Dependence of Direct Carbon Intensities on Input Coefficients in the Uncertainty Quantification of Carbon Footprints

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Research question

Is is necessary to incorporate the dependence of direct carbon intensities (**d**) on input coefficient matrix (**A**), when we consider the uncertainty of the IO model?

$$\mathbf{A} \longrightarrow \mathbf{d}$$

The answer is **YES**, otherwise the uncertainty of carbon footprint will be underestimated.

Uncertainty analysis of IO models in literature

- Analytical approach

Evans (1954); Simonovits (1975); Bullard and Sebald (1977); West (1986)

- Approach by Monte Carlo simulations (uncertainty quantification)
  - Parametric approach:
    - Distribution of input coefficients: normal (Nagashima, 2018); Beta (ten Raa & Steel, 1994); Dirichlet (Nakajima et al., 2013)
    - Distribution of entries of the IO table: normal (Dietzenbacher, 2006); lognormal(Lenzen et al., 2010)
  - Nonparametric approach: Rodrigues et al. (2018); Schulte et al. (2021)

EEIO model for the carbon footprint

$\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$

$\mathbf{I}$: Identity matrix

$\mathbf{L}$: Leontief inverse matrix

$\mathbf{A}$: input coefficient matrix

Carbon footprint C of final demand $\mathbf{f}$ is:

$$C = \mathbf{dLf} \text{ (EEIO model)}$$

$\mathbf{d}$: direct emission intensity

Wassily Leontief (1905-1999)

Uncertainty quantification of the EEIO model

- Entries of the IO follow indep. lognormal dist. (Lenzen et al., 2010)

$$\mathbf{d}^{dep} = \varphi(\mathbf{A}) = \mathbf{s}(\mathbf{R}^1 \odot \mathbf{R}^2 \odot F(\mathbf{A}))$$

- $\mathbf{s}$: summation vector
- $\mathbf{R}^1$: CO_2 emission factor
- $\mathbf{R}^2$: share of fuel used as energy
- F: Extract fossil fuel related intermediate input coefficients

Embodied emission intensity

- Compare

$$\mathbf{e}^{dep} = \mathbf{d}^{dep}(\mathbf{I} - \mathbf{A})^{-1} \text{ vs. } \mathbf{e}^{indep} = \mathbf{d}^{indep}(\mathbf{I} - \mathbf{A})^{-1}$$

- $d_j^{dep} \overset{d}{=} d_j^{indep},\ j=1,2,\ldots,n$
- $\mathbf{d}^{dep} = \varphi(\mathbf{A}),\ \mathbf{d}^{indep} \perp \mathbf{A}$
- Data: Japan 2015 national IO table, 509 sector; 3EID for φ
- # of random variables: 67656 IO entries, 503 direct emission intensities
- # of iterations of the Monte Carlo simulation: 10000

