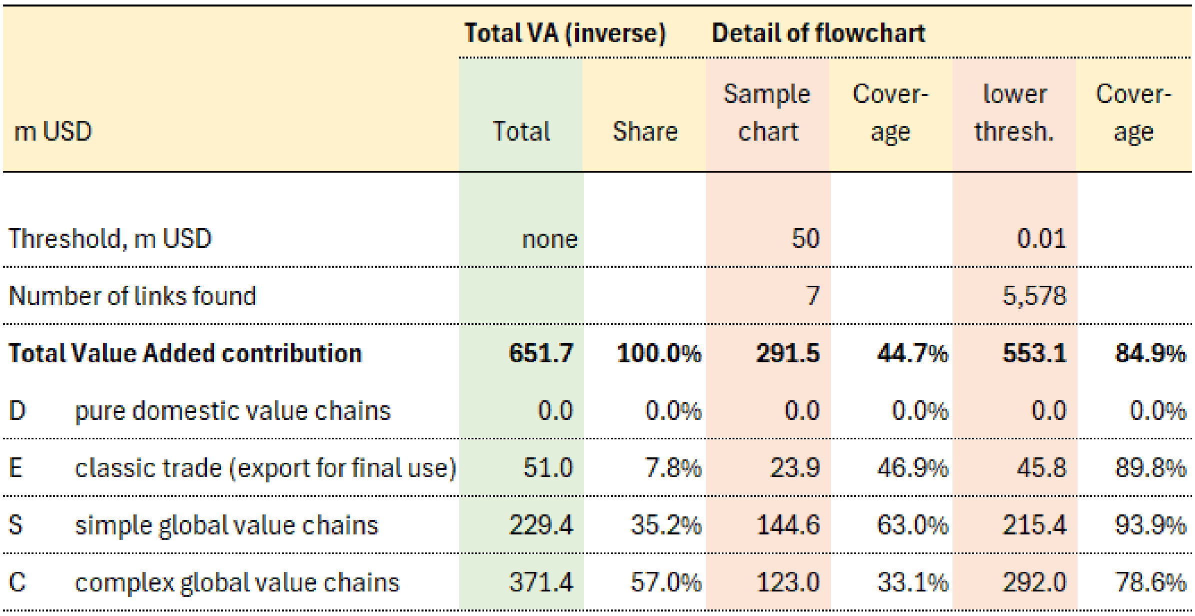
Figure 29 *- CN (domestic) Electronic products for US (domestic) Cars, total SCA types*



Source: own calculations based on OECD (2025), Wang et al. (2021)

When decreasing the threshold to 0.01 M US$, 5578 chains are selected, covering 84.9% of total values. The coverage by SCA type shows a similar pattern as in **Table 24**.

Table 28 *- OECD AMNE SCA types: Total VA contribution vs. selected GVCs*



	Total VA (inverse)		Detail of flowchart			
m USD	Total	Share	Sample chart	Cover-age	lower thresh.	Cover-age
Threshold, m USD	none		50		0.01	
Number of links found			7		5,578	
Total Value Added contribution	**651.7**	**100.0%**	**291.5**	**44.7%**	**553.1**	**84.9%**
D pure domestic value chains	0.0	0.0%	0.0	0.0%	0.0	0.0%
E classic trade (export for final use)	51.0	7.8%	23.9	46.9%	45.8	89.8%
S simple global value chains	229.4	35.2%	144.6	63.0%	215.4	93.9%
C complex global value chains	371.4	57.0%	123.0	33.1%	292.0	78.6%

Source: own calculations based on OECD (2025), Wang et al. (2021)

3.5 Endogenizing Value Added embodied in capital goods

The initial approach of Input-Output analysis was focusing on intermediate material consumption, while Gross Fixed Capital Formation (GFCF) as part of final demand and Consumption of Fixed Capital (CDC) as a component of value added were considered as exogenous outputs and inputs. With increasing attention to energy use, it soon became apparent that the **energy content of final demand is undervalued if it does not also consider the energy embodied in capital goods used in the various economic sectors**.

For the US, proposals to extend Input-Output analysis by including the use of capital goods were already referring to capital flow matrices published by the US Bureau of Economic analysis, e.g. for 1963, 1967 and 1972 (see, e.g. Casler, 1981).

Environmental extensions of Input-Output analysis were affected by the same potential bias when neglecting, e.g., the carbon footprint of capital goods used in production.

Lenzen and Treloar (2004) compared two methods of capital endogenization[6] - the augmentation method with just one additional row and column for capital goods, and the flow matrix method with a full distinction of the use of capital goods by supplying and using industry. The latter takes much better account of the different factor intensity across sectors, "but has the disadvantage of high data requirements" (p. 1).

Södersten et al. (2018) have implemented the flow matrix method for the EXIOBASE MRIO model. A capital flow matrix was created with the column sums aligned with CFC.

A recent study of Ye et al. (2025) applies the flow matrix approach to the calculation of Trade in Value Added (TiVA), using the OECD ICIO with newly constructed capital flow matrices.

In the context of capital endogenization, visual network decomposition again provides deeper additional insights about the driving global value chains, now including those caused by the use of capital goods.

The method will be illustrated using the eurostat FIGARO database for 2021. For capital endogenization, a capital flow matrix for 2021 is used which was constructed and published by Blain (2026) and which is compatible with the FIGARO database.

This **capital flow matrix $\boldsymbol{C}$** is a square table with rows and columns by industry per country, just like the matrix of intermediate material consumption. In this version, column totals are aligned with the values of CFC in value added. Row totals correspond, but are usually not equal to GFCF as part of final demand by suppling industry. In this way, the matrix provides a hypothetical breakdown of depreciation by industry supplying the respective capital goods.

---

[6] The term "capital endogenization" as it is used here must be distinguished from "capital endogenization" in dynamic economic models, where it links economic growth to the demand for capital as a production factor.

Dividing each column of matrix C by the output x of the respective industry in the different countries gives the **matrix of direct capital requirements**:

$$K = C\hat{x}^{-1} \tag{26}$$

Column sums of K are equal to the ratio v_k of depreciation to output, while the row sums of matrix C are related, but not necessarily equal to the contribution y_k of the different supplying industries to GFCF. The remaining elements of value added coefficients v and final demand y are referred to as v_r and y_r respectively:

$$v^T = v_k^T + v_r^T \tag{27}$$

$$y = y_k + y_r \tag{28}$$

$$v_k^T = e^T K \tag{29}$$

$$y_k = C \cdot e = Kx \tag{30}$$

where e and e^T are column and row vectors with ones.

The basic Input-Output equation can now be transformed as follows:

$$x = Ax + y \tag{31}$$

$$x = Ax + y_k + y_r \qquad \text{substituting (28)} \tag{32}$$

$$x = Ax + Kx + y_r \qquad \text{substituting (30)} \tag{33}$$

$$\big(I - (A + K)\big)\, x = y_r \qquad \text{factoring out} \tag{34}$$

$$\boxed{x = \big(I - (A + K)\big)^{-1} y_r} \qquad \text{multiplying with inverse} \tag{35}$$

An analog transformation can be applied to the price equation:

$$e^T = e^T A + v^T \qquad \text{standard price equation} \tag{36}$$

$$e^T = e^T A + v_k^T + v_r^T \qquad \text{substituting (27)} \tag{37}$$

$$e^T = e^T A + e^T K + v_r^T \qquad \text{substituting (29)} \tag{38}$$

$$e^T\big(I - (A + K)\big) = v_r^T \qquad \text{factoring out} \tag{39}$$

$$e^T = v_r^T\big(I - (A + K)\big)^{-1} \qquad \text{multiplying with inverse} \tag{40}$$

