Visual Network Decomposition of the Leontief Inverse

TiVA, Environmental Footprint, Ownership, Endogenized Capital

Jens Kammerath, Königswinter/Germany

32nd IIOA Conference

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jK@iO INPUT·OUTPUT

How can Input-Output analysis have an impact?

If our research is to have an **impact**,
we must be able to **explain**
in the **language** of the different stakeholders
not only the **results**,
but also **how they were obtained**
in order to build and maintain **trust**
in our **methods** and findings.

Input-Output analysis:
move from dry mathematical formulas to an

Interactive

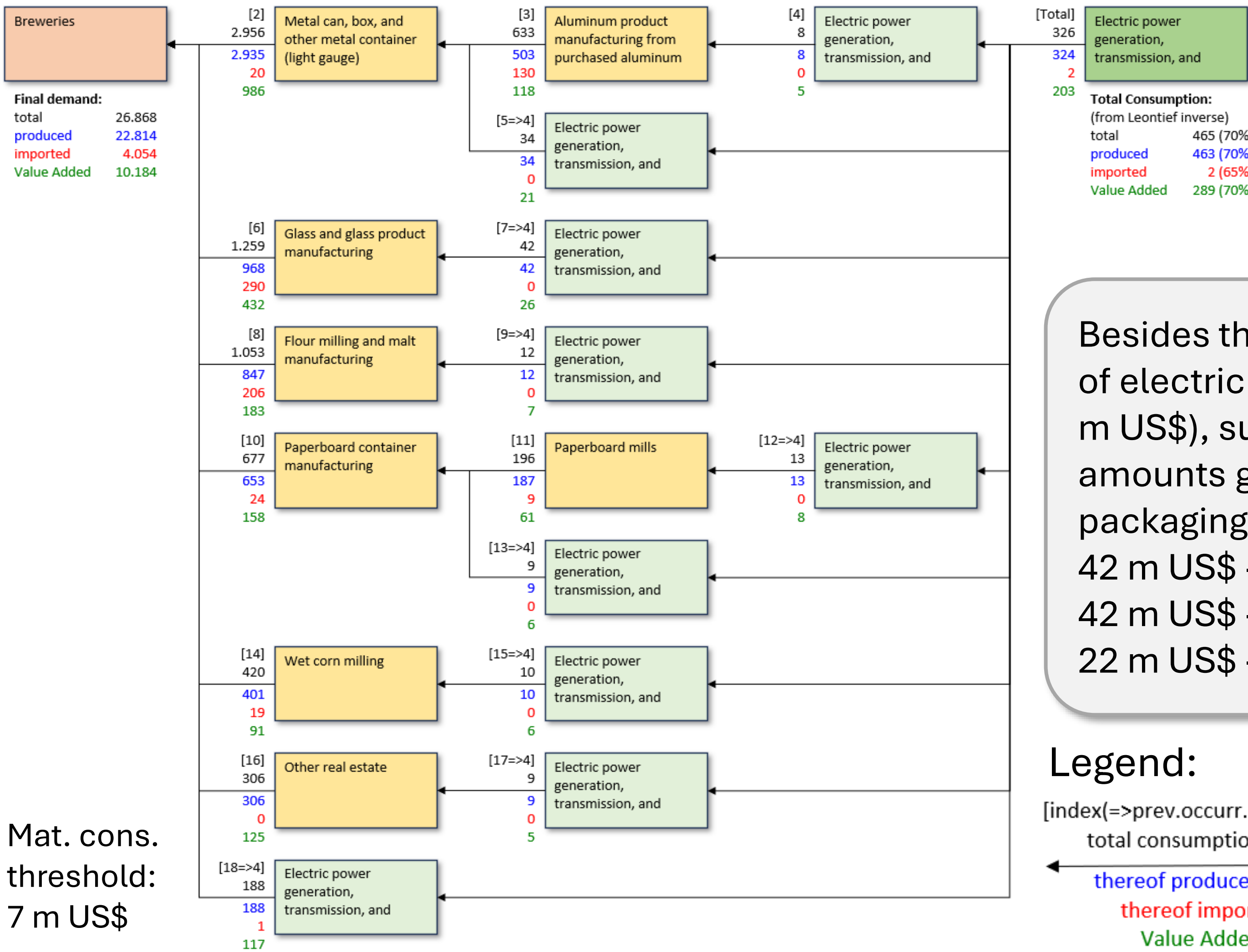
Visual experience

with **traceable calculation**

Is that possible?

And if so, what for?

Visual experience - Electric energy for US beer production



Data source:
BEA US IO table 2017

Besides the direct use of electric power (188 m US$), substantial amounts go into packaging:
42 m US$ - cans
42 m US$ - bottles
22 m US$ - paper boxes

Legend:

[index(=>prev.occurr.)]
total consumption
thereof produced
thereof import
Value Added

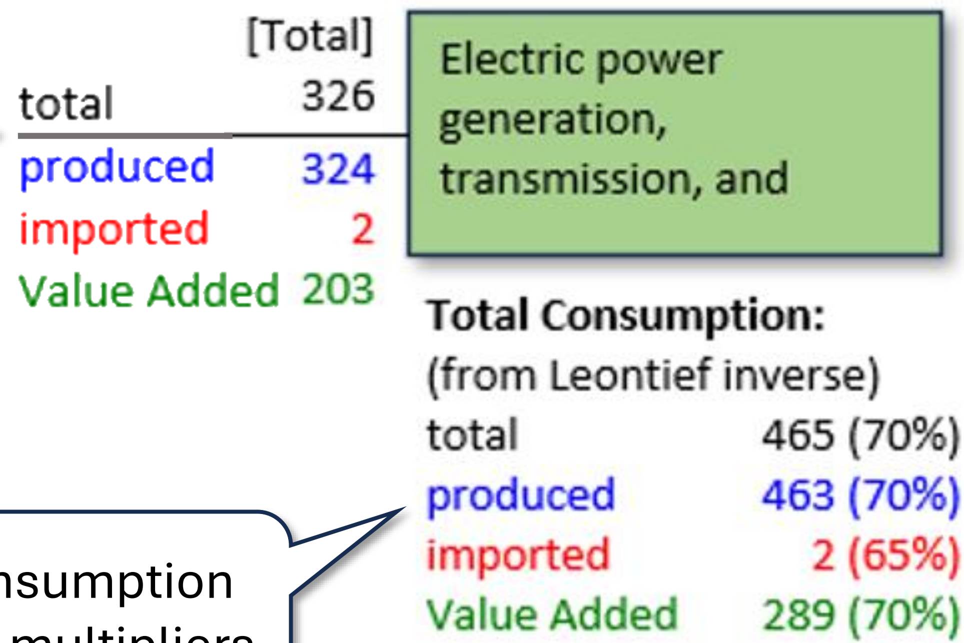
description of material

Mat. cons. threshold:
7 m US$

Source: IOPathFinder with data of U.S. Bureau of Economic Analysis (2026)