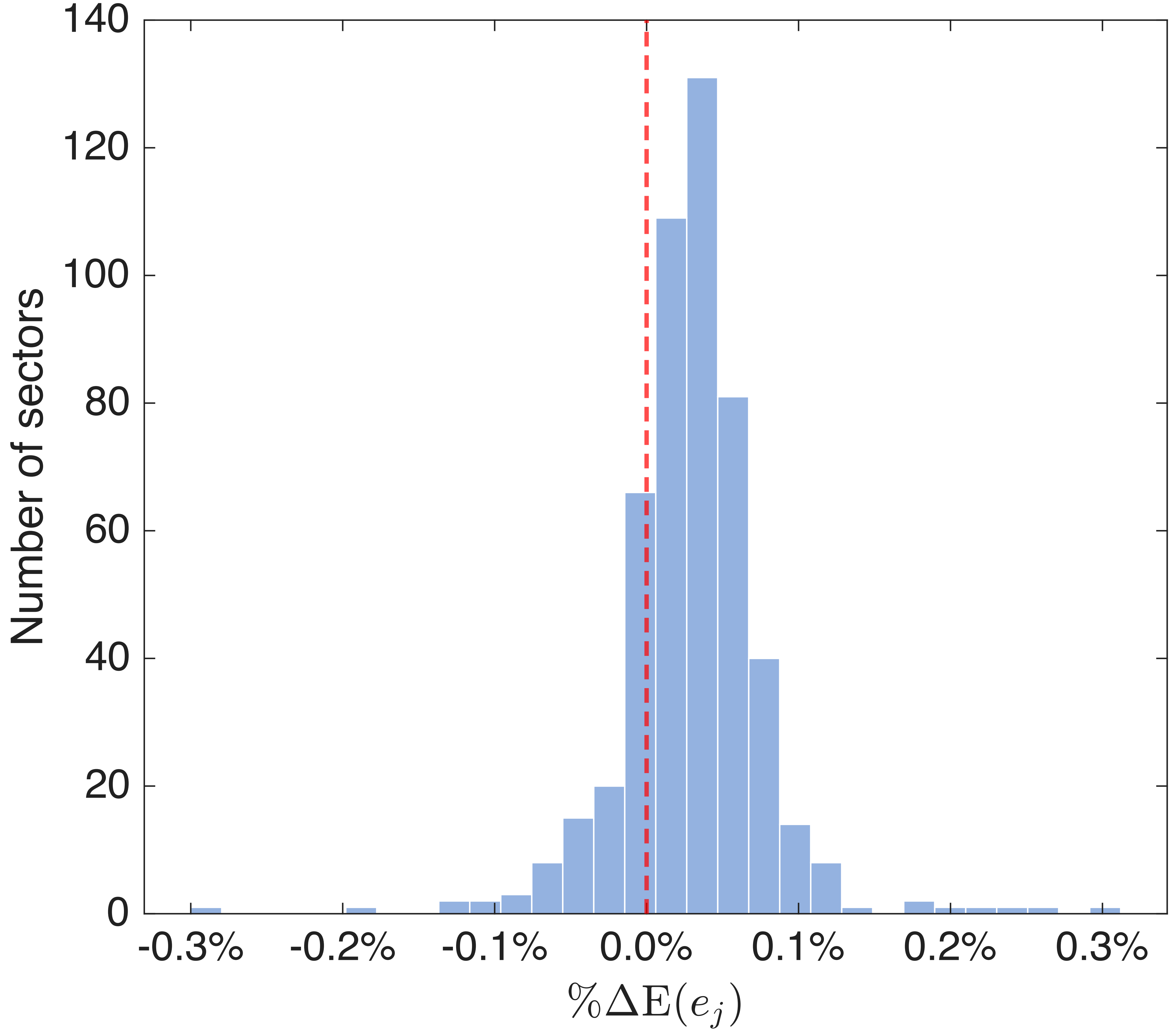
Uncertainty of a carbon footprint

- Carbon footprint as weighted sum of embodied intensities:

