32nd INTERNATIONAL INPUT-OUTPUT CONFERENCE

Global value chains under full employment: An input-output linear optimization

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Introduction: The making of GVCs in the European Union

Industry-level efficiency (status quo):

- **Focus**: Cost-minimization process and labor market arbitrage
- **Mechanism**: Free capital mobility drives production to regions with higher absolute advantages (mainly based on unit labor cost differentials, exchange rates, material cost differentials, etc.)
- **Macro-outcome**: EU policy failures – deindustrialization, unbalanced unemployment, wage deflation, reliance on offshoring

Macroeconomic equilibrium (Pasinetti-Leontief framework):

- **Focus**: Societal well-being over international competitiveness (profits must be bound to social sustainability)
- **Mechanism**: Public planning of investment drives sustained employment of labor force
- **Macro-outcome**: Full employment within each country (preventing exclusion and forced cross-border migration)

What this discussion wants to highlight:

- The restructuring of GVCs in **intermediates** under alternative LP problems through the **multi-regional rectangular choice of techniques (MRIO-based RCOT) framework**, where industries choses the geographic distribution of supply chains

Methodology: MRIO-based RCOT framework

- We have three countries (center, semi-periphery, periphery) and two commodities (1 and 2).
- The objective of the analysis is to determine the **optimal structure of trade in intermediates** within the MRIO matrix, given the techniques into use:

$$\mathbf{A}_{(n\times n)} = \begin{bmatrix} \mathbf{A}_{cc} & \mathbf{A}_{cs} & \mathbf{A}_{cp} \\ \mathbf{A}_{sc} & \mathbf{A}_{ss} & \mathbf{A}_{sp} \\ \mathbf{A}_{pc} & \mathbf{A}_{ps} & \mathbf{A}_{pp} \end{bmatrix}, \qquad \text{where } \mathbf{A}_{rb} = \bar{\mathbf{A}}$$

- We must obtain the **network of trade flows** along with the **quantities**, through alternative LP problems under constraints of final demand and available resources
- The MRIO vector of nominal hourly wages is:

$$\mathbf{w}^{\mathbf{T}}_{(1\times n)} = [\mathbf{w}^{\mathbf{T}}_{\boldsymbol{c}(1\times m)} \quad \mathbf{w}^{\mathbf{T}}_{\boldsymbol{s}(1\times m)} \quad \mathbf{w}^{\mathbf{T}}_{\boldsymbol{p}(1\times m)}]$$

- The MRIO vector of direct labor coefficients is:

$$\boldsymbol{\eta}^{\mathbf{T}}_{(1\times n)} = [\boldsymbol{\eta}^{\mathbf{T}}_{\boldsymbol{c}(1\times m)} \quad \boldsymbol{\eta}^{\mathbf{T}}_{\boldsymbol{s}(1\times m)} \quad \boldsymbol{\eta}^{\mathbf{T}}_{\boldsymbol{p}(1\times m)}]$$

- **Hypothesis: moving from the center to the periphery, unit labor costs decrease**

Augmented Matrices and Vectors (1/2)

- The computation requires that all the available alternatives are displayed in the initial data. Hence, the MRIO matrix and the factor requirement matrix have to be augmented in order to include all the options
- The augmented MRIO "sub-matrix" for product i produced in country r is:

$$\mathbf{A}^*_{\boldsymbol{ir}(n\times k)} = \begin{bmatrix} a_{11} & a_{11} & a_{11} & 0 & 0 & 0 & 0 & 0 & 0 \\ a_{21} & 0 & 0 & a_{21} & 0 & 0 & a_{21} & 0 & 0 \\ 0 & 0 & 0 & a_{11} & a_{11} & a_{11} & 0 & 0 & 0 \\ 0 & a_{21} & 0 & 0 & a_{21} & 0 & 0 & a_{21} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & a_{11} & a_{11} & a_{11} \\ 0 & 0 & a_{21} & 0 & 0 & a_{21} & 0 & 0 & a_{21} \end{bmatrix}$$

- Calling $\mathbf{F}_{(n\times n)} = \widehat{\boldsymbol{\eta}_{(1\times n)}}$, the augmented factor requirement matrix is:

$$\mathbf{F}^*_{[n\times(k\times n)]} = \begin{bmatrix} \eta_1^c & \eta_1^c & \cdots & \eta_1^c & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & \eta_2^c & \eta_2^c & \cdots & \eta_2^c & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \ddots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \ddots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & \eta_2^p & \eta_2^p & \cdots & \eta_2^p \end{bmatrix}$$

- Labor is not homogenous, we have as many different types of labor as there are sectors.