Traceable calculation - Decomposition of multipliers

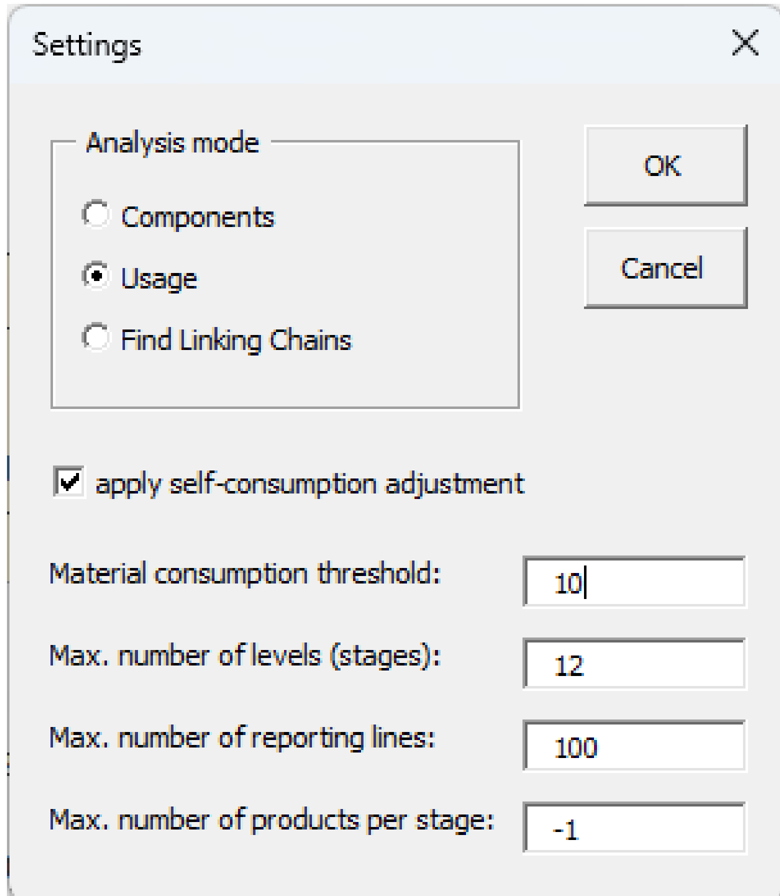
Consumption across the chains covered in the diagram



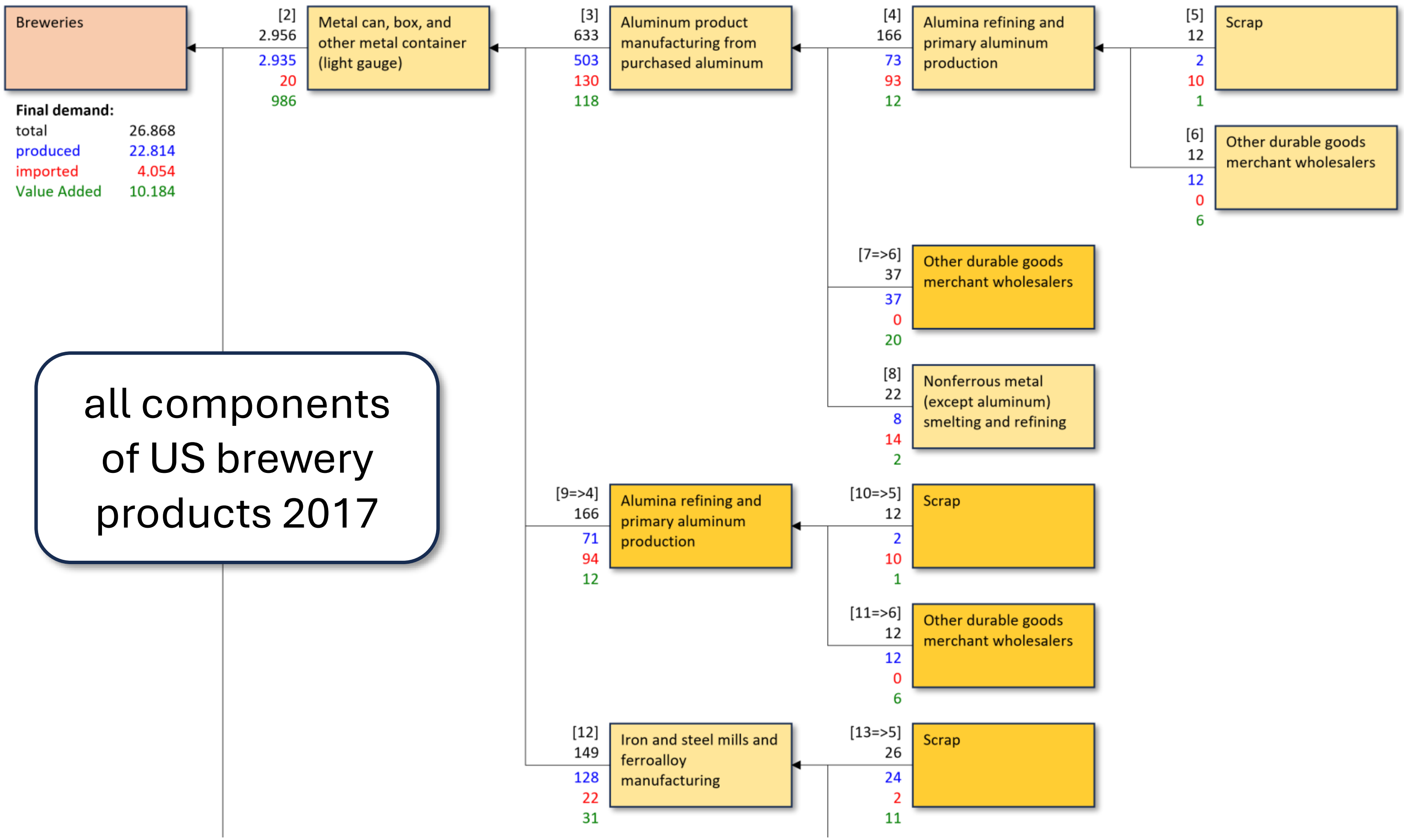
Total consumption based on multipliers

Source: IOPathFinder with data of U.S. Bureau of Economic Analysis (2026)