From (40), the **modified value added content matrix** can be derived by right-hand multiplication with the new final demand diagonal and switching to the diagonal of the value added coefficient vector. It will replace W introduced in equation (13):

$$\boxed{W_k = \hat{v}_r\big(I - (A + K)\big)^{-1}\, \hat{y}_r} \tag{41}$$

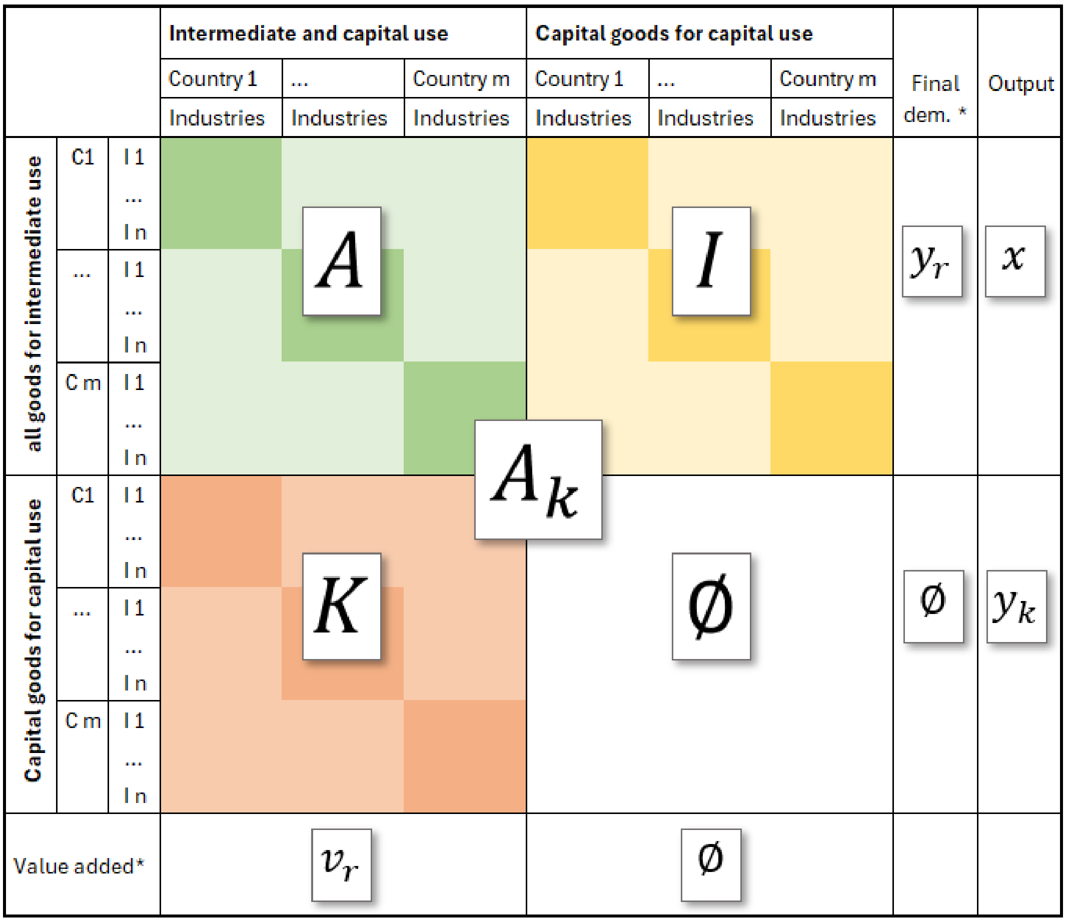
It should be mentioned, that with this notation, value added embodied in exports for direct final use is assigned to the producing country.

How to set up the information for visual network decomposition? For the exploration of the effect of capital endogenization, GVCs caused by intermediate demand should be separated from those induced by the use of capital goods and the respective intermediate demands for the production of capital goods.

For this reason, matrices A for direct intermediate material requirements and K for direct capital requirements will not just be added, but kept separate. To this end, additional columns and rows are inserted representing the supply and the use of capital goods by supplying (rows) and using (columns) country and industry.

Figure 30 shows the extended MRIO coefficient matrix A_k that will be used by the visual decomposition algorithm instead of the original matrix A of direct material requirements. "I" and "Ø" denote the identity matrix and a matrix filled with zeros.

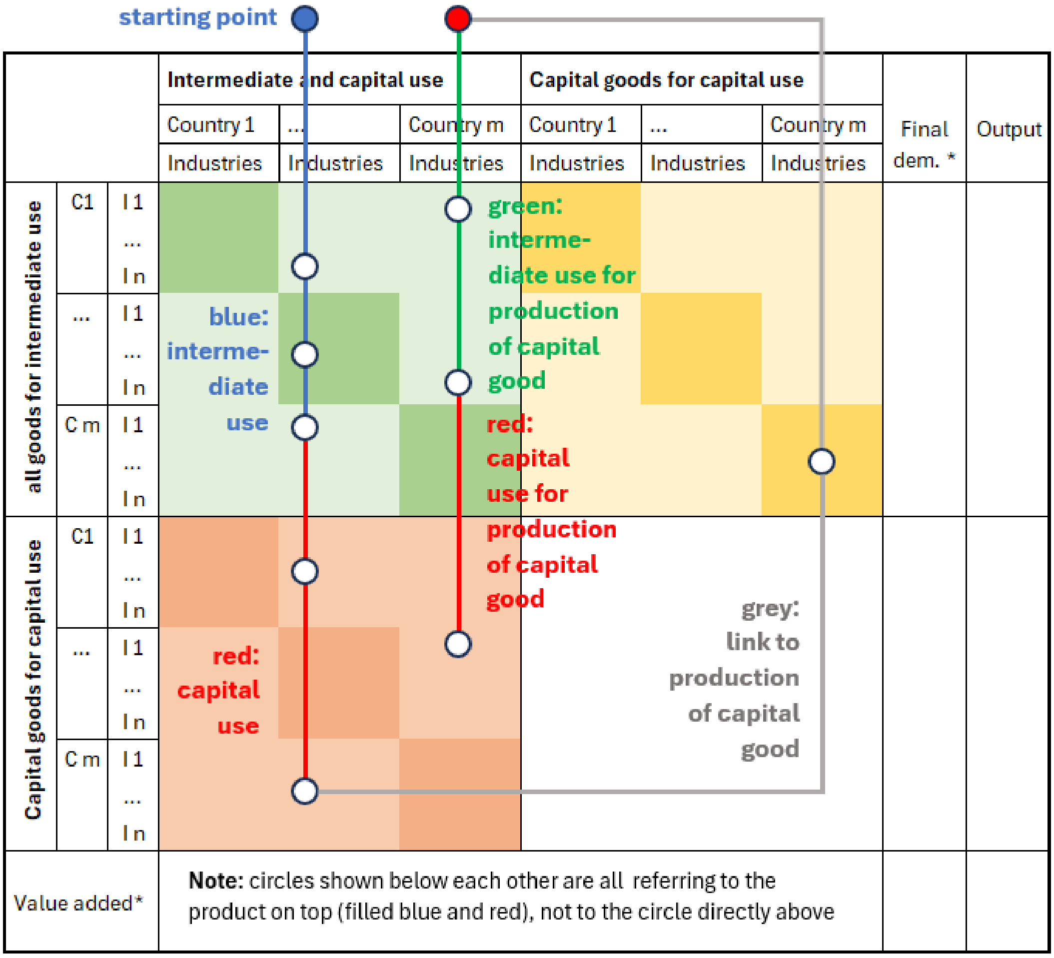
Figure 30 *- Extended MRIO table with separate matrices A and K*



* except GFCF aligned with CFC ** except CFC

As shown in **Figure 31**, the search for backward linkages is not restricted any more to intermediate inputs (blue line), but includes the use of capital goods (red line). The grey line to the right establishes the connection between the use of a capital good and its production via the identity matrix in the upper right part. The production of the capital good, in turn, creates further demands for intermediates and, again, other capital goods.

Figure 31 *- Component search in extended MRIO with endogenized capital*



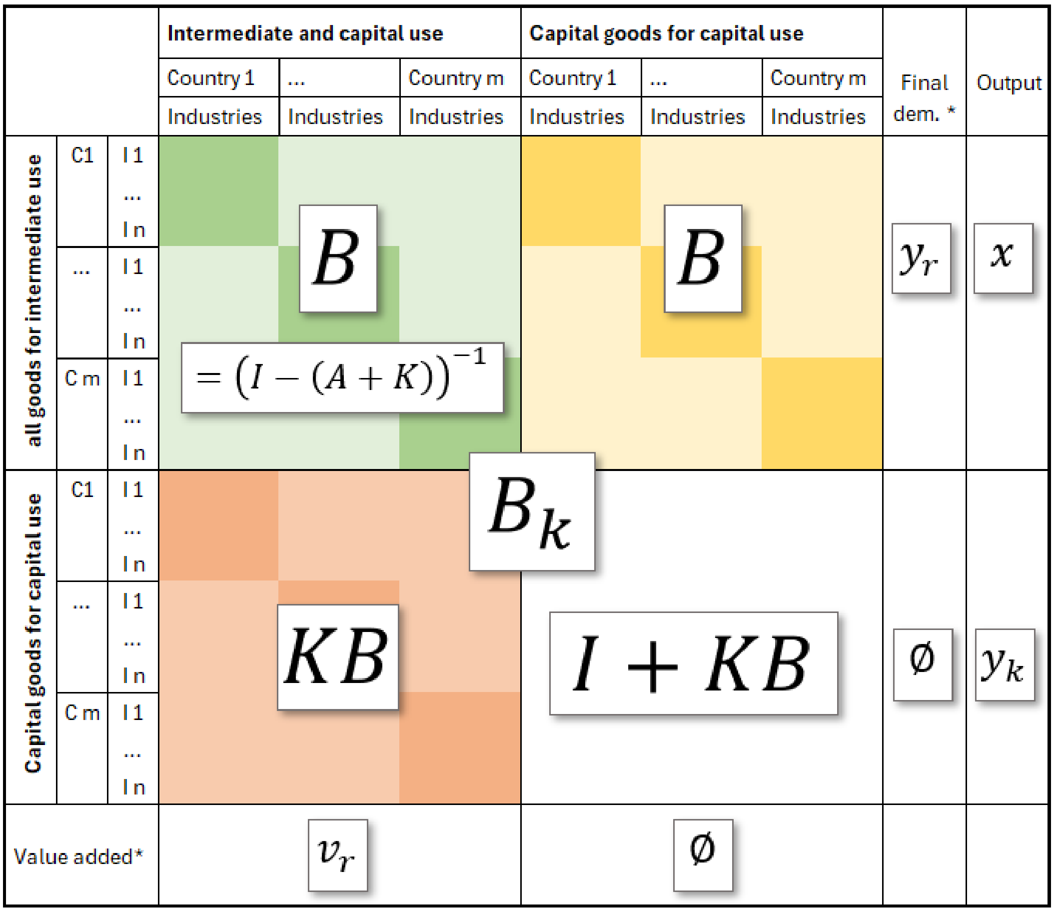
Note: circles shown below each other are all referring to the product on top (filled blue and red), not to the circle directly above