$$C = \mathbf{ef} = e_1 f_1 + e_2 f_2 + \cdots + e_n f_n$$

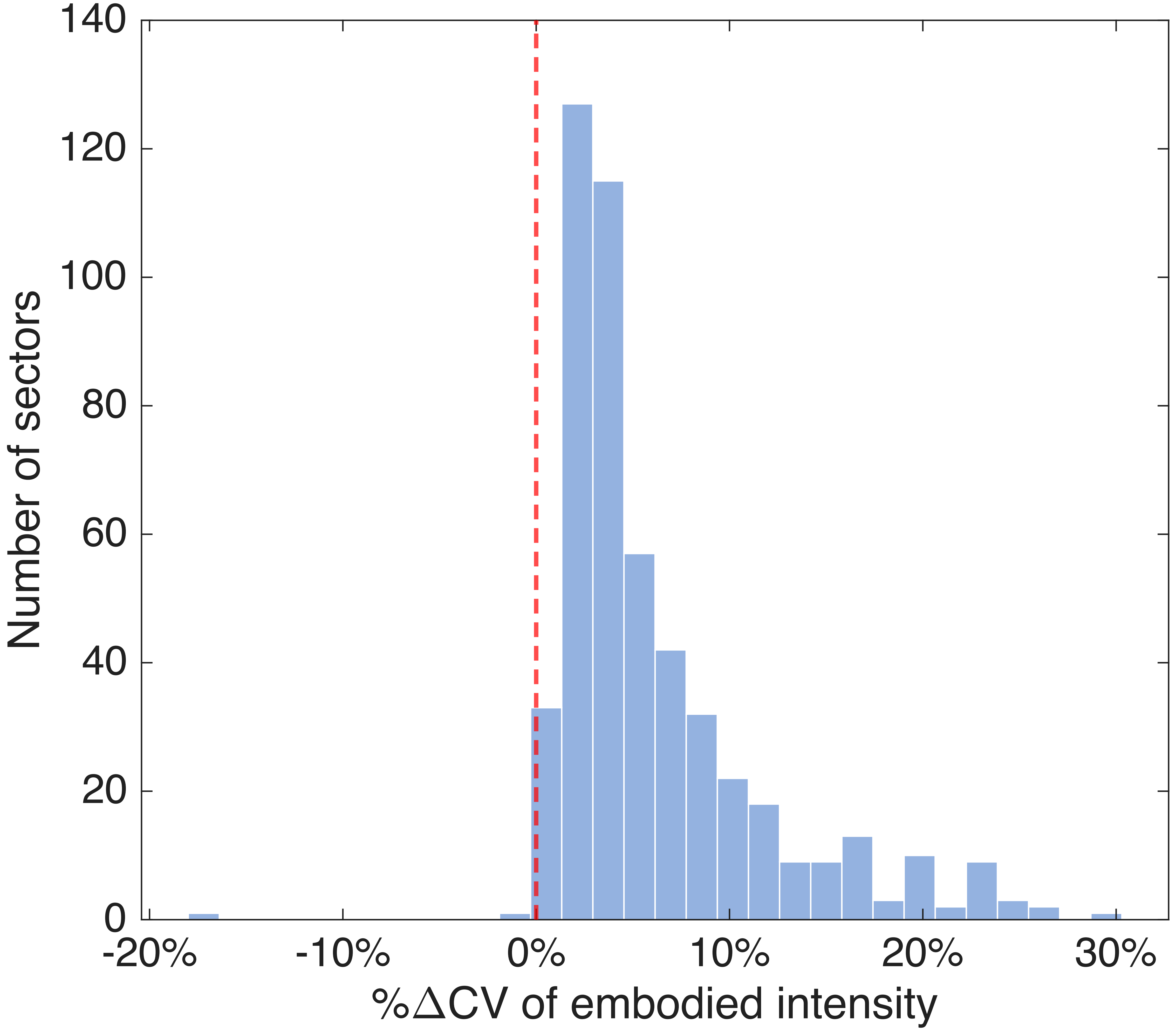
- What factors affect CV(C) (coefficient of variation):
  - CV of embodied intensity: $\mathrm{CV}(e_j) \uparrow \Rightarrow \mathrm{CV}(C) \uparrow$
  - Correlation coefficient between embodied intensities: $\rho^e_{ij} \uparrow \Rightarrow \mathrm{CV}(C) \uparrow$
  - Mean of embodied intensity: *when* $\mathrm{E}(e_j) \uparrow$, $\mathrm{CV}(C)$ *depends*

Dependent vs. Independent: $\mathrm{E}(e_j)$



- $\%\,\Delta\mathrm{E}(e_j) = \dfrac{\mathrm{E}(e_j^{dep}) - \mathrm{E}(e_j^{indep})}{\mathrm{E}(e_j^{indep})} \times 100\,\%$
- For any sector, the change in the embodied emission intensity is in the interval of (-0.29%, 0.31%)
- For 482/505 sectors, $\%\,\Delta\mathrm{E}(e_j)$ is in (-0.1%, 0.1%)
- For 422 sectors, $\mathrm{E}(e_j)$ increase
- For 83 sectors, $\mathrm{E}(e_j)$ decrease