Interactive - Click on any box in the flowchart

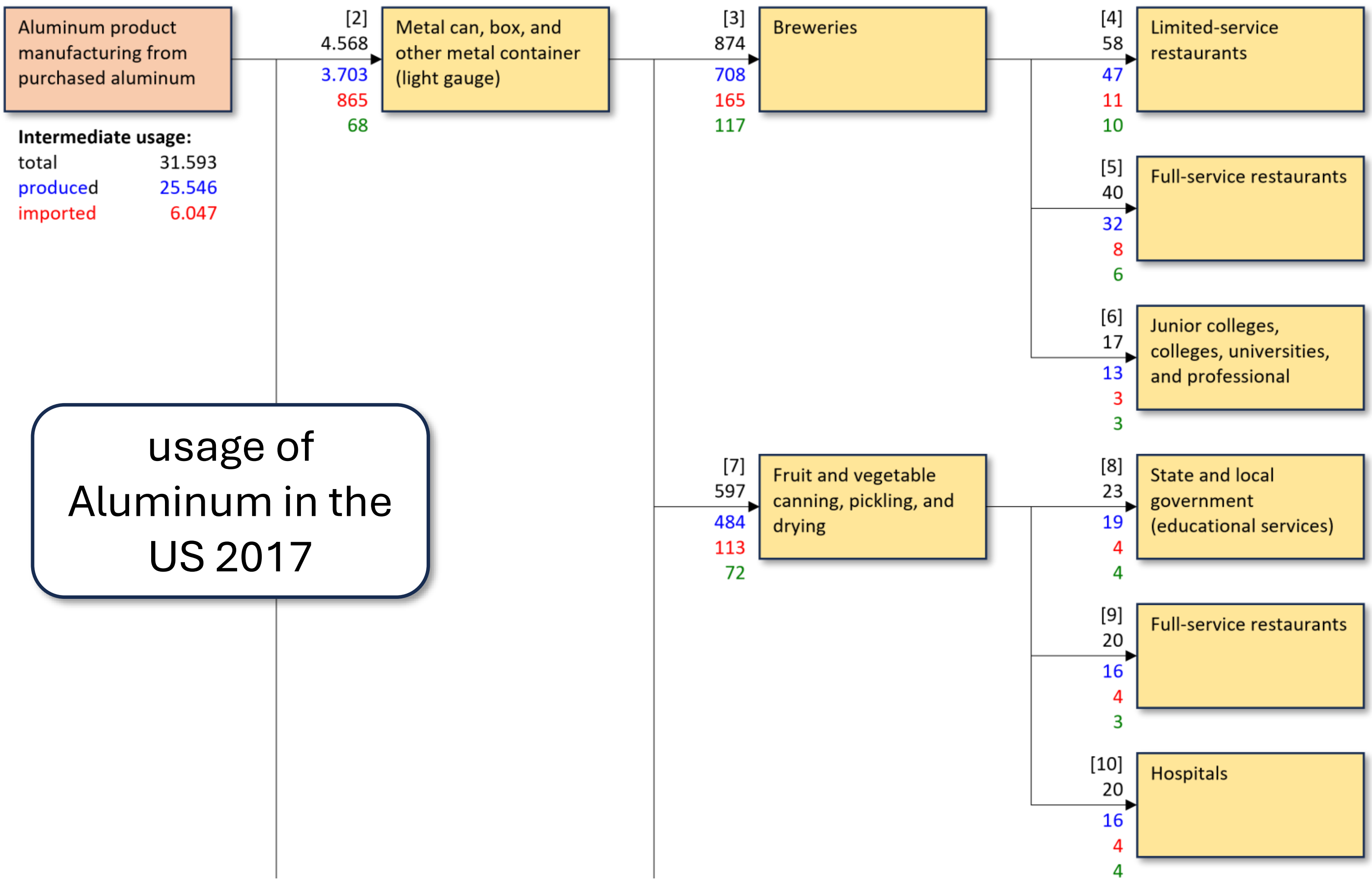


Visual, Interactive - Backward linkages



Source: IOPathFinder with data of U.S. Bureau of Economic Analysis (2026)

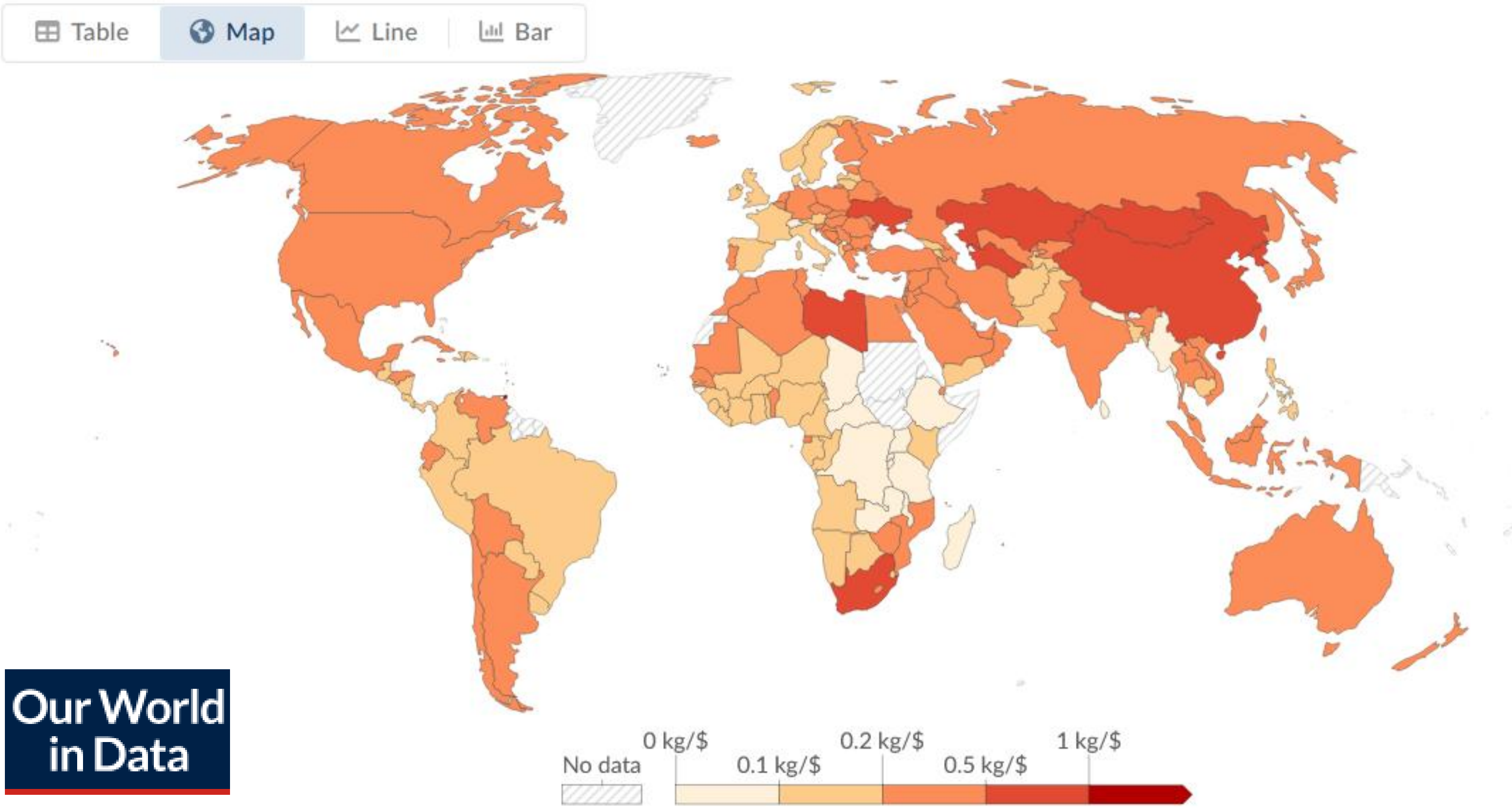
Visual, Interactive - Forward linkages



Source: IOPathFinder with data of U.S. Bureau of Economic Analysis (2026)

Use case 1 - Carbon intensity of GDP 2015

Kilograms of CO_2 emitted per dollar of GDP. Fossil fuel and industry emissions are included. Land-use change emissions are not included. GDP data is adjusted for inflation and differences in living costs between countries.



