

Spectral Structure and the Near-Linearity of Wage–Profit Curves: Evidence from U.S. Input–Output Accounts

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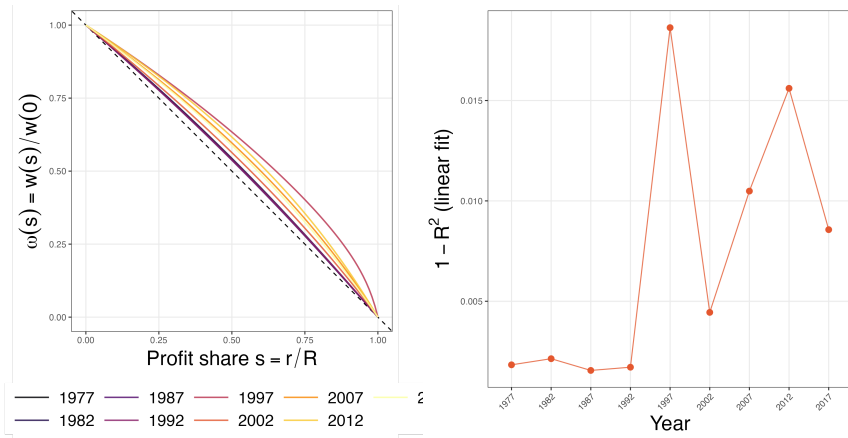
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Introduction I

The motivations of this work:

- Input-Output (IO) accounts estimations of the Wage-Profit Frontier (WPF) are nearly linear (see [Shaikh, 2016](#), ch. 9; [Mariolis and Tsoulfidis, 2018](#), ch. 3; and [Tsoulfidis, 2021](#), ch. 3–4).
- This is a seemingly paradoxical result is at odds with the theoretical results of the *Cambridge capital controversies* of the 1960's and 70's (see [Harcourt, 1972](#); [Sen, 1974](#); [Pasinetti, 1977](#), ch. V; and [Kurz, 1990](#)).
- This paper contributes to this literature by providing a **theoretical and empirical characterization** of the productive-structure restrictions underlying nearly linear WPF's.

Introduction: The Stylized Fact



Normalised wage-profit curves $\omega(s) = w(s)/w(0)$ against the profit share $s = r/R$ (left), and the deviation from a linear fit $1 - R^2$ by year (right) (1977–2017). Own elaboration with data from [BEA \(2022, 2025\)](#).

Introduction II

We evaluate these restrictions using U.S. Benchmark IO accounts (1977–2017, quinquennially) with disintegration levels ranging from 399 to 528 sectors (BEA, 2022, 2025). **We identify three structural regularities:**

- ① A close alignment between the vertically integrated labour coefficient vector ($\mathbf{v}^T$) and the left-hand Perron–Frobenius (PF) eigenvector ($\mathbf{q}_L$) of matrix $\mathbf{A}$.
- ② A close alignment between the net output ($\mathbf{y}$) and the right-hand PF eigenvector ($\mathbf{q}_R$) of matrix $\mathbf{A}$.
- ③ With $\mathbf{A} = \mathbf{Q}\mathbf{\Lambda}\mathbf{Q}^{-1}$, a clustering around zero of:
 - (i) The subdominant spectrum of $\mathbf{A}$;
 - (ii) *Eigenlabours* $\boldsymbol{\alpha}^T = \mathbf{v}^T \mathbf{Q}$;
 - (iii) and *eigenoutputs* $\boldsymbol{\chi} = \mathbf{Q}^{-1} \mathbf{q}$.

Assumptions

Consider a Sraffian price model of an economy producing $n \in \mathbb{N}$ commodities by means of commodities (see [Pasinetti, 1977](#), ch. V). The assumptions are:

- ($\mathfrak{A}_1$) Long-period reproducible positions ([Kurz & Salvadori, 1995](#); [Petri, 2004](#));
- ($\mathfrak{A}_2$) no joint production, no fixed capital, wages are above subsistence and not advanced;
- ($\mathfrak{A}_3$) one given technique of production and social output;
- ($\mathfrak{A}_4$) $\mathbf{l} \in \mathbb{R}_{++}^n$ (labour is indispensable);
- ($\mathfrak{A}_5$) $\mathbf{A} \in \mathbb{R}_{+}^{n \times n}$ is primitive (hence irreducible, i.e., only *basic commodities*) and diagonalizable;
- ($\mathfrak{A}_6$) and $\rho(\mathbf{A}) = \lambda_{\mathbf{A},1} < 1$ (the system is viable, i.e., it is a *surplus economy*).