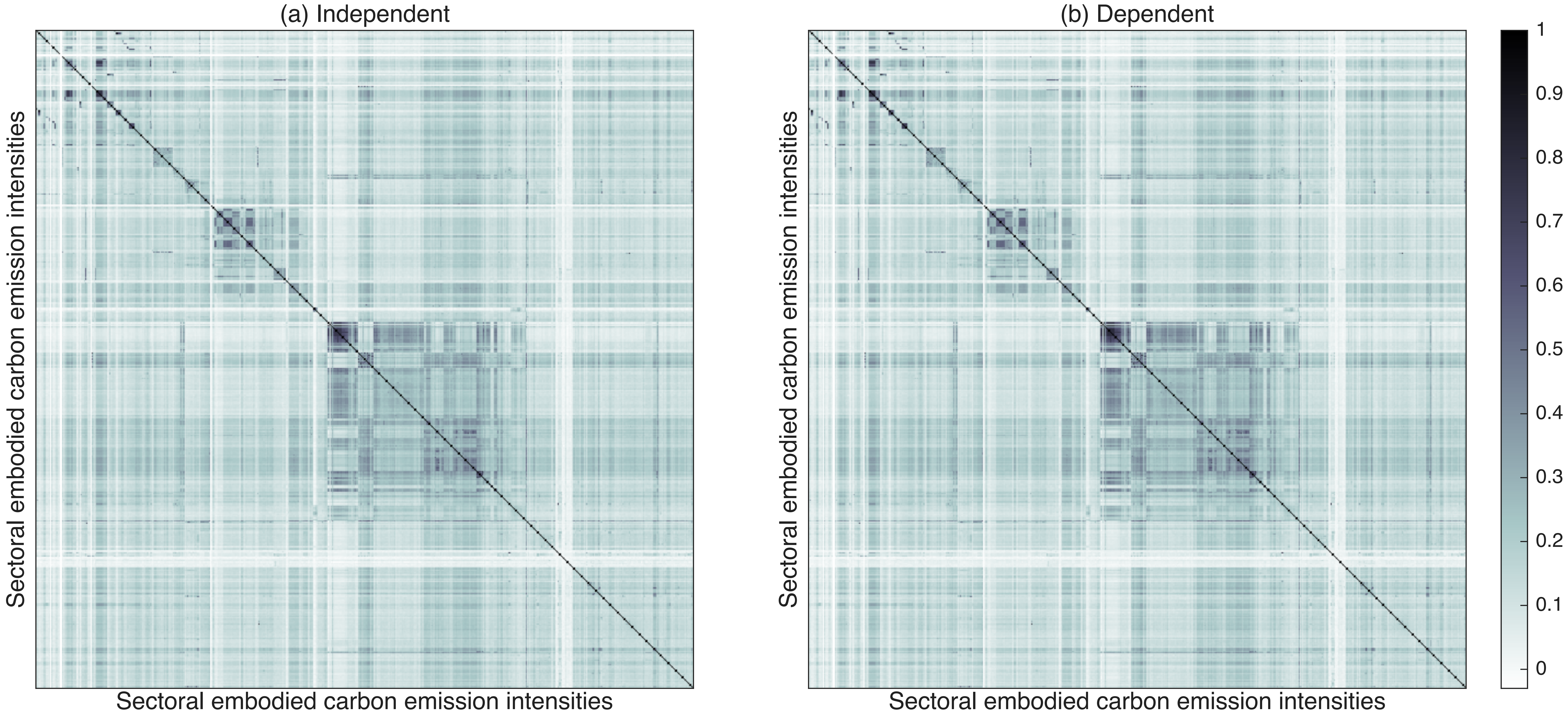
Dependent vs. Independent: $\mathrm{CV}(e_i)$