Source: https://ourworldindata.org/grapher/co2-intensity

Heatmap - Carbon intensity of VA contribution by country

sorted by CO_2 footprint

kg CO2 / € VA 2015 selling country: / buying country:		World Ø	1 China	2 United States	3 Rest of the World	4 India	5 Japan	6 Russia	7 Germany	8 South Korea	9 Saudi Arabia	10 Brazil
	World Ø	**0.509**	**0.948**	**0.349**	**0.556**	**1.176**	**0.365**	**0.967**	**0.301**	**0.573**	**1.006**	**0.380**
1	China	1.031	0.974	1.201	1.255	1.545	1.156	1.243	1.152	1.409	1.388	1.391
2	Rest of the world	0.637	1.165	0.682	0.483	1.636	1.414	1.037	1.026	1.652	1.133	1.920
3	United States	0.311	0.375	0.305	0.353	0.368	0.324	0.314	0.340	0.386	0.299	0.391
4	India	1.207	1.809	0.891	1.542	1.193	1.290	1.217	1.234	1.352	1.293	1.427
5	Russia	1.320	1.925	1.894	2.003	2.065	1.716	1.030	2.274	2.143	1.909	2.740
6	Japan	0.311	0.567	0.386	0.544	0.771	0.278	0.475	0.524	0.659	0.621	0.585
7	Germany	0.240	0.299	0.237	0.314	0.417	0.324	0.293	0.214	0.313	0.308	0.359
8	South Korea	0.517	0.685	0.569	0.695	1.001	0.798	0.678	0.692	0.445	0.676	0.732
9	Saudi Arabia	1.043	0.736	0.757	1.012	0.445	0.741	0.996	0.862	0.997	1.148	0.874
10	Canada	0.416	0.795	0.756	0.721	0.774	0.648	0.694	0.607	0.619	0.626	0.823

Source: own calculations based on FIGARO-E3 2015 (EC, JRC)

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CO_2 footprint by source country and country of final use

sorted by CO_2 footprint

mt CO2 2015 — selling country: / buying country:		World	1 China	2 United States	3 Rest of the World	4 India	5 Japan	6 Russia	7 Germany	8 South Korea	9 Saudi Arabia	10 Brazil
	World Ø	**33,040**	**9,064**	**5,764**	**4,796**	**2,190**	**1,468**	**1,018**	**762.4**	**667.8**	**631.2**	**564.8**
1	China	10,149	7,946	422.3	685.0	85.4	144.6	43.8	64.2	82.3	38.9	38.5
2	Rest of the world	5,371	486.1	314.4	3,133	154.8	188.8	50.4	73.9	89.0	48.1	62.8
3	United States	4,991	83.8	4,379	139.0	15.4	40.2	6.2	19.2	18.3	9.4	18.3
4	India	2,191	37.8	77.3	132.3	1,813	10.8	3.9	9.3	6.6	9.4	6.1
5	Russia	1,506	75.8	46.8	147.5	16.5	27.8	870.9	36.4	17.4	6.6	7.3
6	Japan	1,239	65.1	60.6	72.1	8.3	951.4	2.8	5.6	14.4	5.3	2.8
7	Germany	670.0	24.6	39.0	31.5	5.0	5.9	4.9	418.4	4.1	3.4	4.4
8	South Korea	643.5	68.2	41.5	53.5	9.4	16.7	3.0	4.5	392.2	4.4	3.0
9	Saudi Arabia	613.3	28.3	10.8	35.4	10.0	4.0	0.9	2.3	12.3	474.8	2.5
10	Canada	557.1	27.4	127.3	24.4	5.3	9.2	1.0	3.6	3.3	1.6	2.6

Source: own calculations based on FIGARO-E3 2015 (EC, JRC)

Heatmap - CO_2 footprint India → Germany by product

sorted by CO_2 footprint

kt CO2

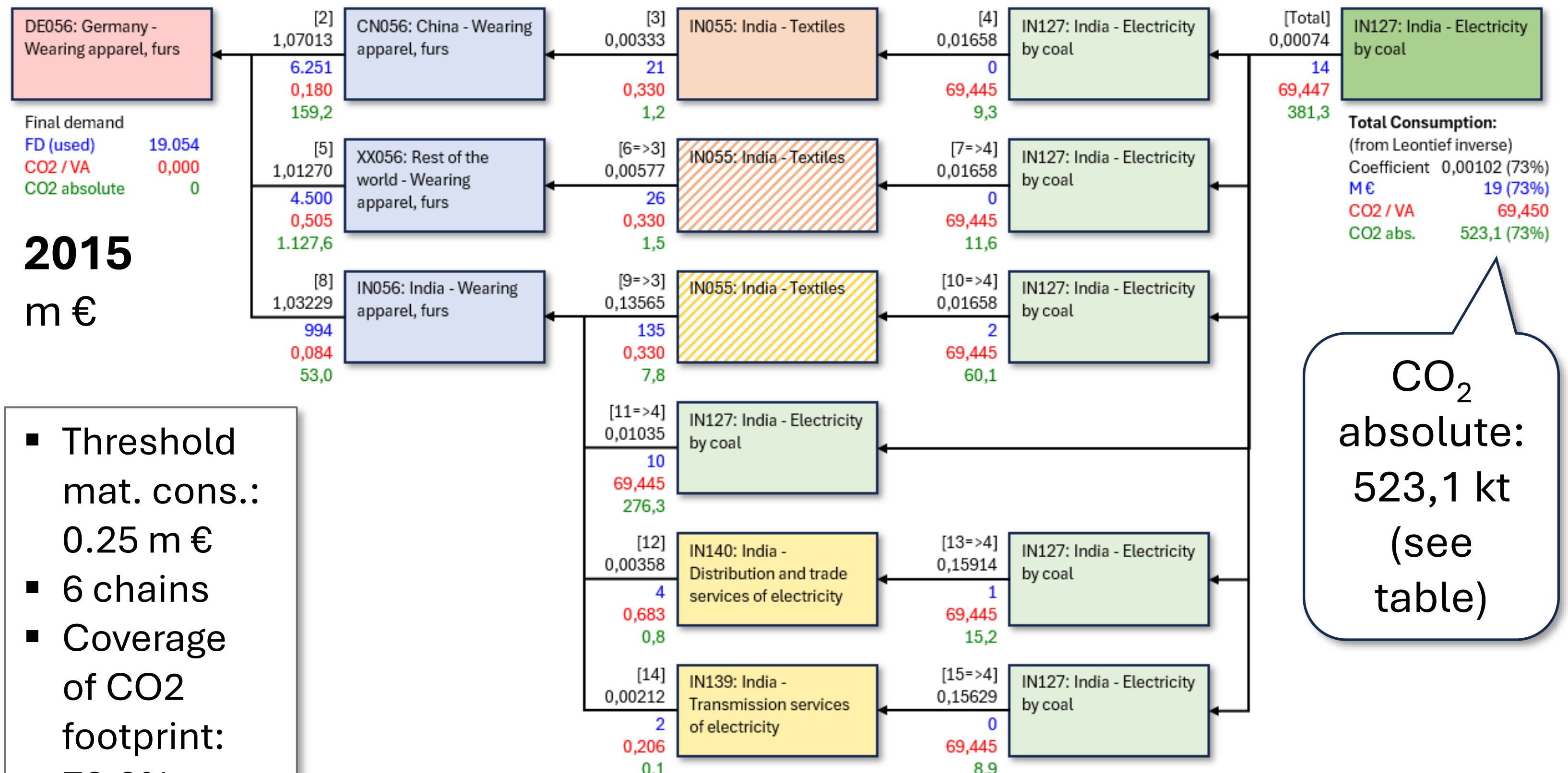
2015

Double-click on detail cell for linking chains!