* except GFCF aligned with CFC   ** except CFC

When inverting the extended coefficient matrix A_k, the upper left submatrix of the resulting inverse B_k is equal to the modified Leontief inverse $\left(I-(A+K)\right)^{-1}$in equations (35) and (41) on page 59. The structure of the extended inverse is presented in **Figure 32**.

The visual network decomposition algorithm uses this inverse as a benchmark for the calculation of the coverage achieved with the selected global value chains.

Figure 32 - *Inverse of the extended matrix with endogenized capital*



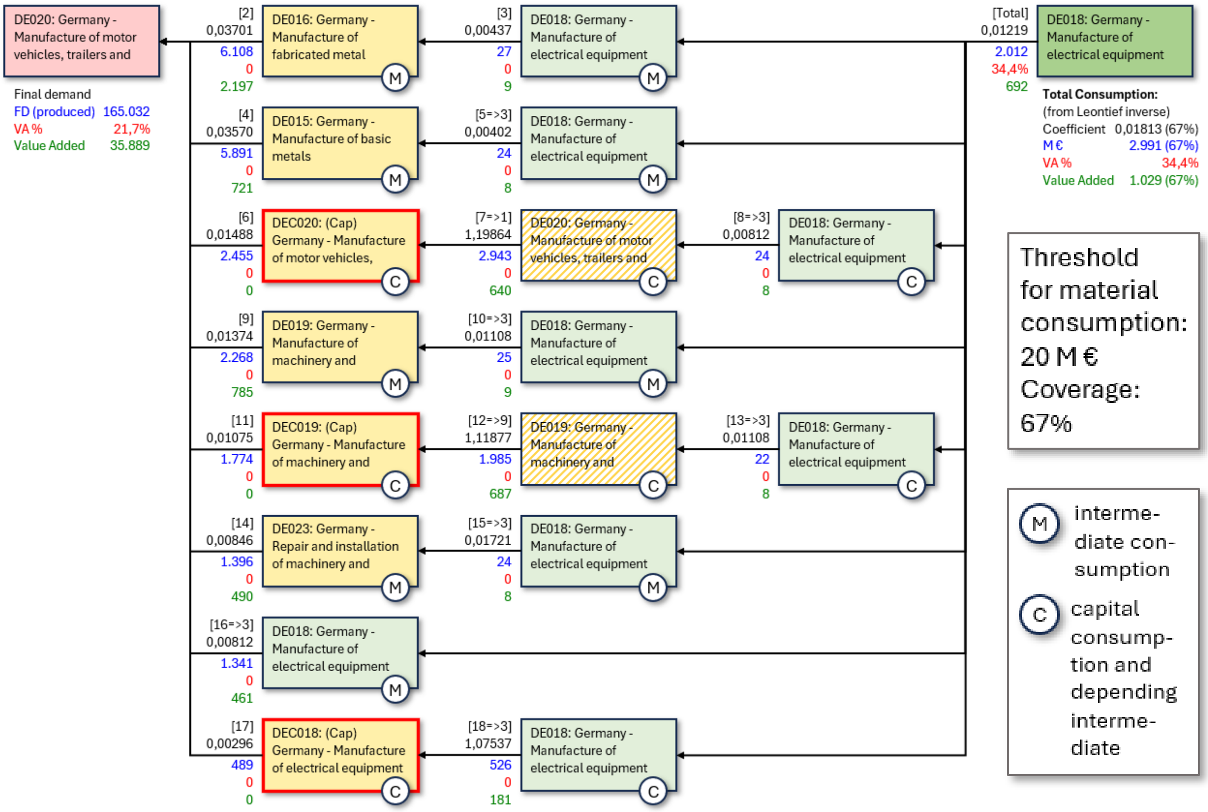
* except GFCF aligned with CFC ** except CFC

As an example, **Figure 33** shows the direct an indirect consumption of electrical equipment from Germany for the "manufacture of motor vehicles, trailers and semitrailers" (in the following "motor vehicles") in Germany. Due to the endogenization of capital consumption, the flowchart includes:

- direct and indirect intermediate consumption - marked with "(M)"
- direct and indirect capital consumption - represented by red-bordered boxes marked with "(C)"
- direct and indirect intermediate consumption for this capital consumption - represented by black-bordered boxes marked with "(C)"

Since the analysis is based on an industry by industry perspective, all boxes refer to industries, not products. Although the underlying FIGARO table as well as the capital flow matrix are multi-country databases, with the threshold of 20 M € only domestic backward linkages have been selected.

Figure 33 *- DE: electrical equipment for motor vehicles, with capital endogenization*



Source: own calculations based on data sources Eurostat (2025) and Blain (2026)

For the explanation of the flowchart elements, please refer to **Figure 20** on page 37.

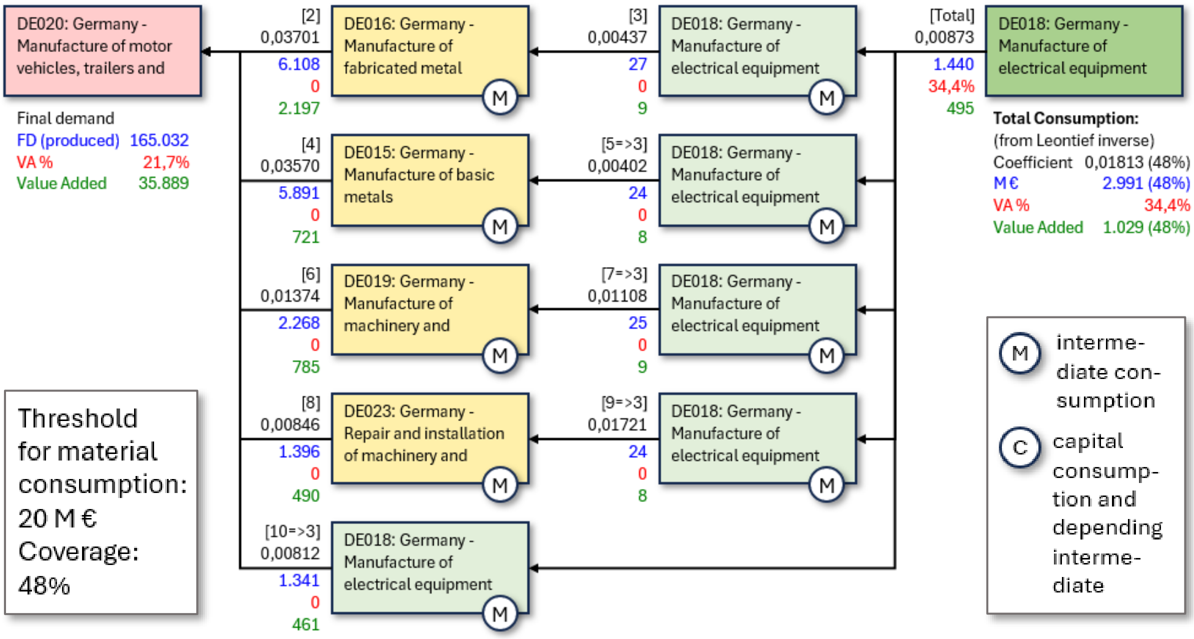
There are three cases of capital consumption (see red-bordered boxes) - motor vehicles themselves (box 6), machinery and equipment (box 11) and electrical equipment (box 17).

In each of these cases, the first box to the right (boxes 6, 12 and 18 respectively) represents the production of these capital goods. Due to the instant inversion of the self-consumption (see segment 2.2), the coefficients of direct consumption (to the left of the boxes, above the arrows) are greater than 1 in all three cases.