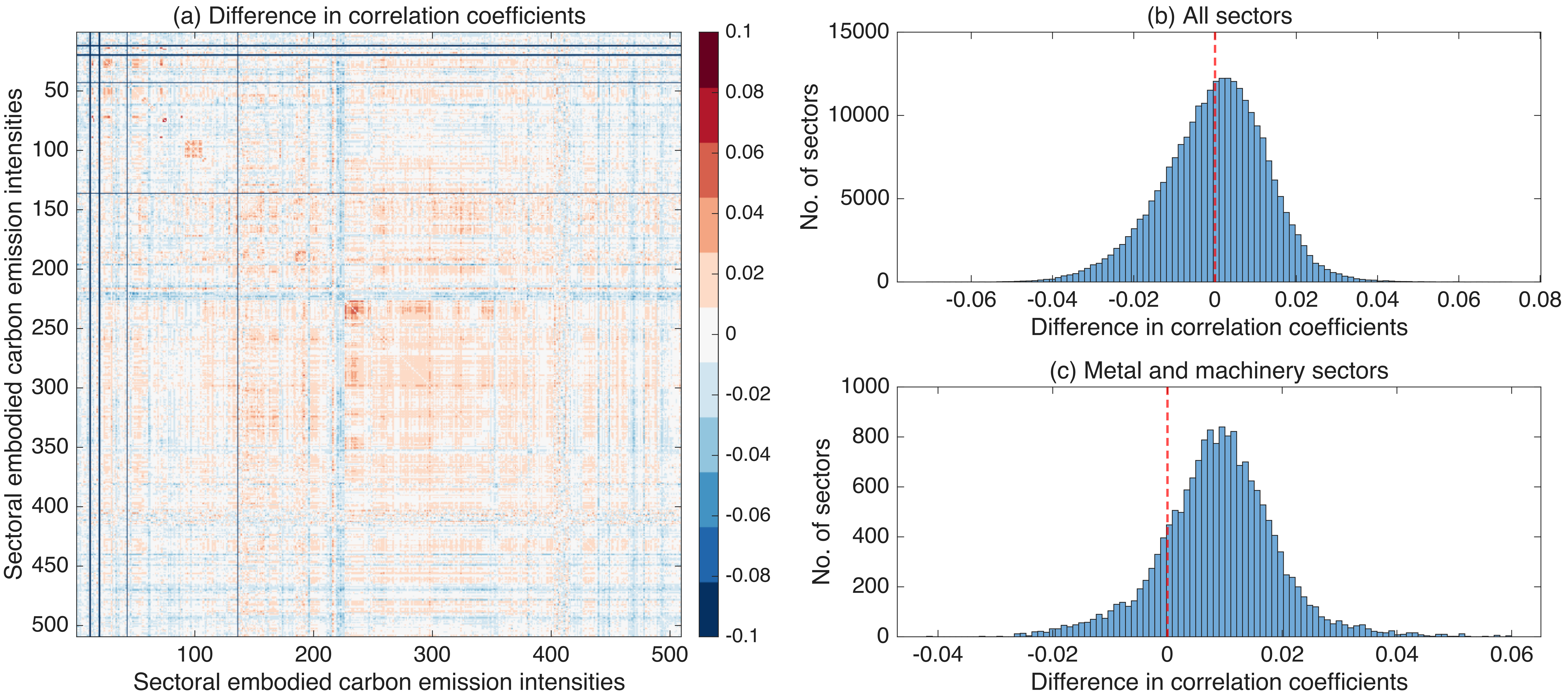
$$\%\,\Delta\mathrm{CV}(e_j) = \frac{\mathrm{CV}(e_j^{dep}) - \mathrm{CV}(e_j^{indep})}{\mathrm{CV}(e_j^{indep})} \times 100\,\%$$

%ΔCV(e_j)	# of sector
>10%	96 sectors
5% ~10%	118 sectors
-5% ~ 5%	290 sectors
< -5%	1 sector

Dependent vs. Independent: ρ_{ij}^{e}



Dependent vs. Independent: ρ_{ij}^{e}



Conclusion

- In the independent case, the uncertainty of carbon footprints are underestimated
- When dependent, the uncertainty of a carbon footprint increases because of the increase in the uncertainty of each sectoral embodied emission intensity
- Means and correlation coefficient between embodied emission intensities almost do not change

Referances

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Thanks for your attention

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