Input product		Output product	1	2	3	4	5	6
			Wearing apparel, furs	Motor vehicles, trailers and semi-trailers	Machinery and equipment n.e.c.	Furniture and other manufactured goods	Construction work	Textiles
		9,274.7	**832.2**	**782.8**	**611.9**	**601.3**	**585.6**	**504.8**
1	Electricity by coal	**3,004.3**	523.1	218.0	148.0	137.7	142.7	306.8
2	Basic iron and steel and of ferro-alloy	**1,332.6**	14.3	215.4	221.2	71.6	96.8	10.0
3	Other Bituminous Coal	**587.3**	36.0	68.3	65.9	36.9	37.9	21.6
4	Rubber and plastic products	**528.3**	16.7	60.7	22.7	19.4	64.5	13.7
5	P- and other fertiliser	**311.5**	12.9	12.3	7.4	9.7	22.8	14.6
6	Furniture and other manufactured go	**288.2**	6.2	6.9	3.6	201.6	3.7	3.1
7	Plastics, basic	**228.4**	12.5	20.0	8.4	8.4	16.0	13.8
8	Railway transportation services	**226.3**	14.8	20.7	15.4	13.8	12.9	8.2
9	Transportation services via pipelines	**198.9**	6.6	10.9	7.7	7.0	13.3	4.4
10	Sea and coastal water transportation	**171.7**	2.4	6.3	4.1	3.2	5.7	1.3

Source: own calculations based on FIGARO-E3 2015 (EC, JRC)

CO_2 footprint: **IN** Electricity by coal for **DE** Wearing apparel



- Threshold mat. cons.: 0.25 m €
- 6 chains
- Coverage of CO2 footprint: 72.9%

Source: IOPathFinder based on FIGARO-E3 2015 (EC, JRC)

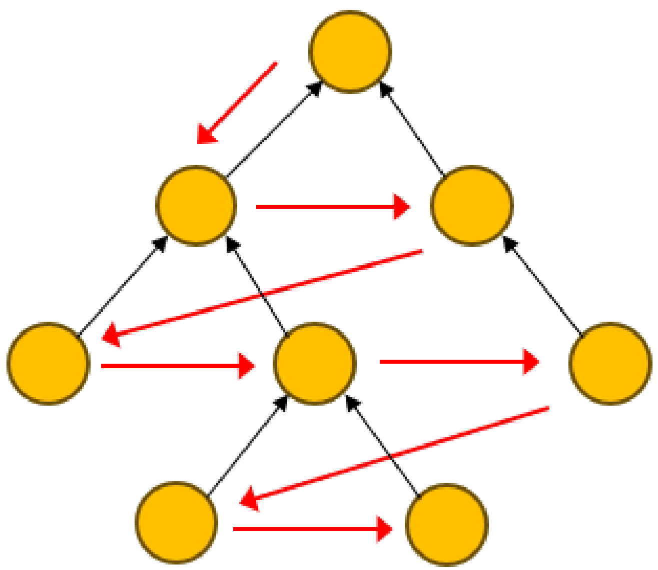
What can Visual Network Decomposition be used for?

Input-Output researchers ❯❯❯❯❯❯ Decision makers

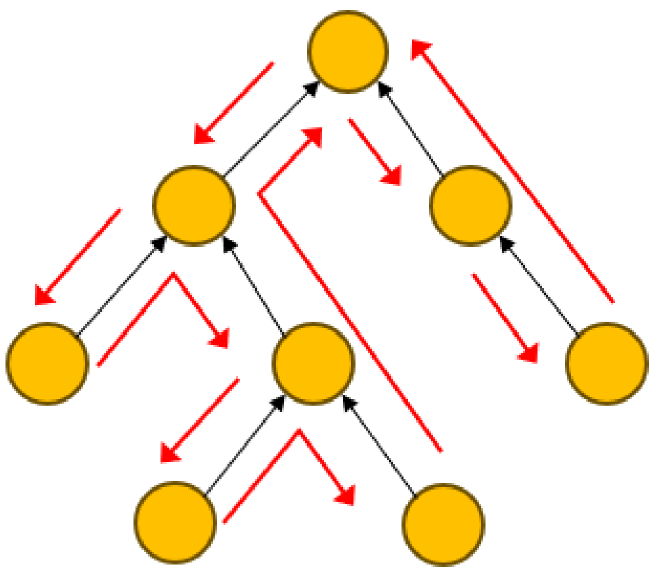
validate	explain	analyze	address
▪ identify bias due to aggre-gation, SUT, other model assumptions ▪ find data errors ▪ compare dif-ferent MRIOs in detail	▪ shed light on the black box ▪ illustrate the concept of input-output analysis and extensions ▪ find com-pelling GVC examples	▪ reveal driving GVCs behind multipliers and derived KPIs ▪ identify key causes of model projec-tions ▪ dynamics	▪ assign measures and respon-sibilities to agents along GVCs ▪ derive policy recommen-dations

The algorithm behind - How to scan Input-Output networks

- Breadth-first (BFS): algorithm scans graph level by level
- Input-Output case: series expansion of Leontief inverse



- Depth-first (DFS): recursive traversal along branches
- Input-Output: e. g. recursive network decomposition