For comparison, **Figure 34** shows the same relationship without including capital consumption. All red-bordered boxes representing capital consumption and the depending intermediate inputs disappeared. For comparability, the same final demand (produced) of motor vehicles was used, i.e. excluding GFCF aligned with CFC.

The total direct and in direct consumption of electrical equipment shown in the flowchart with endogenized capital (692 M €) is 39.8% higher than in the chart with just intermediate consumption (495 M €).

Figure 34 *- DE: electrical equipment for motor vehicles, w/o capital endogenization*



Source: own calculations based on data sources Eurostat (2025) and Blain (2026)

The application of Visual Network Decomposition to the case of an Input-Output framework with endogenized capital consumption confirms once again the advantages of this approach in terms of additional transparency and deeper understanding of the underlying value chain detail.

4 Conclusions and prospects

Visual Network Decomposition of the Leontief inverse and derived indicators can be effectively used to gain deeper insights into the underlying network of backward and forward linkages. The interactive application enables a flexible, exploratory approach to reveal the main driving value chains behind multipliers.

The concept can be applied to both national and multiregional input-output frameworks. In addition to the standard input-output tables, the methodology can also be used to investigate extensions such as carbon footprint, decomposition by type of supply chain activity, by foreign and domestic ownership or the endogenization of capital consumption.

There are a number of possible directions for future research:

- **Dynamic analysis**: For most national and global input-output tables, comparable time series allow for the examination of changes in the input-output network over time. Visual Network Decomposition can help identify those value chains that have the greatest impact on changes in multipliers.
- While the starting point for the analysis so far has been the Value Added Contribution Matrix, the approach can also be applied to the **decomposition of "Trade in Value Added (TiVA)"** indicators (see section 3.1.2). The typical cases of double-counting can be identified and visualized in the flowcharts of backward linkages.
- **National (Social) Accounting Matrices (NAM/SAM)**: Eurostat has recently started publishing National Accounting Matrices compatible with the FIGARO MRIO database (FIGARO-NAM, Pedauga et al., 2025). First tests with the NAM tables for selected countries have shown, that Visual Network Decomposition can provide a visual presentation of flows in a NAM framework and help to understand the driving paths behind NAM multipliers and their changes.
- Revealing **driving causal chains in econometric models**: The application for NAM already goes beyond just intermediate and capital consumption by including income distribution, capital and financial flows. The biggest challenge in applying the Visual Network Decomposition approach to econometric models is that the latter contain a number of nonlinear equations.
- **Extending web-based dashboards**: The Visual Network Decomposition application "IOPathFinder" is currently a local stand-alone solution. A similar approach could be implemented as an extension of existing online dashboards, e.g., for TiVA indicators.

References

Amadei, A., Venturelli, S. and Manfredi, S. (2025), Plastics materials flows in the EU-27 and their environmental impacts - Unveiling the European plastic value chain, Publications Office of the European Union, Luxembourg, 2025, https://data.europa.eu/doi/10.2760/6579757, JRC142860.

Aray, H., Pedauga, L., & Velázquez, A. (2017), Financial Social Accounting Matrix: a useful tool for understanding the macro-financial linkages of an economy. Economic Systems Research, 29(4), p. 502

Arto, I., Dietzenbacher, E., Rueda Cantuche, J. (2019) Measuring bilateral trade in terms of value added, EUR 29751 EN, Publications Office of the European Union, Luxembourg, 2019, ISBN 978-92-76-03846-7, doi:10.2760/639612, JRC116694.

Ayres, R. U., Kneese, A. V. (1969), Production, Consumption, and Externalities. The American Economic Review, Vol. 59, No. 3 (June 1969), pp. 282-297

Blain J. (2026) Implementing a capital-use matrix in FIGARO: an environmental application to the ICT sector, Working Paper, La Société Nouvelle, Lille, France, 2026

Cadestin, C. et al. (2018), Multinational enterprises and global value chains: the OECD analytical AMNE database, OECD Trade Policy Papers, No. 211, OECD Publishing, 2018-02-27, Paris. http://dx.doi.org/10.1787/d9de288d-en

Casler, S. (1981), Correcting Input-Output Coefficients for Capital Depreciation. Energy Research Group of the University of Illinois, Urbana. *Theory and Methods*. 71. https://researchrepository.wvu.edu/rri_iotheorymethods/71

Cazcarro, I., Usubiaga-Liaño, A., Román, M.V. *et al.* (2025). FIGARO-E3: a high-resolution extended multi-regional input-output database consistent with official statistics. *Sci Data* 12, 575 (2025). https://doi.org/10.1038/s41597-025-04431-z

Defourny J, Thorbecke E (1984) Structural path analysis and multiplier decomposition within a social accounting matrix framework. The Economic Journal, Vol. 94, No. 373 (Mar., 1984), pp. 111-136

Kammerath, J. (1989), Analyse von Erzeugnisketten mit interaktiver Grafik, rechentechnik/datenverarbeitung 26 (1989) 7, Verlag Die Wirtschaft, Berlin, 1989

Kammerath, J. (2025). IOPathFinder - Revealing the Driving Value Chains behind the Leontief Inverse, Presentation at the 16th Input-Output Workshop, Groningen, March 26-27, 2025

Kammerath, J. (2026). Exploring GVC Detail with IOPathFinder: FIGARO-E3, OECD AMNE, AIPNET & National IO Tables, Presentation at the 17th Input-Output Workshop, Maastricht, March 4-5, 2026

Köhler, G., Fülle, H., Pinkau, K. et al. (1981). Materielle Bilanzierung - Die Bilanzierung materieller Proportionen und Verflechtungsbeziehungen, Verlag Die Wirtschaft, Berlin, 1981

Koopman, R., Wang, Z., Wei, S.-J. (2014). Tracing Value-Added and Double Counting in Gross Exports. American Economic Review 2014, 104(2): 459–494, http://dx.doi.org/10.1257/aer.104.2.459

Lenzen, M., Treloar G. J. (2004). Endogenising Capital: A comparison of Two Methods, Journal of Applied Input-Output Analysis, Vol. 10, 2004

Leontief, W. W. (1951). Input-output economics. Scientific American, vol. 185, no. 4, Oct. 1951, pp. 15–21.

Marrucci L., Daddi T., Iraldo F. (2024), Identifying the most sustainable beer packaging through a Life Cycle Assessment, Science of The Total Environment, Volume 948,2024,174941, ISSN 0048-9697, https://doi.org/10.1016/j.scitotenv.2024.174941.

Miller, R. E., Blair, P. D. (2022). Input-output analysis: Foundations and extensions (3rd ed.). Cambridge University Press, 2022

Pedauga, L., Ferreira, V., Banacloche, S., Romero. K., Rueda-Cantuche, J.M., Montaigne, F. (2025). FIGARO-NAM: Multi-country National Accounting Matrices based on FIGARO. Unveiling the Structure of EU Economies through National Accounting Matrices. EUROSTAT Statistical Working Paper. https://ec.europa.eu/eurostat/product?code=KS-01-25-050

Rémond-Tiedrez, I., & Rueda-Cantuche, J. M. (Eds.). (2019). Full International and Global Accounts for Research in Input-Output Analysis (FIGARO): Methodological Framework. Luxembourg: Publications Office of the European Union

Sargent T. J., Stachurski J. (2024). Economic Networks – Theory and Computations. Cambridge University Press, 2024
Downloaded on April 1, 2026 from https://networks.quantecon.org/ (free access)

Södersten, C.-J., Wood, R., Hertwich E. G. (2018). Endogenizing Capital in MRIO Models: The Implications for Consumption-Based Accounting, Environmental Science & Technology 2018 52 (22), 13250-13259

Timmer, M. P., Dietzenbacher, E., Los, B., Stehrer, R., de Vries, G. J. (2015). An Illustrated User Guide to the World Input–Output Database: the Case of Global Automotive Production. Review of International Economics, 23(3), 575–605

Wang, Z., Wei, S.-J., Yu, X., Zhu, K. (2017). Measures of Participation in Global Value Chains and Global Business Cycles, NBER Working Paper No. 23222, Cambridge, MA, March 2017

Wang, Z., Wei, S.-J., Yu, X., Zhu, K. (2021). Tracing Value Added in the Presence of Foreign Direct Investment, NBER Working Paper No. 29335, Cambridge, MA, October 2021

Xie R., Zhao Y., Chen L. (2020) Structural path analysis and its applications: literature review. National Accounting Review, 2020, 2(1): 83-9, doi: 10.3934/NAR.2020005

Ye M., Miroudot S., Meng B. (2025) Endogenizing Capital in Measuring Trade in Value-Added, IDE Discussion Paper No. 982, Institute of Developing Economies, JETRO, Chiba, Japan, December 2025

Data Sources

Blain, J. (2026). Capital-use matrices compatible with FIGARO (2010-2021) [Data set]. Zenodo. https://doi.org/10.5281/zenodo.19494656, accessed April 20, 2026, file CAPITAL_USE_2021.parquet

