Impact of Automation on Employment and Productivity: A Nonlinear Input-output Model

Wolfgang Koller

Institute for Industrial Research, Vienna

Mikulas Luptacik

Institute for Industrial Research, Vienna
and
University of Business and Economics, Vienna

Eduard Nezinsky

University of Economics in Bratislava

International Input-Output Association Conference 2026
June 22nd-26th, Seville

Overview

The competitiveness of the EU economy is not the result of merely aggregating individual industries' performance but the result of a complex network of relationships between them.

European Commission, 2005, p.33

Structure of the presentation:

1. Introduction
2. An open input-output model with substitution between primary factors
3. Nonlinear employment multiplier
4. Impact of automation on employment
5. Application: Austria, 2019
6. Conclusions and further research

Introduction

- Background:
  - Different manifestations of automation: digitalization, robotization, artificial intelligence
  - Fundamental implications for the labour market by substitution of machines for labour
- Previous studies:
  - Mostly at the macroeconomic level
  - Studies at the sectoral level (multi-industry setting): Walheer (2016, 2016a), Eder/Koller/Mahlberg (2025): interconnections between industries are not taken into account.
- Aim of this contribution:
  - Extend the input-output model by postulating a substitution between employment, traditional (physical) capital and automation capital
  - What is the impact of (increasing) automation capital in one industry for the employment in the other industries and the whole economy?

An open input-output model with substitution (1)

Following earlier research paths (Johansen 1960; Schumann 1968; Luptacik 2010), we extend the input-output model by substitution possibilities for primary factors according to a Cobb-Douglas production function:

$$x_j = \epsilon_j L_j^{\alpha_j} K_j^{\beta_j} P_j^{\gamma_j}, \quad j = 1, 2, \ldots, n \tag{1}$$

where x_j gross output L_j employment K_j capital stock P_j automation capital in industry j .

Balance equations (as inequality): $(\mathbf{E} - \mathbf{A})\mathbf{x} \geqq \mathbf{y}$ with $\mathbf{y}$ final demand $\mathbf{A}$ matrix of input coeff.

or $$\sum_{j=1}^{n} (\delta_{ij} - a_{ij}) x_j \geq y_i \qquad i = 1, 2, \ldots, n$$

with $\delta_{ij} = 0$ for $i \neq j$ and $\delta_{ij} = 1$ for $i = j$ (Kronecker delta)

An open input-output model with substitution (2)

Substituting (1) for x_j in the balance equations and transforming:

$$\sum_{j \neq i} d_{ij} L_j^{\alpha_j} K_j^{\beta_j} P_j^{\gamma_j} L_i^{-\alpha_i} K_i^{-\beta_i} P_i^{-\gamma_i} + c_i L_i^{-\alpha_i} K_i^{-\beta_i} P_i^{-\gamma_i} \leqq 1 \qquad (2)$$

where

$$d_{ij} = \frac{a_{ij}\epsilon_j}{(1 - a_{ii})\epsilon_i} \quad \text{and} \quad c_i = \frac{y_i}{(1 - a_{ii})\epsilon_i} \quad (i, j = 1, 2, \ldots, n)$$

Constraints on physical and automation capital (non-transferability):

$$K_j \leqq \bar{K}_j \quad \text{or} \quad \frac{1}{\bar{K}_j} K_j \leqq 1 \quad (i, j = 1, 2, \ldots, n) \qquad (3)$$

$$P_j \leqq \bar{P}_j \quad \text{or} \quad \frac{1}{\bar{P}_j} P_j \leqq 1 \quad (i, j = 1, 2, \ldots, n) \qquad (4)$$

An open input-output model with substitution (3)

Objective function: minimizing labour input

$$\min L = \sum_{j=i}^{n} L_j \qquad (5)$$

For given final demand implies maximization of labour productivity.

We have an optimization problem: minimize (5) subject to constraints (2), (3) and (4) and positivity constraints:

$$L_j > 0, K_j > 0, P_j > 0 \quad (j = 1, 2, \ldots, n) \qquad (6)$$

This is a geometric programming problem.