Augmented Matrices and Vectors (2/2)

- The augmented vector of gross quantities resulting from the linear optimization is:

$$\mathbf{q}^*_{[(k\times n)\times 1]} = \begin{bmatrix} q_1^{c(1)} \\ \vdots \\ q_1^{c(9)} \\ q_2^{c(1)} \\ \vdots \\ q_2^{c(9)} \\ \vdots \\ q_2^{p(1)} \\ \vdots \\ q_2^{p(9)} \end{bmatrix}$$

where $q_i^{r(z)}$ is the gross quantity of commodity i produced in country r under the $z\text{th}$ alternative (out of k). The matrices and vector are augmented such that each $n\text{th}$ sector has k alternatives to choose from, hence $k \times n = 9 \times 6 = 54$

- By eliminating the **inactive options**, we can transform the rectangular MRIO matrix ($\mathbf{A}^*$) into the optimal square matrix ($\mathbf{A}$) and the augmented quantity vector ($\mathbf{q}^*$) into the optimal quantity vector ($\mathbf{q}$).

IO-Linear Programming Problems

- Problem (1): cost-minimization under final demand constraints:

$$\begin{aligned} \underset{\mathbf{q}^*}{\text{minimize}}\, W &= \mathbf{w}^{\mathbf{T}}_{(1\times n)}\mathbf{F}^*_{[n\times(k\times n)]}\mathbf{q}^*_{[(k\times n)\times 1]} \\ \text{subject to } & \left[\mathbf{I}^*_{[n\times(k\times n)]} - \mathbf{A}^*_{[n\times(k\times n)]}\right]\mathbf{q}^*_{[(k\times n)\times 1]} \geq \bar{\mathbf{y}}_{(n\times 1)} \end{aligned}$$

- Problem (2): income-maximization under multi-regional full employment constraints

$$\begin{aligned} \underset{\mathbf{q}^*}{\text{maximize}}\, V &= \mathbf{w}^{\mathbf{T}}_{(1\times n)}\mathbf{F}^*_{[n\times(k\times n)]}\mathbf{M}_{[(k\times n)\times n]}\left[\mathbf{I}^*_{[n\times(k\times n)]} - \mathbf{A}^*_{[n\times(k\times n)]}\right]\mathbf{q}^*_{[(k\times n)\times 1]} \\ \text{subject to } & \left[\mathbf{I}^*_{[n\times(k\times n)]} - \mathbf{A}^*_{[n\times(k\times n)]}\right]\mathbf{q}^*_{[(k\times n)\times 1]} \geq \bar{\mathbf{y}}_{(n\times 1)} \\ & \mathbf{E}_{(t\times n)}\mathbf{F}^*_{[n\times(k\times n)]}\mathbf{q}^*_{[(k\times n)\times 1]} = \mathbf{Q}_{\boldsymbol{N}(t\times 1)} \end{aligned}$$

Numerical Applications (1/2)

- Executing model (1) yields the following results, namely the trade structure, total quantities, final demand and employment that allow for the realization of the minimum production cost:

$$\mathbf{Aq} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.4 & 0.3 & 0.4 & 0.3 & 0.4 & 0.3 \\ 0.2 & 0.1 & 0.2 & 0.1 & 0.2 & 0.1 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 550 \\ 300 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 450 \\ 200 \end{bmatrix}, \qquad \mathbf{L} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0.87 & 0.62 & 0.87 & 0.62 & 1.87 & 0.62 \\ 0.41 & 0.25 & 0.41 & 0.25 & 0.41 & 1.25 \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix},$$

$$\mathbf{EFq} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 100 \\ 100 \\ 100 \\ 100 \\ 550 \\ 300 \end{bmatrix} = \begin{bmatrix} 900 \\ 1100 \\ 5400 \end{bmatrix}, \qquad \mathbf{Q}_{\boldsymbol{N}} = \begin{bmatrix} Q_N^c \\ Q_N^s \\ Q_N^p \end{bmatrix} = \begin{bmatrix} 3500 \\ 4500 \\ 5500 \end{bmatrix}$$