EC, Joint Research Centre. (2025). Cazcarro, I.; Usubiaga-Liaño, A.; Román, M. V.; Piñero, P.; Dietzenbacher, E.; Rueda-Cantuche, J. M.; Arto I.
(2025a) Inter-Country Input-Output product by product (Model B) Matrix. European Commission, Joint Research Centre [Dataset]
Doi: 10.2905/JRC.0865P8G; 10.2905/1e9cedd0-aac1-41c3-9754-41e1c0a2be69
PID: http://data.europa.eu/89h/1e9cedd0-aac1-41c3-9754-41e1c0a2be69,
file matrix_FIGARO-e-ic-io-B_prod-by-prod_millionEUR_2015.xlsx,
accessed on January 9, 2025
(2025b) Air Emissions Accounts Flatfile (Gg). European Commission, Joint Research Centre [Dataset]
Doi: 10.2905/JRC.0YHDGN0; 10.2905/0466566d-3e48-4537-a6cf-ad70921e56e5
PID: http://data.europa.eu/89h/0466566d-3e48-4537-a6cf-ad70921e56e5
filematrix_FIGARO-e_AEM_Gg_CO2e_2015.csv
accessed on February 7, 2025

Eurostat (2025). EU inter-country supply, use and input-output tables (2025 edition). Input-Output tables industry by industry
https://circabc.europa.eu/ui/group/cec66924-a924-4f91-a0ef-600a0531e3ba /library/6736dea8-da14-450f-b212-a791baf238c8/details
file matrix_eu-ic-io_ind-by-ind_25ed_2021.csv,
accessed on April 20, 2026

OECD (2025). Multinational enterprises and global value chains, https://www.oecd.org/en/data/datasets/multinational-enterprises-and-global-value-chains.html, file AAMNE_ICIO2020.zip, accessed on September 3, 2025

U.S. Bureau of Economic Analysis (2026). Interactive Access to Input-Output Accounts Data. https://apps.bea.gov/iTable/?reqid=1602&step=2&Categories=Core&isURI=1, files Supply-Use (SUPPLY-USE.zip), Total and Domestic Requirements: (TOTAL AND DOMESTIC REQUIREMENTS.zip), accessed on April 1, 2026

Appendix A – Implementation in Excel with Python integration

From the very beginning in the 1980ies, then still as "Product Chain Analysis", the application was intended for **use by researchers and analysts as well as economists and experts in the different industries as an interactive tool** (Kammerath, 1989).

The early versions were developed in the C language under MS DOS. At that time it already used sparse matrix techniques similar to the CSC and CSR formats in today's "scipy.sparse" Python library. This made it possible to store the program with all data on a single 1.44 MB floppy disk for the Input-Output model with 625 products and later the Ownership tool with more than 50,000 companies and owners. The program was running on standard personal computers with 1 MB of RAM and less with sufficient response times, often less than a second.

The version for the analysis of ownership and control soon became a Windows application using the Windows API for flowcharts. This version was used by a renowned German business data publisher on their "Corporate Group database CD" over 14 years with more than 100,000 entries. Among the users were several consultancies, banks, insurance companies and industrial enterprises.

For the analysis of global bills of materials (BOM) on corporate level, MS Excel was used with VBA for both flowchart generation and part of the calculations. Excel as a frontend very much contributed to the user acceptance. Due to the high number of products and materials (ca. 300,000), the traversal of the supply chains was executed mostly with recursive SQL statements on fast servers with direct access, invoked from the Excel workbook.

The "IOPathFinder" development started in 2023 for the detailed US BEA tables with this Excel/VBA setup. When moving to MRIOs like the WIOD, then FIGARO and FIGARO-E3, the increasing matrix dimensions provided a clear motivation for switching to Python. After starting with standard numpy arrays, the scripts were soon recoded to sparse techniques of the scipy.sparse library. The efficient implementation of these libraries gives an amazing performance, thereby keeping the tool truly interactive with minimal response times.

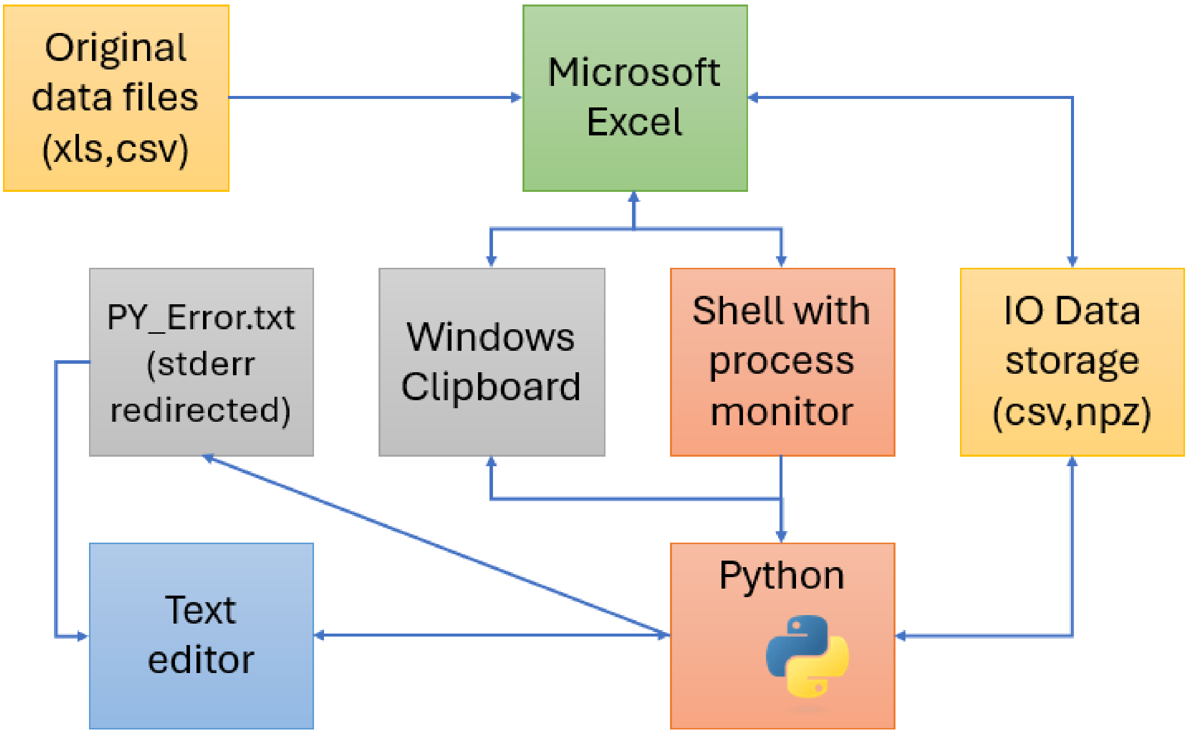
It was a deliberate decision to stay with MS Excel as the frontend to make the tool as user-friendly as possible. The standard view is always a report, not an IDE like PyCharm or VS code. This keeps the focus on the content, with a flexible and transparent step-by-step analysis path.

The **core function of flowchart generation** is implemented in **VBA** with more than 800 lines of code. There is also a version with Python libraries networkx and matplotlib (see example in Appendix B). However, VBA offers much more flexibility when creating interactive, pixel-perfect flowcharts with embedded complex numeric information, fully integrated in MS Excel as collections of moveable connected shapes.

The application is calling Python via a Windows process execution call with a check for process completion (CreateProcessA). Beforehand, the location of Python, either standalone or with an Anaconda distribution, is detected, which works for most local installation setups.

In each report, the respective Python scripts can be viewed. Error handling is supported by redirecting and displaying the output for both syntax and runtime errors. Even for the development standard IDE's were used only in exceptional cases.