An open input-output model with substitution (4)

Basics of geometric programming

Primal problem:

$$\begin{aligned} \underset{\mathbf{x}}{\text{minimize}} \quad & g_0(\mathbf{x}) & \\ \text{subject to} \quad & g_k(\mathbf{x}) \leqq 1 & (k = 1, 2, \ldots, m) \\ & x_j > 0 & (j = 1, 2, \ldots, n) \end{aligned}$$

where

$$g_k(\mathbf{x}) = \sum_{t=1}^{T_k} c_{kt} \prod_{j=1}^{n} x_j^{a_{ktj}} \qquad (k = 0, 1, \ldots, m)$$

Industriewissenschaftliches Institut

An open input-output model with substitution (5)

The dual problem D corresponding to the primal model P:

$$\max \quad v(\boldsymbol{\delta}) = \prod_{k=0}^{m} \prod_{t=1}^{T_k} \left(\frac{c_{kt}}{\delta_{kt}}\right)^{\delta_{kt}} \prod_{k=1}^{m} \lambda_k(\boldsymbol{\delta})^{\lambda_k(\boldsymbol{\delta})}$$

where

$$\lambda_k(\boldsymbol{\delta}) = \sum_{t=1}^{T_k} \delta_{kt} \qquad (k = 1, 2, \ldots, m)$$

subject to

$$\sum_{k=0}^{m} \sum_{t=1}^{T_k} a_{ktj}\delta_{kt} = 0 \qquad (j = 1, 2, \ldots, n)$$

$$\sum_{t=1}^{T_0} \delta_{0t} = 1 \qquad \delta_{kt} \geqq 0 \quad \begin{pmatrix} k = 0, 1, \ldots, m \\ t = 1, 2, \ldots, T_k \end{pmatrix}$$

Main lemma of geometric programming: $g_0(\mathbf{x}) \geqq v(\boldsymbol{\delta})$ for any feasible solution.

An open input-output model with substitution (6)

For simplicity but without loss of generality we reduce the number of sectors to two (n=2): the dual model is:

$$
\begin{aligned}
\max v(\boldsymbol{\delta}) = & \left(\frac{1}{\delta_{01}}\right)^{\delta_{01}} \left(\frac{1}{\delta_{02}}\right)^{\delta_{02}} \left(\frac{d_{12}}{\delta_{11}}\right)^{\delta_{11}} \left(\frac{c_1}{\delta_{12}}\right)^{\delta_{12}} \left(\frac{d_{21}}{\delta_{21}}\right)^{\delta_{21}} \left(\frac{c_2}{\delta_{22}}\right)^{\delta_{22}} \\
& \times \left(\frac{\frac{1}{K_1}}{\delta_{31}}\right)^{\delta_{31}} \left(\frac{\frac{1}{K_2}}{\delta_{41}}\right)^{\delta_{41}} \left(\frac{\frac{1}{P_1}}{\delta_{51}}\right)^{\delta_{51}} \left(\frac{\frac{1}{P_2}}{\delta_{61}}\right)^{\delta_{61}} \\
& \times \lambda_1(\boldsymbol{\delta})^{\lambda_1(\boldsymbol{\delta})} \lambda_2(\boldsymbol{\delta})^{\lambda_2(\boldsymbol{\delta})} \lambda_3(\boldsymbol{\delta})^{\lambda_3(\boldsymbol{\delta})} \lambda_4(\boldsymbol{\delta})^{\lambda_4(\boldsymbol{\delta})} \lambda_5(\boldsymbol{\delta})^{\lambda_5(\boldsymbol{\delta})} \lambda_6(\boldsymbol{\delta})^{\lambda_6(\boldsymbol{\delta})} \qquad (7)
\end{aligned}
$$

subject to:

$$\delta_{01} \geqq 0, \delta_{02} \geqq 0, \ldots, \delta_{61} \geqq 0 \qquad (8)$$

$$
\begin{aligned}
&\delta_{01} + \delta_{02} && = 1, && (9) \\
&\delta_{01} - \alpha_1\delta_{11} - \alpha_1\delta_{12} + \alpha_1\delta_{21} && = 0, && (10) \\
&\delta_{02} + \alpha_2\delta_{11} - \alpha_2\delta_{21} - \alpha_2\delta_{22} && = 0, && (11) \\
&- \beta_1\delta_{11} - \beta_1\delta_{12} + \beta_1\delta_{21} + \delta_{31} && = 0, && (12) \\
&\beta_2\delta_{11} - \quad - \beta_2\delta_{21} - \beta_2\delta_{22} + \delta_{41} && = 0, && (13) \\
&- \gamma_1\delta_{11} - \gamma_1\delta_{12} + \gamma_1\delta_{21} + \delta_{51} && = 0, && (14) \\
&\gamma_2\delta_{11} - \gamma_2\delta_{21} - \gamma_2\delta_{22} + \delta_{61} && = 0 && (15)
\end{aligned}
$$