Results from Problem (1)

- The choice of suppliers falls entirely on the producers in the periphery
- The central and semi-peripheral economies experience **unemployment**
- The periphery falls into **economic dependence** on exports and hence on FDI decisions made by multinationals from other countries
- The trade structure can only change in response to a **shift in the labor cost** structure.
  This would cause **pressure on wage bargaining** both in center/semi-periphery and the periphery
- The optimal structure of international trade based on autonomous industry-level decisions proves to be **suboptimal** when considering the conditions of the labor market

Numerical Applications (2/2)

- Executing model (2) yields the following results that allow for the realization of the maximum global income constrained to the achievement of multi-regional full employment of labor force:

$$\mathbf{Aq} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.15 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.15 & 0.3 & 0.4 & 0.3 & 0.4 & 0.15 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0.25 & 0 & 0 & 0 & 0 & 0 \\ 0.2 & 0.1 & 0.2 & 0.1 & 0.2 & 0.1 \end{bmatrix} \begin{bmatrix} 750 \\ 100 \\ 780 \\ 100 \\ 289.41 \\ 537.64 \end{bmatrix} = \begin{bmatrix} 79.64 \\ 0 \\ 680 \\ 0 \\ 189.41 \\ 437.64 \end{bmatrix}, \qquad \mathbf{L} = \begin{bmatrix} 1.06 & 0.03 & 0.06 & 0.03 & 0.06 & 0.18 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0.54 & 0.57 & 1.79 & 0.57 & 0.79 & 0.39 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0.26 & 0.01 & 0.01 & 0.01 & 1.01 & 0.04 \\ 0.41 & 0.25 & 0.41 & 0.25 & 0.41 & 1.25 \end{bmatrix}$$

$$\mathbf{y} = \begin{bmatrix} 670.35 \\ 100 \\ 100 \\ 100 \\ 100 \\ 100 \end{bmatrix}$$

$$\mathbf{EFq} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} 4 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 \end{bmatrix} \begin{bmatrix} 750 \\ 100 \\ 780 \\ 100 \\ 289.41 \\ 537.64 \end{bmatrix} = \begin{bmatrix} 3500 \\ 4500 \\ 5500 \end{bmatrix}, \qquad \mathbf{Q_N} = \begin{bmatrix} Q_N^c \\ Q_N^s \\ Q_N^p \end{bmatrix} = \begin{bmatrix} 3500 \\ 4500 \\ 5500 \end{bmatrix}$$

Results from Problem (2)

- Unemployment is not only addressed at the global level but it is minimized within each singular country.
- An **increase in final demand** beyond the initially expected level might be necessary. The model provides the **volume and composition** of the additional final demand required to meet all policy constraints
- The framework allows for determining the **distribution of supply along GVCs** and, consequently, the degree of specialization within each economy. Indeed, industries are not completely left to operate within a specific area of expertise, but the supply of intermediates can be coordinated among industries located in different countries
- No worker remains unemployed or is forced to migrate from one country to another

Concluding Remarks

- This is a **simple exercise** that shows one of the many possibilities of planning GVCs at the supranational level. However, these results may prompt reflection on the nature of the European Union’s policies
- EU witnessed an economic **integration model** based on **capital mobility** and **labor market flexibility**. The combination of these two elements has led to the **offshoring** of certain stages of GVCs to peripheral areas (Germany to Slovakia in automotive, Italy to Poland in automotive and home appliances, etc.), which has resulted in a process of **wage stagnation** across the region.
  This was due both to rising **unemployment** caused by deindustrialization and to the supply of products imported at **lower prices**
- The need for a decisive **shift in EU economic policies** emerges, one that can drive the region towards integration based on **coordination** rather than market competition among countries
- This paper is an attempt to move in that direction, placing **full employment as the primary objective**. Increasing competitiveness through wage moderation must be rejected. Instead, a “higher cost” with the objective of social and economic recovery and integration must be supported

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