Figure 35 *- Architecture of the Excel/Python interface*



Source: own development

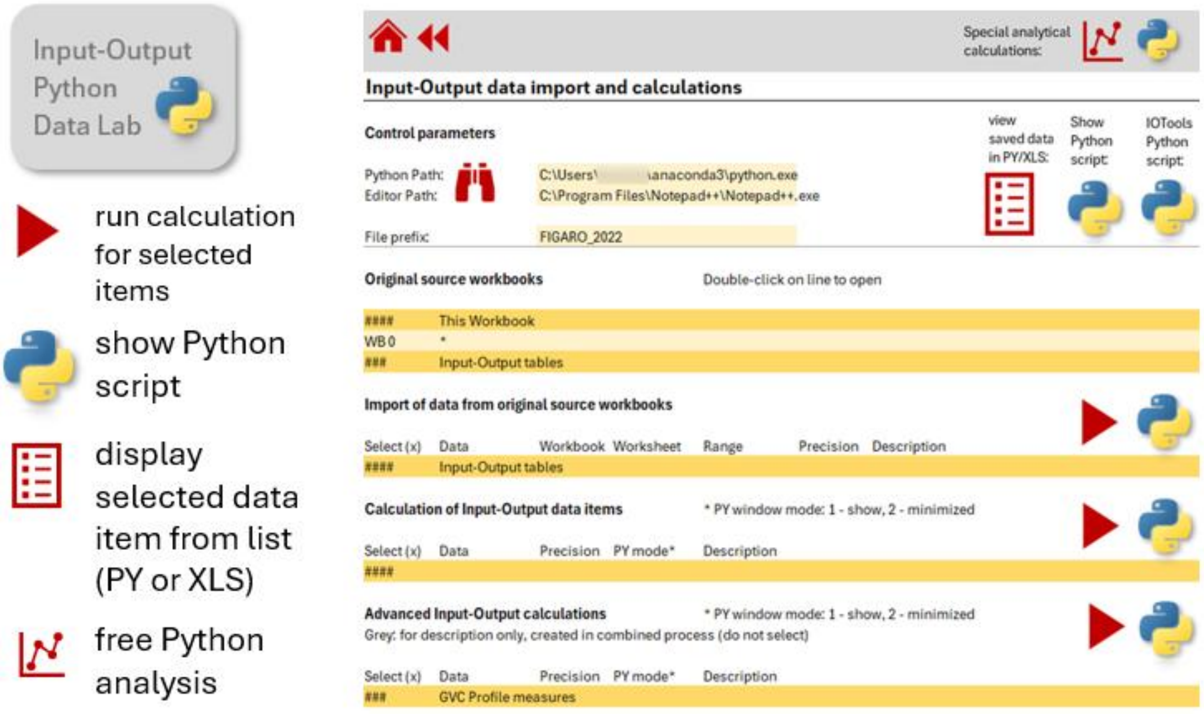
As a text editor, a program with syntax-highlighting like "NotePad++" is recommended.

Microsoft recently released an Excel/Python Add-In. It allows to enter Python scripts in cells of a spreadsheet. The light version of this Add-In is free to use. For professional use with more complex scripts and, first of all, the necessary control of the sequence of calculations, a separate subscription is required. Therefore, that was not an option.

Nevertheless, the Microsoft Excel/Python Add-In is used by the author for the demonstration of certain Input-Output techniques with a Sample World Input-Output table (SWIOT) with 3 countries and 3 products (see Appendix B).

The whole process of data preparation, starting with the download of original source files from data providers, is also included in the workbook.

Figure 36 *- The Python Data Lab, part of the application workbook*



Input-Output Python Data Lab

run calculation for selected items

show Python script

display selected data item from list (PY or XLS)

free Python analysis

Special analytical calculations:

Input-Output data import and calculations

Control parameters

view saved data in PY/XLS: | Show Python script: | IOTools Python script:

Python Path: C:\Users\[redacted]\anaconda3\python.exe
Editor Path: C:\Program Files\Notepad++\Notepad++.exe

File prefix: FIGARO_2022

Original source workbooks Double-click on line to open

####	This Workbook
WB 0	*
###	Input-Output tables

Import of data from original source workbooks

Select (x)	Data	Workbook	Worksheet	Range	Precision	Description
####	Input-Output tables					

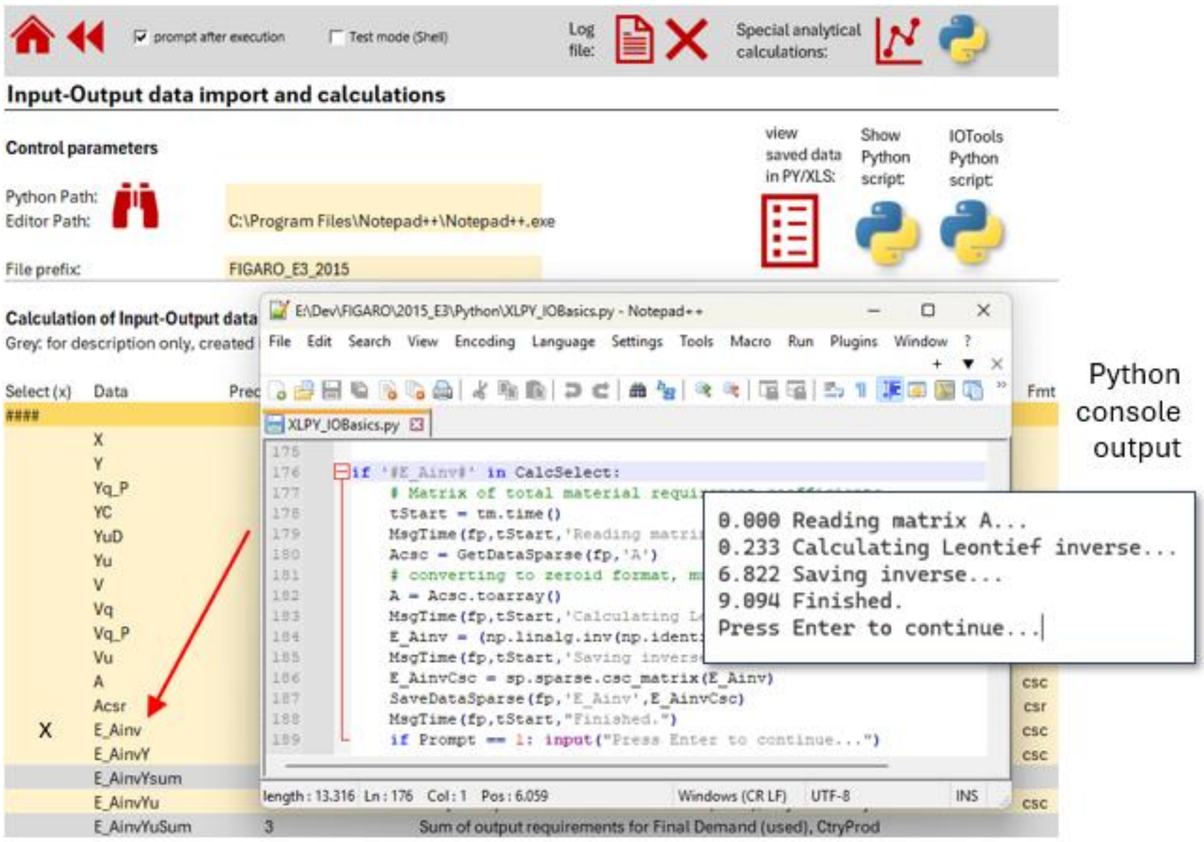
Calculation of Input-Output data items * PY window mode: 1 - show, 2 - minimized

Select (x)	Data	Precision	PY mode*	Description
####				

Advanced Input-Output calculations * PY window mode: 1 - show, 2 - minimized
Grey: for description only, created in combined process (do not select)

Select (x)	Data	Precision	PY mode*	Description
###	GVC Profile measures			

Figure 37 *- Running the calculation of the Leontief inverse from the Data Lab*



☑ prompt after execution ☐ Test mode (Shell) Log file: Special analytical calculations:

Input-Output data import and calculations

Control parameters

view saved data in PY/XLS: | Show Python script: | IOTools Python script:

Python Path:
Editor Path: C:\Program Files\Notepad++\Notepad++.exe

File prefix: FIGARO_E3_2015

Calculation of Input-Output data
Grey: for description only, created

Select (x) Data Prec Fmt

####
X
Y
Yq_P
YC
YuD
Yu
V
Vq
Vq_P
Vu
A
Acsr csc
X E_Ainv csr
E_AinvY csc
E_AinvYsum csc
E_AinvYu csc
E_AinvYuSum 3 Sum of output requirements for Final Demand (used), CtryProd