The Price System

We have the following system of equations:

$$\mathbf{x} = \mathbf{Ax} + \mathbf{y} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{y} \quad (1)$$

$$\mathbf{s} = \mathbf{Ax} \quad (2)$$

$$\mathbf{p}^T = (1 + r)\mathbf{p}^T \mathbf{A} + w\mathbf{l}^T = w\mathbf{l}^T [\mathbf{I} - (1 + r)\mathbf{A}]^{-1} \quad (3)$$

$$Y = \mathbf{p}^T \mathbf{y} = 1 \quad (4)$$

$$L = \mathbf{l}^T \mathbf{x} = 1 \quad (5)$$

$$r = r^* \in \left[0, R = \frac{1 - \lambda_{\mathbf{A},1}}{\lambda_{\mathbf{A},1}}\right) \quad (6)$$

With $\mathbf{x}, \mathbf{p}, \mathbf{l} \in \mathbb{R}_{++}^n$; $\mathbf{y}, \mathbf{s} \in \mathbb{R}_+^n$; and $\mathbf{A} \in \mathbb{R}_+^{n \times n}$. The net product ($\mathbf{y}$) is the *numéraire*, labour is normalized such that $w = \omega$.

Vertically Integrated Price, Wage and Capital Intensities

Consider $\mathbf{v}^T = \mathbf{I}^T(\mathbf{I} - \mathbf{A})^{-1} \in \mathbb{R}_{++}^{1 \times n}$ and $\mathbf{H} = \mathbf{A}(\mathbf{I} - \mathbf{A})^{-1} \in \mathbb{R}_{++}^{n \times n}$. Hence $R \equiv 1/\lambda_{\mathbf{H},1}$. For $r \in [0, R)$ we can write:

$$\mathbf{p}^T(r) = w\mathbf{v}^T(\mathbf{I} - r\mathbf{H})^{-1} = \frac{1}{\mathbf{v}^T(\mathbf{I} - r\mathbf{H})^{-1}\mathbf{y}}\mathbf{v}^T(\mathbf{I} - r\mathbf{H})^{-1} \quad (7)$$

$$w(r) = \frac{1}{\mathbf{v}^T(\mathbf{I} - r\mathbf{H})^{-1}\mathbf{y}} \quad (8)$$

$$K(r) = \mathbf{p}^T\mathbf{s} = \mathbf{p}^T\mathbf{A}\mathbf{x} = \mathbf{p}^T\mathbf{H}\mathbf{y} \quad (9)$$

Note that, for $r \in [0, R)$ matrix $\mathbf{I} - r\mathbf{H}$ is a non-singular $\mathcal{M}$ -matrix, with $(\mathbf{I} - r\mathbf{H})^{-1} > \mathbf{O}$.

Prices, Wages and Capital as Power Series in r

For $r \in [0, R)$, the Neumann expansion of $(\mathbf{I} - r\mathbf{H})^{-1}$ makes every curve a **power series** in r :

$$w(r) = \frac{1}{\sum_{m=0}^{\infty} r^m b_m}, \quad \mathbf{p}^T(r) = w \sum_{m=0}^{\infty} r^m \mathbf{v}^T \mathbf{H}^m, \quad K(r) = w \sum_{m=0}^{\infty} r^m b_{m+1},$$

with coefficients $b_m = \mathbf{v}^T \mathbf{H}^m \mathbf{y}$.

- Generically **non-linear**: the wage curve may exhibit multiple inflections (“wiggles”), i.e., the source of the capital-theoretic paradoxes.
- The whole question reduces to the behaviour of the coefficients b_m .