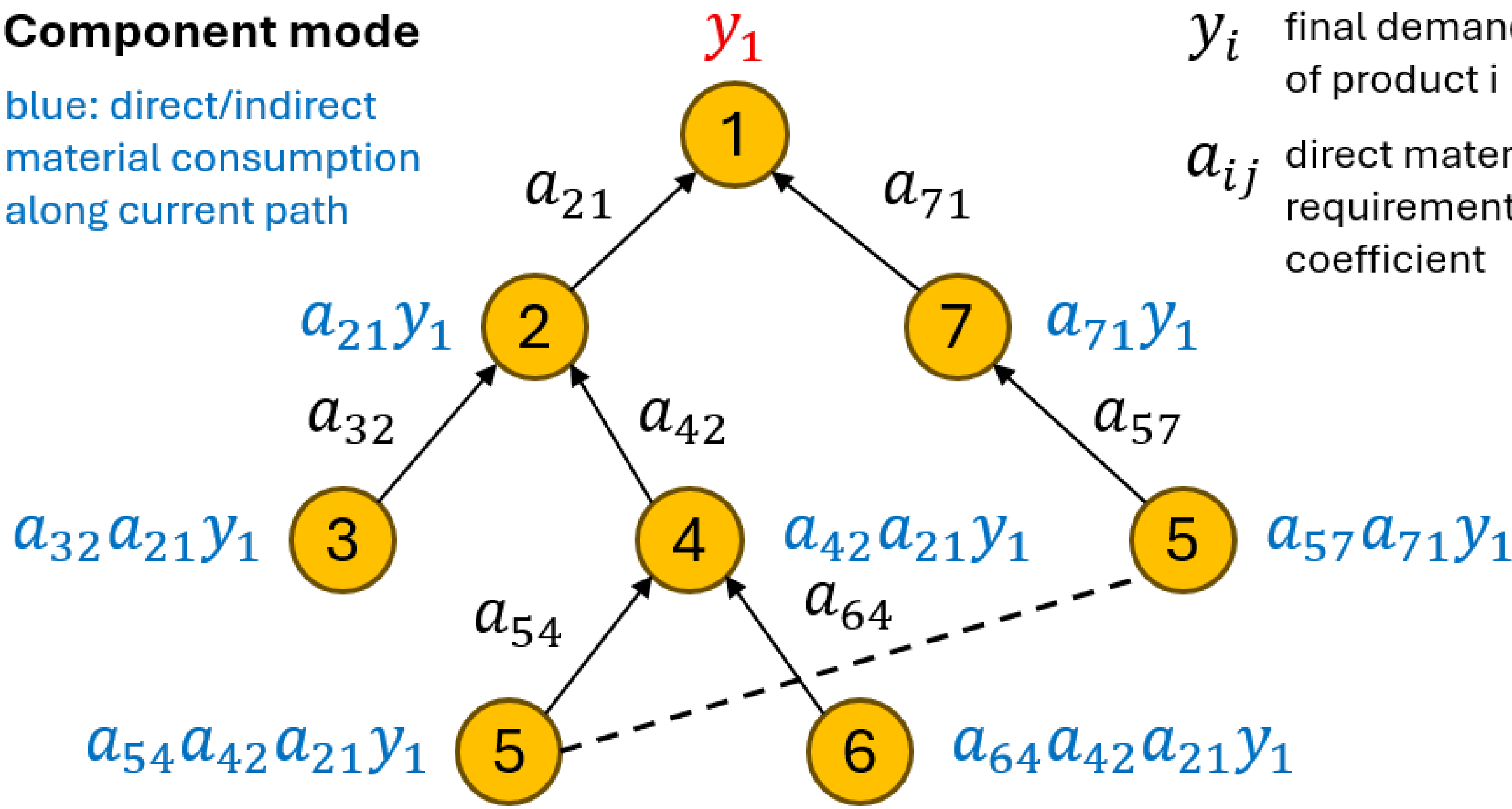
Network decomposition of multipliers in component mode

Component mode

blue: direct/indirect material consumption along current path

y_i final demand of product i

a_{ij} direct material requirement coefficient



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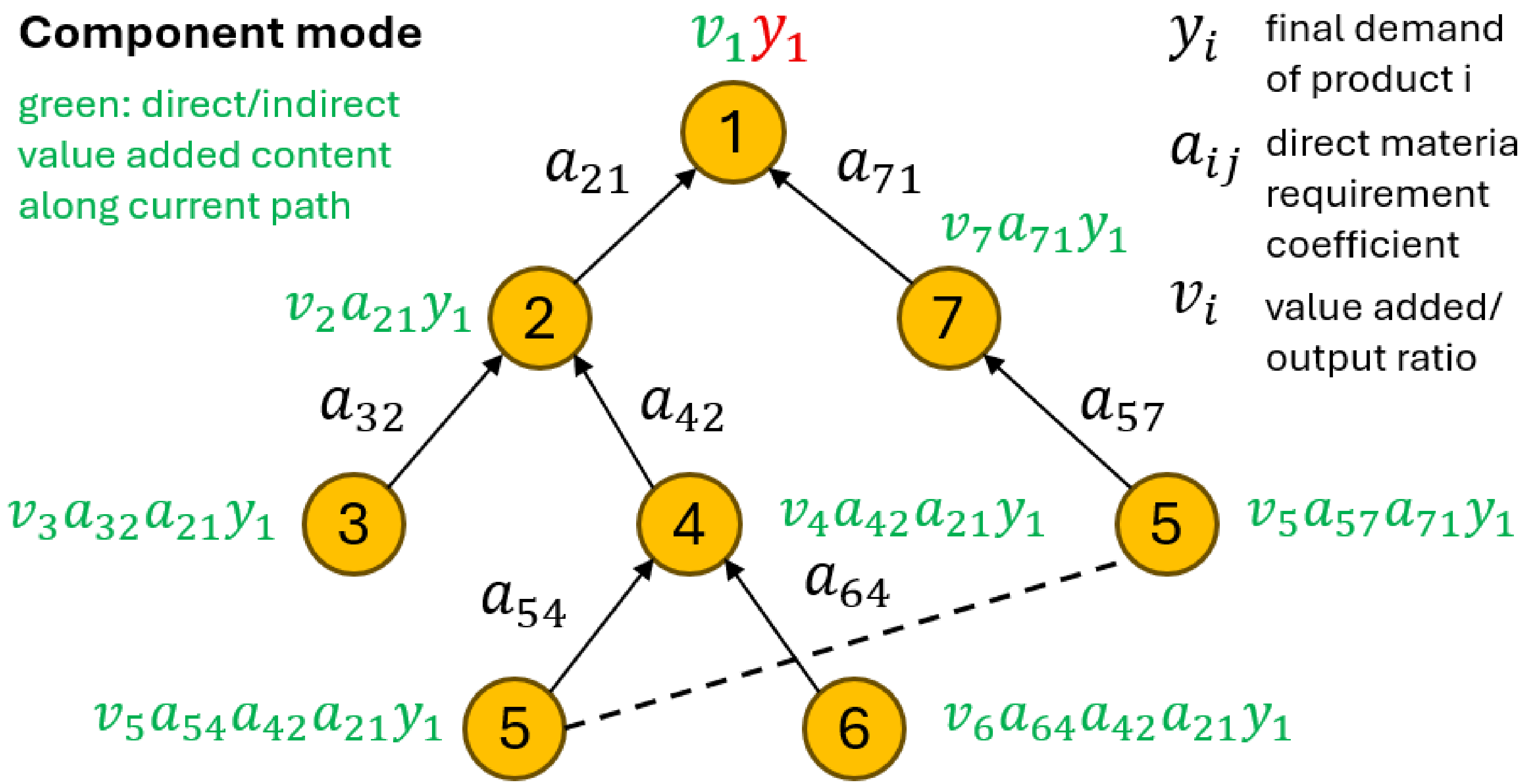
Network decomposition of VA content in component mode