E:\Dev\FIGARO\2015_E3\Python\XLPY_IOBasics.py - Notepad++

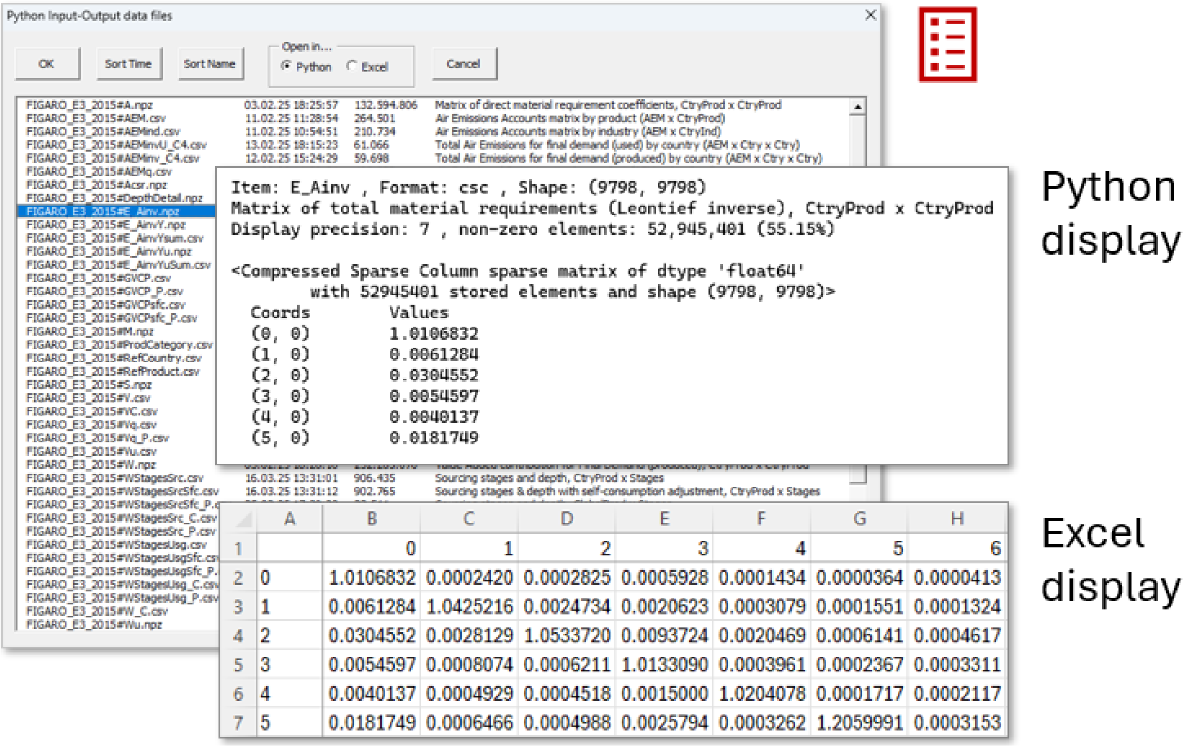
```
if '#E_Ainv#' in CalcSelect:
    # Matrix of total material requi
    tStart = tm.time()
    MsgTime(fp,tStart,'Reading matri
    Acsc = GetDataSparse(fp,'A')
    # converting to zeroid format, m
    A = Acsc.toarray()
    MsgTime(fp,tStart,'Calculating Le
    E_Ainv = (np.linalg.inv(np.identi
    MsgTime(fp,tStart,'Saving inverse
    E_AinvCsc = sp.sparse.csc_matrix(E_Ainv)
    SaveDataSparse(fp,'E_Ainv',E_AinvCsc)
    MsgTime(fp,tStart,"Finished.")
    if Prompt == 1: input("Press Enter to continue...")
```

length : 13.316 Ln : 176 Col : 1 Pos : 6.059 Windows (CR LF) UTF-8 INS

Python console output

```
0.000 Reading matrix A...
0.233 Calculating Leontief inverse...
6.822 Saving inverse...
9.094 Finished.
Press Enter to continue...
```

***Figure 38** - Input-Output File-Explorer, part of the Python Data Lab*



FIGARO_E3_2015#A.npz	03.02.25 18:25:57	132.594.806	Matrix of direct material requirement coefficients, CtryProd x CtryProd
FIGARO_E3_2015#AEM.csv	11.02.25 11:28:54	264.501	Air Emissions Accounts matrix by product (AEM x CtryProd)
FIGARO_E3_2015#AEMind.csv	11.02.25 10:54:51	210.734	Air Emissions Accounts matrix by industry (AEM x CtryInd)
FIGARO_E3_2015#AEMinvU_C4.csv	13.02.25 18:15:23	61.066	Total Air Emissions for final demand (used) by country (AEM x Ctry x Ctry)
FIGARO_E3_2015#AEMinv_C4.csv	12.02.25 15:24:29	59.698	Total Air Emissions for final demand (produced) by country (AEM x Ctry x Ctry)
FIGARO_E3_2015#AEMq.csv			
FIGARO_E3_2015#Acsr.npz			
FIGARO_E3_2015#DepthDetail.npz			
FIGARO_E3_2015#E_Ainv.npz			
FIGARO_E3_2015#E_AinvY.npz			
FIGARO_E3_2015#E_AinvYsum.csv			
FIGARO_E3_2015#E_AinvYu.npz			
FIGARO_E3_2015#E_AinvYuSum.csv			
FIGARO_E3_2015#GVCP.csv			
FIGARO_E3_2015#GVCP_P.csv			
FIGARO_E3_2015#GVCPsfc.csv			
FIGARO_E3_2015#GVCPsfc_P.csv			
FIGARO_E3_2015#M.npz			
FIGARO_E3_2015#ProdCategory.csv			
FIGARO_E3_2015#RefCountry.csv			
FIGARO_E3_2015#RefProduct.csv			
FIGARO_E3_2015#S.npz			
FIGARO_E3_2015#V.csv			
FIGARO_E3_2015#VC.csv			
FIGARO_E3_2015#Vq.csv			
FIGARO_E3_2015#Vq_P.csv			
FIGARO_E3_2015#Vu.csv			
FIGARO_E3_2015#W.npz			
FIGARO_E3_2015#WStagesSrc.csv	16.03.25 13:31:01	906.435	Sourcing stages and depth, CtryProd x Stages
FIGARO_E3_2015#WStagesSrcSfc.csv	16.03.25 13:31:12	902.765	Sourcing stages & depth with self-consumption adjustment, CtryProd x Stages
FIGARO_E3_2015#WStagesSrcSfc_P.			
FIGARO_E3_2015#WStagesSrc_C.csv			
FIGARO_E3_2015#WStagesSrc_P.csv			
FIGARO_E3_2015#WStagesUsg.csv			
FIGARO_E3_2015#WStagesUsgSfc.cs			
FIGARO_E3_2015#WStagesUsgSfc_P.			
FIGARO_E3_2015#WStagesUsg_C.csv			
FIGARO_E3_2015#WStagesUsg_P.csv			
FIGARO_E3_2015#W_C.csv			
FIGARO_E3_2015#Wu.npz			

```
Item: E_Ainv , Format: csc , Shape: (9798, 9798)
Matrix of total material requirements (Leontief inverse), CtryProd x CtryProd
Display precision: 7 , non-zero elements: 52,945,401 (55.15%)

<Compressed Sparse Column sparse matrix of dtype 'float64'
        with 52945401 stored elements and shape (9798, 9798)>
  Coords        Values
  (0, 0)        1.0106832
  (1, 0)        0.0061284
  (2, 0)        0.0304552
  (3, 0)        0.0054597
  (4, 0)        0.0040137
  (5, 0)        0.0181749
```

	A	B	C	D	E	F	G	H
1		0	1	2	3	4	5	6
2	0	1.0106832	0.0002420	0.0002825	0.0005928	0.0001434	0.0000364	0.0000413
3	1	0.0061284	1.0425216	0.0024734	0.0020623	0.0003079	0.0001551	0.0001324
4	2	0.0304552	0.0028129	1.0533720	0.0093724	0.0020469	0.0006141	0.0004617
5	3	0.0054597	0.0008074	0.0006211	1.0133090	0.0003961	0.0002367	0.0003311
6	4	0.0040137	0.0004929	0.0004518	0.0015000	1.0204078	0.0001717	0.0002117
7	5	0.0181749	0.0006466	0.0004988	0.0025794	0.0003262	1.2059991	0.0003153

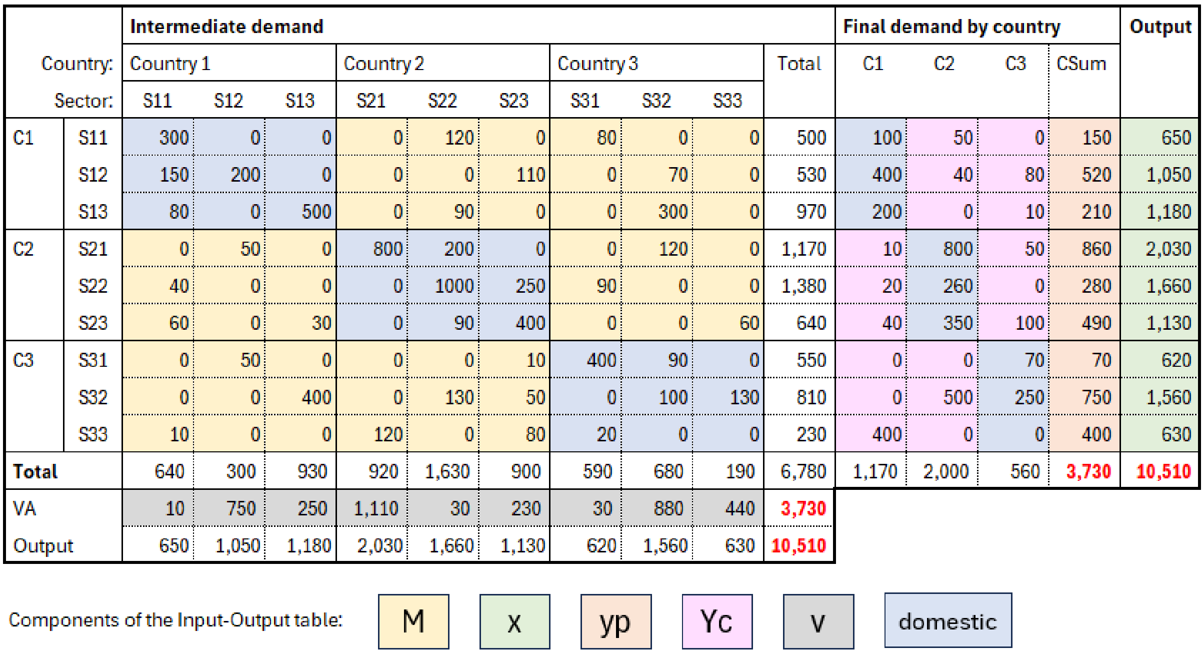
The process of data preparation can look a bit different depending on the data source. For the US BEA table, e.g., the BEA web site in the past offered the total and domestic Leontief inverses, which had to be reinverted first to obtain the matrices of direct material requirements. For another Input-Output database, some data has to be converted from MATLAB "mat" files using the respective Python libraries.