Two Classical Routes to a Linear Wage Curve

Both special cases collapse the coefficients to a geometric sequence $b_m = \lambda_{H,1}^m$, yielding $w(r) = 1 - r/R$ and constant capital $K(r) = 1/R$:

	<i>Standard commodity</i>	<i>Standard labour</i>
Alignment	$\mathbf{y} \propto \mathbf{q}_R$	$\mathbf{v} \propto \mathbf{q}_L$
Meaning	equal s_i/y_i ratios	equal capital intensities k_j
Wage curve	$1 - r/R$	$1 - r/R$
Capital $K(r)$	$1/R$	$1/R$
Prices $\mathbf{p}^T(r)$	vary with r	<i>constant</i> ($= \mathbf{v}^T$)

Long dismissed as “flukes” ([Schefold, 1976](#)). We show they are the two *corner* cases of a single, more general condition.

Spectral Representation of the Curves

Diagonalise $\mathbf{A} = \mathbf{Q}\mathbf{\Lambda}_\mathbf{A}\mathbf{Q}^{-1}$ ($\mathbf{H} = \mathbf{Q}\mathbf{\Lambda}_\mathbf{H}\mathbf{Q}^{-1}$) and change to the eigenbasis of $\mathbf{A}$:

$$\text{eigenlabours: } \boldsymbol{\alpha}^T = \mathbf{v}^T \mathbf{Q}, \quad \text{eigenoutputs: } \boldsymbol{\chi} = \mathbf{Q}^{-1} \mathbf{y}.$$

The coefficients and the wage curve become

$$b_m = \underbrace{\lambda_{\mathbf{H},1}^m \alpha_1 \chi_1}_{\text{linear part}} + \sum_{k=2}^e \lambda_{\mathbf{H},k}^m \boldsymbol{\alpha}_k^T \boldsymbol{\chi}_k, \quad w(r) = \frac{1}{\sum_{k=1}^e \frac{\boldsymbol{\alpha}_k^T \boldsymbol{\chi}_k}{1 - r \lambda_{\mathbf{H},k}}}.$$

Where $e \leq n$ is the number of distinct eigenvalues of $\mathbf{A}$. Curvature is driven by the **subdominant** terms ($k \geq 2$), each weighted by the inner product $\sigma_k \equiv \boldsymbol{\alpha}_k^T \boldsymbol{\chi}_k$.

Spectral Characterisation of Linearity

Proposition

Under $\mathfrak{A}_1$ – $\mathfrak{A}_6$, the wage curve is exactly linear, $w(r) = 1 - r/R$, if and only if

$$\sigma_k = \boldsymbol{\alpha}_k^T \boldsymbol{\chi}_k = 0 \quad \forall k = 2, 3, \dots, e.$$

- Each σ_k vanishes in two ways: $\boldsymbol{\alpha}_k = \mathbf{0}$ (standard *labour*) or $\boldsymbol{\chi}_k = \mathbf{0}$ (standard *commodity*).
- $\Rightarrow$ these two situations are the *corner* cases of one orthogonality condition.
- Linearity may also arise from *cancellation* between non-zero $\boldsymbol{\alpha}_k$ and $\boldsymbol{\chi}_k$.

Near-Linearity: Three Structural Channels

Exact alignment is not expected empirically. Near-linearity follows when the σ_k (or their dynamic weight) are small. By Cauchy–Schwarz, $|\sigma_k| \leq \|\alpha_k\| \|\chi_k\|$:

(C1) Labour channel $\|\alpha_k\|$ small $\Leftrightarrow \mathbf{v}^T \approx \alpha_1 \mathbf{q}_{\mathcal{L}}$

(C2) Output channel $\|\chi_k\|$ small $\Leftrightarrow \mathbf{y} \approx \chi_1 \mathbf{q}_{\mathcal{R}}$

(C3) Spectral channel $|\lambda_{\mathbf{H},k}|$ small (large spectral gap, a property of $\mathbf{A}$ *alone*)

Any single channel $\Rightarrow$ small deviation from $1 - r/R$; several at once $\Rightarrow$ deviation small to second order. **The empirical section tests all three.**

Empirical Strategy and Data

Data: U.S. Benchmark Input–Output Accounts ([BEA, 2022, 2025](#)), 1977–2017 (quinquennial, 9 years), $n = 399$ to 528 sectors.