An open input-output model with substitution (7)

Overview:

	Primal Problem P	**Dual Problem D**
$T = \sum_{k=0}^{m} T_k$	Number of terms	Number of variables
n	Number of variables	Number of orthogonality constraints

Dual variables:

δ_{01}, δ_{02}	two terms in the objective function
δ_{11}, δ_{12}	two terms in the first interindustry constraint
δ_{21}, δ_{22}	two terms in the second interindustry constraint
δ_{31}, δ_{41}	one term in each of two capital constraints
δ_{51}, δ_{61}	one term in each of two automation capital constraints

Nonlinear employment multiplier

How to estimate the impact of increasing final demand on total employment in the economy under the assumption of substitution possibilities between labour, physical and automation capital?

Following the main lemma of geometric programming

$$L^* \geqq v^*(\boldsymbol{\delta}^0) \tag{16}$$

where L^* denotes the new level of employment and $v^*(\boldsymbol{\delta}^0)$ the value of the dual objective function corresponding to the (feasible) solution $\boldsymbol{\delta}^0$ with the new coefficients $c_i^* \quad (i = 1, 2, \ldots, n)$ in the dual objective function.

Dividing both sides of (16) by L^0 we have

$$\frac{L^*}{L^0} \geqq \frac{v^*(\boldsymbol{\delta}^0)}{v(\boldsymbol{\delta}^0)} = (\frac{c_1^*}{c_1})^{\delta_{12}} (\frac{c_2^*}{c_2})^{\delta_{22}} = (\frac{y_1^*}{y_1})^{\delta_{12}} (\frac{y_2^*}{y_2})^{\delta_{22}} \tag{17}$$

$$L^* \geqq L^0 (\frac{y_1^*}{y_1})^{\delta_{12}} (\frac{y_2^*}{y_2})^{\delta_{22}} \tag{18}$$

Impact of Automation on Employment (1)

To analyse the substitution processes between the three primary factors and estimate the impact of automation we rewrite (10 – (15):

$$\begin{aligned}
\delta_{01} &= \alpha_1(\delta_{11}+\delta_{12}-\delta_{21}) && (10^*)\\
\delta_{02} &= \alpha_2(\delta_{21}+\delta_{22}-\delta_{11}) && (11^*)\\
\delta_{31} &= \beta_1(\delta_{11}+\delta_{12}-\delta_{21}) && (12^*)\\
\delta_{41} &= \beta_2(\delta_{21}+\delta_{22}-\delta_{11}) && (13^*)\\
\delta_{51} &= \gamma_1(\delta_{11}+\delta_{12}-\delta_{21}) && (14^*)\\
\delta_{61} &= \gamma_2\underbrace{(\delta_{21}+\delta_{22}-\delta_{11})}_{\text{Interindustry effect}} && (15^*)
\end{aligned}$$

δ_{01}, δ_{02} shares of employment of industry 1 (2) in total employment

δ_{31}, δ_{41} δ_{51} δ_{61} elasticity coefficients (impact of increasing capital stock in industry 1 (2) and of increasing automation capital in industry 1 (2) on total employment

Impact of Automation on Employment (2)

Due to the non-negativity of the dual variables $\delta_{01}, \ldots, \delta_{61}$ and the positivity of the parameters α_j, β_j and γ_j we have

$$\delta_{11} + \delta_{12} - \delta_{21} \geqq 0 \tag{19}$$

$$\delta_{21} + \delta_{22} - \delta_{11} \geqq 0 \tag{20}$$

It can be shown that the non-negativity of the interindustry effects is ensured by the workability of the Leontief models (Hawkins-Simon conditions, Brauer-Solow conditions).