The Excel/Python interface with the Python Data Lab has proven to be very efficient. A new database can be integrated within a few days, if it follows the standard structure of an MRIO or of a national table with distinction of domestic and import material requirements.

Appendix B - A Sample World Input-Output Table

For demonstration of different algorithms, a Sample World Input-Output Table with 3 countries and 3 products was created. It follows the standard MRIO structure like WIOD, FIGARO, OECD-ICIO or EXIOBASE:

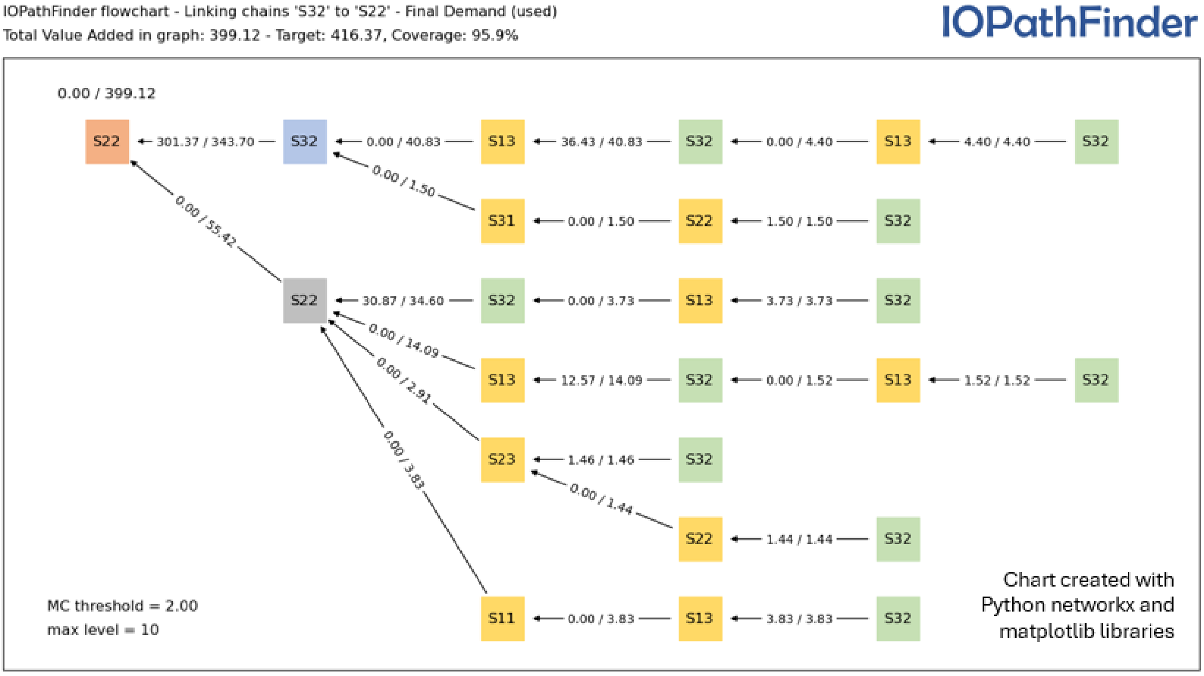
***Figure 39** - A Sample World Input-Output Table*



		Intermediate demand										Final demand by country				Output
	Country:	Country 1			Country 2			Country 3			Total	C1	C2	C3	CSum	
	Sector:	S11	S12	S13	S21	S22	S23	S31	S32	S33						
C1	S11	300	0	0	0	120	0	80	0	0	500	100	50	0	150	650
	S12	150	200	0	0	0	110	0	70	0	530	400	40	80	520	1,050
	S13	80	0	500	0	90	0	0	300	0	970	200	0	10	210	1,180
C2	S21	0	50	0	800	200	0	0	120	0	1,170	10	800	50	860	2,030
	S22	40	0	0	0	1000	250	90	0	0	1,380	20	260	0	280	1,660
	S23	60	0	30	0	90	400	0	0	60	640	40	350	100	490	1,130
C3	S31	0	50	0	0	0	10	400	90	0	550	0	0	70	70	620
	S32	0	0	400	0	130	50	0	100	130	810	0	500	250	750	1,560
	S33	10	0	0	120	0	80	20	0	0	230	400	0	0	400	630
Total		640	300	930	920	1,630	900	590	680	190	6,780	1,170	2,000	560	**3,730**	**10,510**
VA		10	750	250	1,110	30	230	30	880	440	**3,730**					
Output		650	1,050	1,180	2,030	1,660	1,130	620	1,560	630	**10,510**					

Components of the Input-Output table: M x yp Yc v domestic

***Figure 40** - Find Linking Chains with the Sample World Input-Output Table*



Appendix C - VA contribution in a national Input-Output table

This Appendix provides the formulas that underlie the tables related to the US model in section 2.4.

In many standard national Input-Output tables, the matrix of direct material requirement coefficients is split into a domestic and an import part:

$$x = Ax + y = Ax + y_p + y_i - i \qquad (42)$$

$$x + i = (A_d + A_i)x + y_p + y_i \qquad (43)$$

Where

x	Vector of output by commodity
y	Vector of final demand by commodity
y_p	Vector of produced final demand by commodity
y_i	Vector of imported final demand by commodity
i	Vector of total import by commodity

Matrices of direct material requirement coefficients:

A	total (domestic and import)
A_d	domestic (produced)
A_i	import requirements

Split of equation (2) into the domestic (produced) and the imported part:

$$x = A_d x + y_p \qquad (44)$$

$$i = A_i x + y_i \qquad (45)$$

Leontief inverse and Value Added contribution matrix for produced final demand:

$$(I - A_d)x = y_p \qquad (46)$$

$$x = (I - A_d)^{-1} y_p \qquad (47)$$

$$\boxed{W_d = \hat{v}(I - A_d)^{-1}\hat{y}_p} \qquad (48)$$

Where

$\hat{v}$	diagonal matrix of value added coefficients

Import requirements for final demand of each commodity:

$$\boxed{\begin{aligned} i &= A_i(I - A_d)^{-1} y_p + y_i \\ W_i &= A_i(I - A_d)^{-1} y_p \end{aligned}} \qquad \begin{aligned} &(49) \\ &(50) \end{aligned}$$

About the author

The author has been involved with Input-Output Analysis for more then four decades.

After graduating in Mathematical Economics and Computer Sciences, he worked for ten years on a detailed Input-Output planning model. During this time, the author contributed to linear programming and decomposition approaches with this model and developed “Product Chain Analysis”, an early predecessor of the approach presented in this paper.

Afterwards, the author created an ownership network analysis and display framework based on IO concepts which was used by the German Privatization agency with more than 8500 affiliates in its daily operations. Analysis of ownership and control was also the subject of several publicly funded research projects and a commercial database of German and international business groups.

He was responsible for corporate reporting, budgeting and Business Intelligence in three international groups of companies, using IO techniques for value chain management, group margins and group ownership structures.

After retirement, the author resumed Input-Output research activities and applied his visual decomposition approach to a number of MRIO and national IO tables.

Disclaimer

The author bears the sole responsibility for all statements, reports, methods of calculation and definitions of indicators presented in this document unless otherwise stated.

Multiregional and national Input-Output tables used as the database for this presentation are copyright of the publishing organizations referred to in the different sections (JRC of the EC, eurostat, OECD, US BEA, La Société Nouvelle).

The publication of this paper does not imply endorsement by any of the organizations that published this underlying data.

Images with no source specified are created by and copyright of the author.

The software application "IOPathFinder" for Visual Network Decomposition was developed by the author independently, without any external assistance.