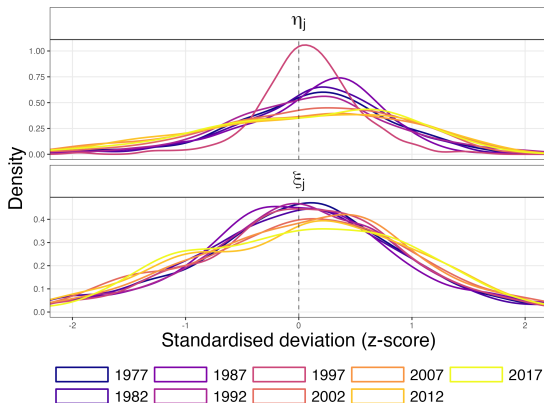
For each year we recover $\mathcal{T} = \{(\mathbf{A}, \mathbf{l}); \mathbf{x}\}$ and $\mathcal{V} = \{(\mathbf{H}, \mathbf{v}); \mathbf{y}\}$, and compute $\mathbf{A} = \mathbf{Q}\mathbf{\Lambda}_\mathbf{A}\mathbf{Q}^{-1}$ with $\boldsymbol{\alpha}^T = \mathbf{v}^T\mathbf{Q}$, $\boldsymbol{\chi} = \mathbf{Q}^{-1}\mathbf{y}$.

Deviations from exact alignment (standardised to z -scores per year):

$$\begin{aligned} \text{Labour (C1): } \boldsymbol{\eta}^T &= \mathbf{v}^T - \alpha_1 \mathbf{q}_\mathcal{L}, & \boldsymbol{\xi}^T &= \mathbf{v}^T \mathbf{H} - \lambda_{\mathbf{H},1} \mathbf{v}^T, \\ \text{Output (C2): } \boldsymbol{\mu} &= \mathbf{y} - \chi_1 \mathbf{q}_\mathcal{R}, & \boldsymbol{\zeta} &= \mathbf{H}\mathbf{y} - \lambda_{\mathbf{H},1} \mathbf{y}. \end{aligned}$$

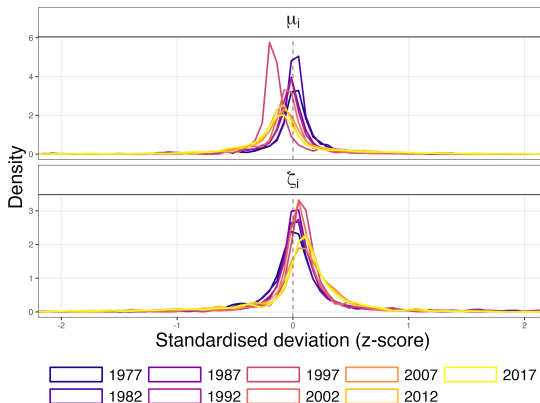
$\boldsymbol{\eta}, \boldsymbol{\mu}$ measure *geometry* (proximity to a P-F eigenray); $\boldsymbol{\xi}, \boldsymbol{\zeta}$ measure *dynamics* (being fixed by $\mathbf{H}$).

Channel C1: Alignment of $\mathbf{v} \propto \mathbf{q}_{\mathcal{L}}$



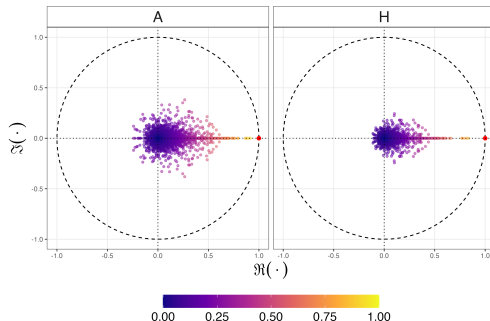
Kernel densities of standardised residuals, η_j and ξ_j (1977–2017). Own elaboration with data from [BEA \(2022, 2025\)](#).

Channel C2: Alignment of $y \propto q_{\mathcal{R}}$



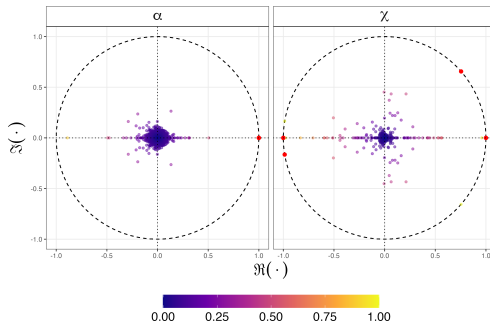
Kernel densities of standardised residuals, μ_i and ζ_i (1977–2017). Own elaboration with data from [BEA \(2022, 2025\)](#).