Impact of Automation on Employment (3)

From equation (10*) – (15*) it can be seen

$$\frac{\partial \delta_{0j}}{\partial \alpha_j} \geqq 0, \quad \frac{\partial \delta_{ij}}{\partial \beta_j} \geqq 0, \quad \frac{\partial \delta_{kj}}{\partial \gamma_j} \geqq 0 \quad j = 1, 2, i = 3, 4, k = 5, 6 \qquad (21)$$

Combining equation (12*) with (14*) and (13*) with (15*) we get

Proposition 1

Higher (lower) output elasticity of automation capital relative to traditional capital implies higher (lower) impact of increasing automation capital on total employment than the impact from increasing physical capital.

$$\beta_j \leqq (\geqq)\ \gamma_j\ (j = 1, 2)\ \textit{implies}\ \delta_{31} \leqq (\geqq)\ \delta_{51}$$

Impact of Automation on Employment (4)

Combining equation (10*) with (14*) and (11*) with (15*) leads to the following

Proposition 2

When the output elasticity of labour is equal to the output elasticity of automation capital then the share of employment in industry *j* is equal to the employment elasticity of automation capital in industry *j*.

$$\alpha_j = \gamma_j \Longrightarrow \delta_{0j} = \delta_{kj} \;\; (j = 1, 2, \;\; k = 5, 6) \qquad (23)$$

Impact of Automation on Employment (5)

Taking into account the normality condition (9), the share of employment in industry 1 can be expressed as the function of the production function parameters of industry 2:

$$\delta_{01} = 1 - \delta_{61}\frac{\alpha_2}{\gamma_2} \tag{24}$$

Proposition 3

Increasing output elasticity of automation capital in industry 2 leads to an increasing employment share in industry 1. Increasing output elasticity of labour in industry 2 implies decreasing employment share in industry 1.

It follows from (24): $$\frac{\partial \delta_{01}}{\partial \gamma_2} > 0 \quad \text{and} \quad \frac{\partial \delta_{01}}{\partial \alpha_2} < 0.$$

Impact of Automation on Employment (6)

The implication of the technology for the degree of utilization of automation capital is described by proposition 4.

Proposition 4

If the output elasticity of automation capital in industry *j* is higher than the output elasticity of labour then automation capital is fully utilized (the constraint for automation capital is fulfilled as equality).

Following equations (10*) and (14*) of the dual models and Proposition 2 we observe

$$\gamma_1 > \alpha_1 \Longrightarrow \delta_{51} > \delta_{01}$$

Because of $\delta_{01} > 0$, we have $\delta_{51} = \lambda_5(\boldsymbol{\delta}) > 0$ and according the complementary slackness condition the inequalities (4) must be fulfilled as equations.

Industriewissenschaftliches Institut

Impact of Automation on Employment (7)

Proposition 5

Employment elasticity of automation capital of industry j is smaller than the ratio of output elasticity of automation capital to labour in industry j.

$$\delta_{51}\frac{\alpha_1}{\gamma_1} + \delta_{61}\frac{\alpha_2}{\gamma_2} = 1$$

$$\delta_{51} = \frac{\gamma_1}{\alpha_1}\left(1 - \delta_{61}\frac{\alpha_2}{\gamma_2}\right) > 0 \implies \underline{\frac{\gamma_2}{\alpha_2} > \delta_{61}}$$

$$\delta_{61} = \frac{\gamma_2}{\alpha_2}\left(1 - \delta_{51}\frac{\alpha_1}{\gamma_1}\right) > 0 \implies \underline{\frac{\gamma_1}{\alpha_1} > \delta_{51}}$$

Impact of Automation on Employment (8)

Following Proposition 2 and duality theory of geometric programming, the level of employment for a particular industry can be estimated (without the necessity to solve the primal problem).

Proposition 6

The employment in industry j (j=1,2) is a function of technological parameters α_j, γ_j and of interindustry relationships

$$L_1^0 = v(\boldsymbol{\delta}^0)\delta_{51}^0 \frac{\alpha_1}{\gamma_1} \quad \text{and} \quad L_2^0 = v(\boldsymbol{\delta}^0)\delta_{61}^0 \frac{\alpha_2}{\gamma_2}$$

where the superscript 0 denotes the optimal solution.

Impact of Automation on Employment (9)

Finally, the lower bound for employment multiplier of automation will be derived:

$$\frac{L^*}{L^0} \geqq \frac{v^*(\boldsymbol{\delta}^0)}{v(\boldsymbol{\delta}^0)} = (\frac{\bar{P}_1}{\bar{P}_1^*})^{\delta_{51}} (\frac{\bar{P}_2}{\bar{P}_2^*})^{\delta_{61}}$$

or:

$$L^* \geqq L^0 (\frac{\bar{P}_1}{\bar{P}_1^*})^{\delta_{51}} (\frac{\bar{P}_2}{\bar{P}_2^*})^{\delta_{61}}$$

Application (1)