Component mode

green: direct/indirect value added content along current path



y_i final demand of product i

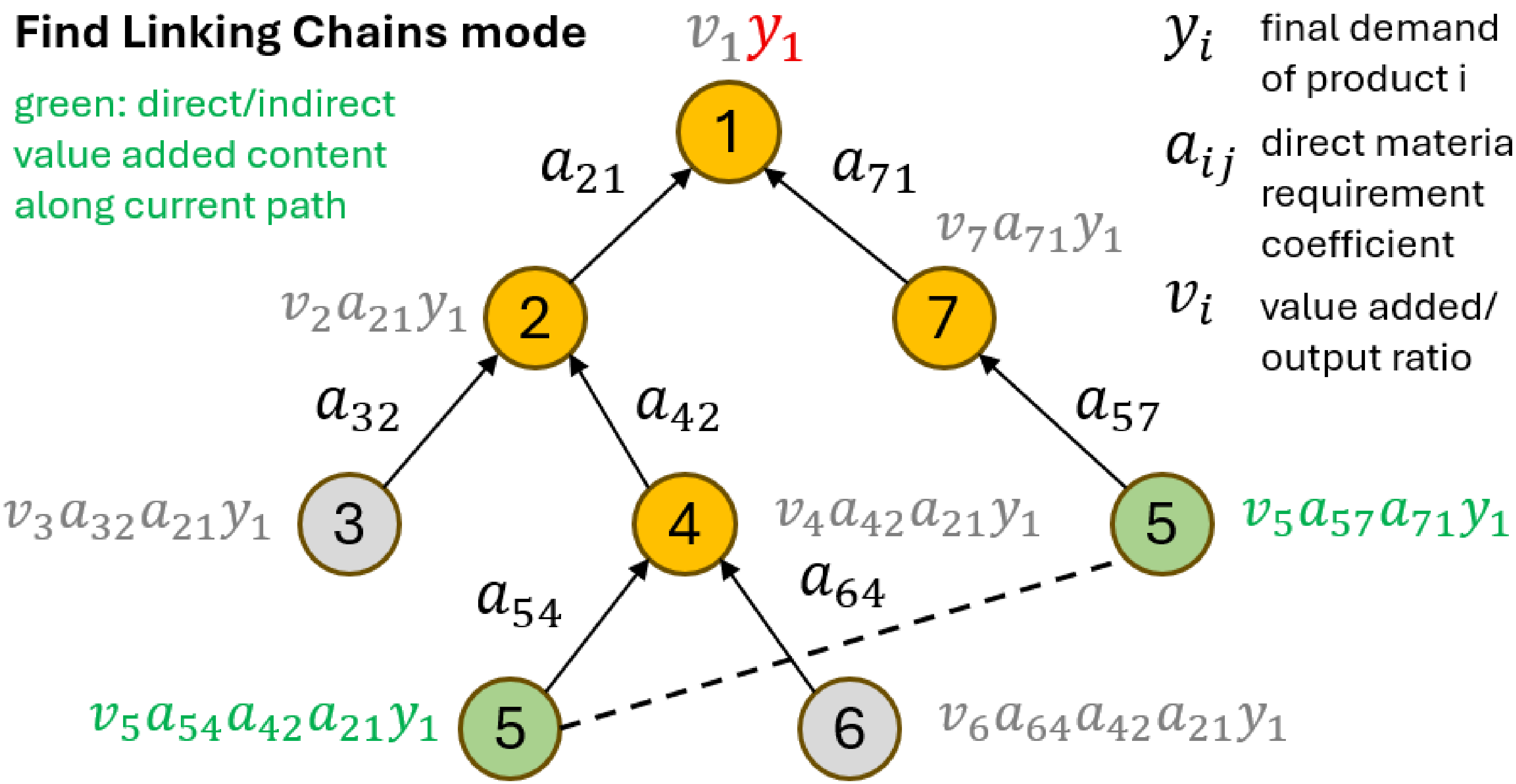
a_{ij} direct material requirement coefficient

v_i value added/ output ratio

Decomposition of VA content in "Find Linking Chains" mode

Find Linking Chains mode

green: direct/indirect value added content along current path



y_i final demand of product i

a_{ij} direct material requirement coefficient

v_i value added/ output ratio

Total value added content of material 5 in product 1: $w_{51} = v_5(a_{54}a_{42}a_{21} + a_{57}a_{71})y_1$

Hierarchical table for tracing the calculation

Find Linking Chains of "Product 5" for "Product 1"

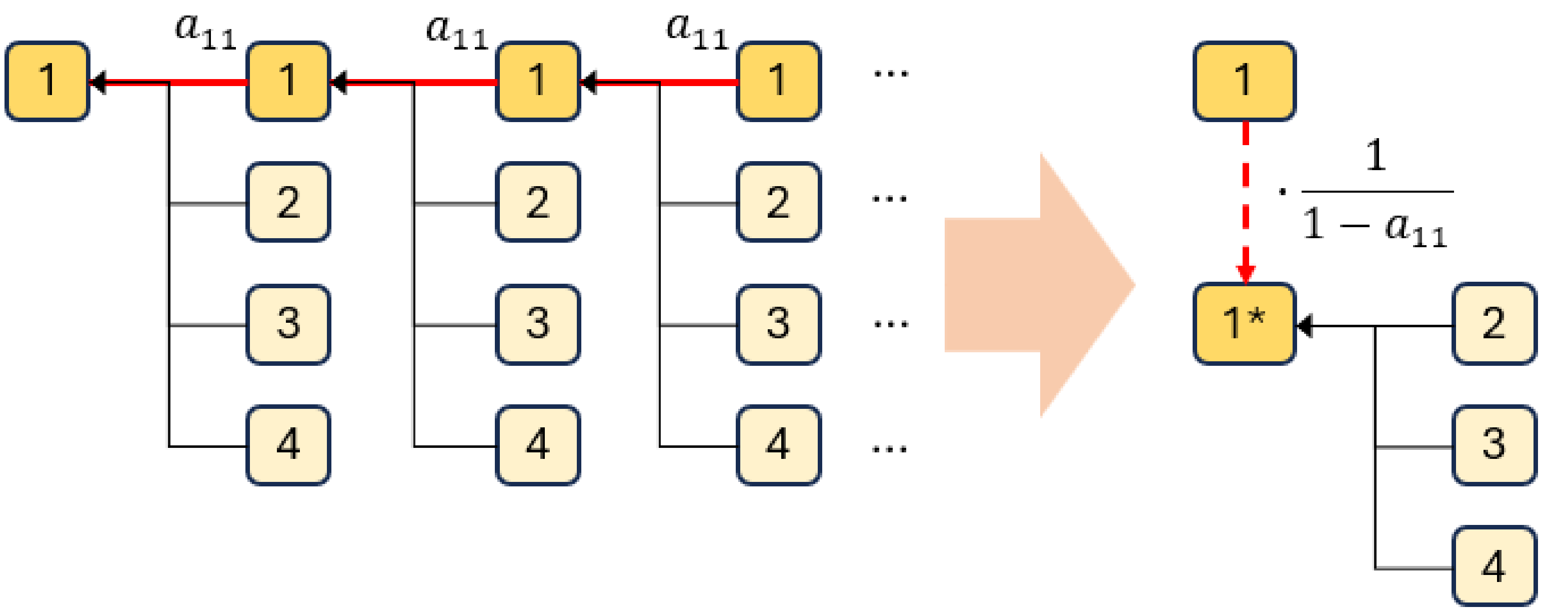
		Material coefficient		Material	VA	Value added
		direct	total	consumption*	ratio	content
		a	*b = b(-1) * a*	*c = b * y1*	*d*	*e = d * c*
Total in report			$a_{54}a_{42}a_{21} + a_{57}a_{71}$	$a_{54}a_{42}a_{21}y_1 + a_{57}a_{71}y_1$		$v_5a_{54}a_{42}a_{21}y_1 + v_5a_{57}a_{71}y_1$
1	Product 1			y_1	v_1	v_1y_1
2	. Product 2	a_{21}	a_{21}	$a_{21}y_1$	v_2	$v_2a_{21}y_1$
3	. . Product 4	a_{42}	$a_{42}a_{21}$	$a_{42}a_{21}y_1$	v_4	$v_4a_{42}a_{21}y_1$
4	. . . Product 5	a_{54}	$a_{54}a_{42}a_{21}$	$a_{54}a_{42}a_{21}y_1$	v_5	$v_5a_{54}a_{42}a_{21}y_1$
5	. Product 7	a_{71}	a_{71}	$a_{71}y_1$	v_7	$v_7a_{71}y_1$
6 → 4	. . Product 5	a_{57}	$a_{57}a_{71}$	$a_{57}a_{71}y_1$	v_5	$v_5a_{57}a_{71}y_1$