Channel C3: Subdominant Spectrum (i)



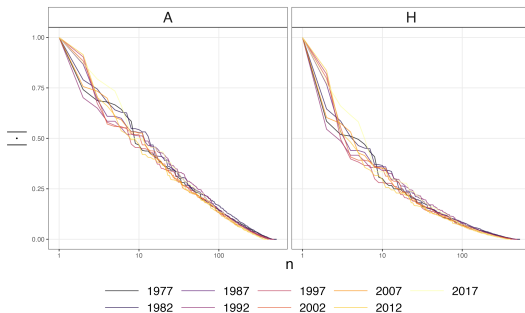
*Eigenvalues of **A** and **H**, pooled over all years (1977-2017) and rescaled to the unit disk. Red markers: dominant (P-F) entry. Own elaboration with data from [BEA \(2022, 2025\)](#).*

Channel C3: Subdominant Spectrum (ii)



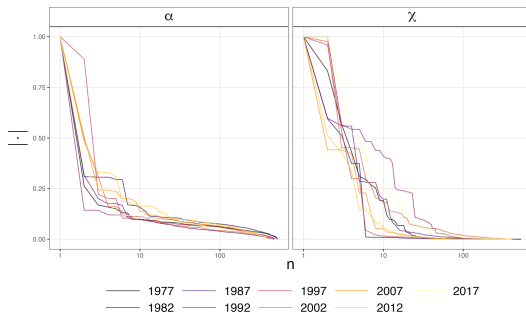
Eigenlaborers of α and eigenoutputs χ , pooled over all years (1977-2017) and rescaled to the unit disk. Red markers: dominant (P-F) entry. Own elaboration with data from [BEA \(2022, 2025\)](#).

Channel C3: Rank Decay of the Spectral Quantities (*i*)



Rank-ordered moduli of $|\lambda_{A,k}|$ and $|\lambda_{H,k}|$ (normalised so the top entry equals one), log rank axis (1977–2017). Own elaboration with data from [BEA \(2022, 2025\)](#).

Channel C3: Rank Decay of the Spectral Quantities (ii)



Rank-ordered moduli of $|\alpha_k|$ and $|\chi_k|$ (normalised so the top entry equals one), log rank axis (1977–2017). Own elaboration with data from [BEA \(2022, 2025\)](#).

What the Evidence Shows

- All three channels (C1)–(C3) are **active** and **stable** across four decades — no secular drift.
- **Asymmetry:** output residuals (μ, ζ) concentrate an order of magnitude more tightly than labour residuals (η, ξ) . The data sit closer to the *standard-commodity* case than to the *standard-labour* case.
- The inner products $\sigma_k = \alpha_k^T \chi_k$ are small in *two independent ways*: small factors *and* small eigenvalue weights $\lambda_{\mathbf{H},k}$.

Near-linearity of the U.S. wage curve is *overdetermined* by the productive structure.

Conclusions

- A unified **spectral framework**: the standard-labour ($\mathbf{v} \propto \mathbf{q}_{\mathcal{L}}$) and standard-commodity ($\mathbf{y} \propto \mathbf{q}_{\mathcal{R}}$) conditions are the two *corner cases* of a single condition, $\sigma_k = \boldsymbol{\alpha}_k^T \boldsymbol{\chi}_k = 0$.
- Near-linearity is governed by the size of the σ_k , decomposing into **three independent channels** (C1)–(C3).
- The U.S. technique activates **all three simultaneously** $\Rightarrow$ overdetermined, stable near-linearity (1977–2017).
- A novel **asymmetry**: the output (demand) side aligns more tightly than the labour side: the literature has focused on the labour side.
- Channel (C3) is a property of **A alone**, we argue that the reasons behind why interindustry structure yields such fast spectral decay is an open question.

Thank you! (¡Muchísimas gracias!)

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