Application: Austria 2019

- input-output table, aggregated to 9 x 9 io-sectors
- Robot capital stock compiled from International Federation of Robotics (IFR), data preparation see Eder/Koller/Mahlberg (2025, Structural Change and Economic Dynamics)

	IFR-Codes	NACE-codes	Description
Manufacturing, low robot-intensity	CA	C10-12	Food products, Beverages, Tobacco
	CB	C13-15	Textiles, Wearing apparel, Leather
	CC+CM	C16-18, C31-C33	Wood, Paper, Printing s., Furniture, Other, Repair s.
	CD+CE+CF	C19-21	Coke and refined petroleum prod., Chemicals, Pharmaceuticals
Manufacturing, medium robot-intensity	CI+CJ	C26-C27	Electronic and optical prod., Electrical equipment
	CK	C28	Machinery
Manufacturing, high robot-intensity	CG	C22-23	Rubber and plastic prod., Other non-metallic mineral prod.
	CH	C24-25	Basic Metals, Metal prod.,
	CL	C29-30	Motor vehicles, Other transport equipment

A	B	C.low	C.medium	C.high	DE	FGHI	JKLMN	OPQRST
0	0	855	985	4659	0	0	0	0

Application (2)

(Preliminary) Estimation of Cobb-Douglas function parameters

- Using data from previous research project (see Eder/Koller/Mahlberg 2025): panel for 18 countries, 10 years (2010-2019)
- Method: deterministic frontier production function estimation, inspired by Aigner/Chu (1968, AER) → minimizing a loss function in an optimizing framework

The estimated epsilon (Total productivity parameter) was not used but calibrated with the io-data.

Method	epsilon	beta_L	beta_K	beta_P
Manufacturing, low robot-intensity				
weighted, w/o CRS	5,225721902	0,393520519	0,554251867	0,054001144
weighted, with CRS	5,315857545	0,394488121	0,554241416	0,051270463
unweighted, w/o CRS	5,225721758	0,393520528	0,554251863	0,054001144
unweighted, with CRS	5,215707099	0,388243291	0,557738423	0,054018286
Manufacturing, low robot-intensity				
weighted, w/o CRS	19,10752995	0,447718902	0,340982782	0,143417706
weighted, with CRS	10,20819449	0,490754139	0,440924584	0,068321277
unweighted, w/o CRS	10,62091146	0,555617137	0,434960086	0,005102072
unweighted, with CRS	10,20819436	0,559075412	0,440924587	0
Manufacturing, low robot-intensity				
weighted, w/o CRS	25,22172193	0,663241816	0,249734697	0,119661031
weighted, with CRS	12,54824267	0,598381734	0,257222201	0,144396065
unweighted, w/o CRS	24,64202714	0,665479947	0,253660421	0,115968002
unweighted, with CRS	12,54823757	0,598381642	0,257222285	0,144396073
Manufacturing, total				
weighted, w/o CRS	6,682471942	0,433430292	0,425008444	0,132965341
weighted, with CRS	5,36854396	0,410985606	0,464982093	0,124032298
unweighted, w/o CRS	4,914572986	0,401927826	0,481113976	0,120427131
unweighted, with CRS	5,368547495	0,410985689	0,464981861	0,124032449

Application (3)

IOGP-Model

- Primal model: 27 target variables (L_j, K_j, P_j for $j = 1, \ldots, 9$)
  - Optimal total employment is smaller than observed employment in real world (due to estimated/calibrated inefficiency)
  - Restrictions (final demand, capital, automation capital) are met
- Dual model: elasticities of total employment on robots use
  - No preliminary results yet available.

Conclusions and further research

- Nonlinear input-output model admitting substitution of primary inputs in every industry
- Interpretation of dual variables as employment shares and employment elasticities
- Deeper insights into substitution processes between primary factors
- Alternative objective functions, e.g. maximization of final demand
- Further empirical work:
  - i) estimation of Cobb-Douglas or CES production function
  - ii) sensitivity analysis for different IO-framworks (domestic vs. total version)
  - Iii) country comparison Austria – Slovakia

Thank you for your attention!

Contact:

Wolfgang Koller
Institute for Industrial Research, Vienna
koller@science.iwi.ac.at

Mikulas Luptacik
Institute for Industrial Research, Vienna
and
University of Business and Economics, Vienna
Mikulas.luptacik@wu.ac.at

Eduard Nezinsky
University of Economics in Bratislava
nezinsky@gmail.com