* line 1: Final Demand

Eliminating the echo effect of self-consumption

strong self-consumption causes redundant replication of sub-networks in flowcharts

instant inversion eliminates effect



Use case 2 - GDP decomposition by supply chain activity type

$$\hat{V}B\hat{Y} = \underbrace{\hat{V}L\hat{Y}^L}_{D} + \underbrace{\hat{V}L\hat{Y}^E}_{E} + \underbrace{\hat{V}LA^E L\hat{Y}^L}_{S} + \underbrace{\hat{V}LA^E\left(B\hat{Y} - L\hat{Y}^L\right)}_{C}$$

$\hat{V}B\hat{Y}$ value added contribution matrix by country/product of origin and country/product of final use

D pure domestic value chains

E classic trade (export for final use)

S simple global value chains

C complex global value chains

B global Leontief inverse

L local Leontief inverse $L = (E - A^L)^{-1}$

$\hat{Y}^L$ diagonal submatrix of $\hat{Y}$

A^L diagonal submatrix of A

$\hat{Y}^E$ off-diagonal submatrix of $\hat{Y}$

A^E off-diagonal submatrix of A

V value-added coefficients

Source: Zhi Wang, Shang-Jin Wei, Xinding Yu, Kunfu Zhu (2021). Tracing Value Added in the Presence of Foreign Direct Investment, NBER Working Paper No. 29335, Cambridge, MA, October 2021, p. 13, formula (1)

Value added contribution matrix by country

$\hat{V} B \hat{Y}$

VA contribution of country of origin to country of final use - bilateral trade **and** via 3rd countries

Selection: type of supply chain activity

Total
Total
local to local Final Use
local to export for direct Final Use
simple GVC
complex GVC

sorting criteria

Row totals: Value added produced in each country

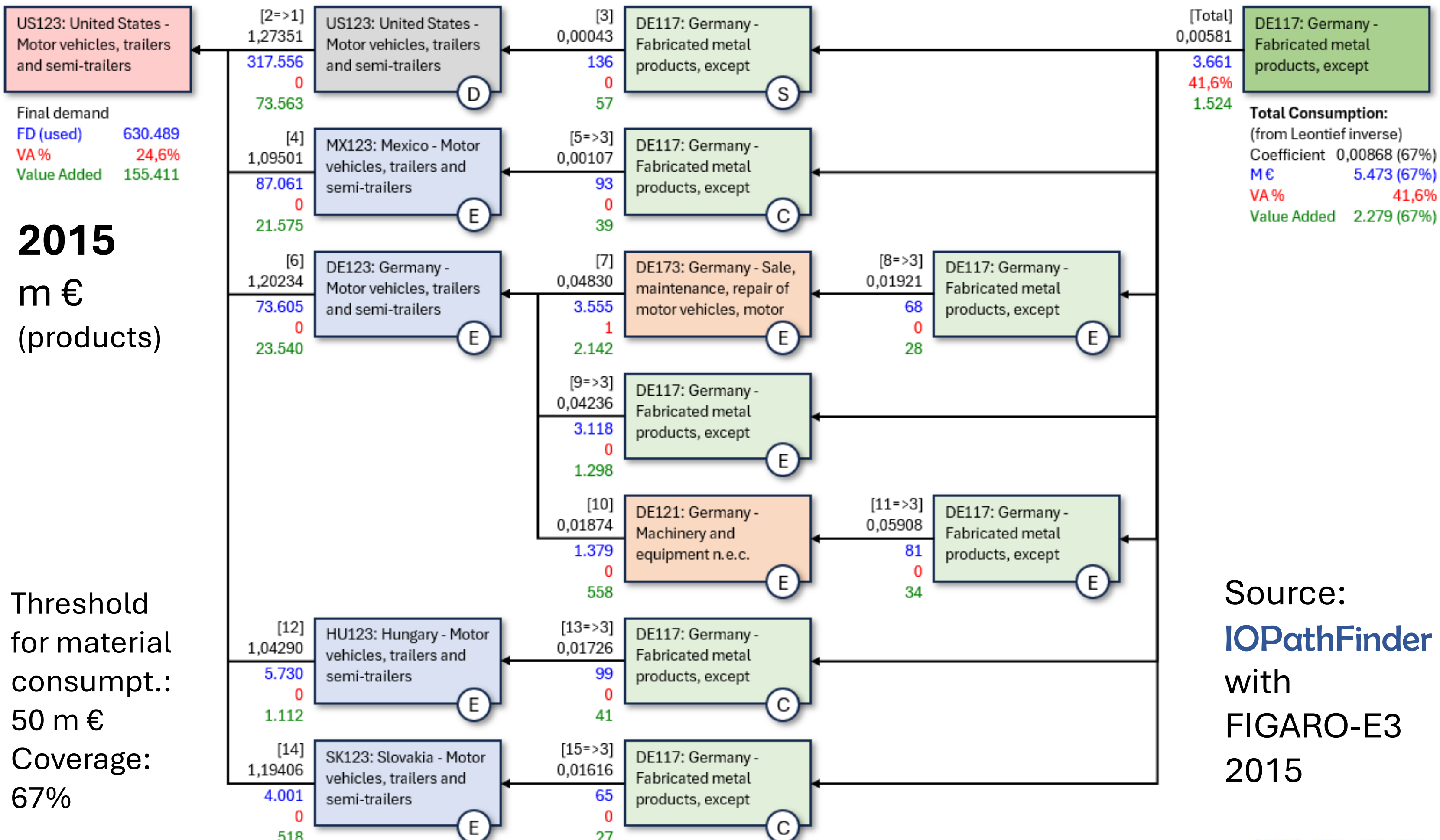
M € 2015 Selling country		Buying country Total	1 Germany	2 United States	3 Rest of the World	4 China	5 France	6 Italy
		64,932,132	**2,534,258**	**16,527,118**	**8,632,743**	**9,558,416**	**2,060,154**	**1,493,504**
1	United States	**16,062,633**	56,457	14,345,284	394,130	223,613	54,792	20,998
2	Rest of the world	**8,436,606**	72,047	460,901	6,484,979	417,128	60,086	35,823
3	China	**9,846,205**	55,765	351,564	545,604	8,159,935	34,517	21,678
4	Canada	**1,340,155**	5,949	168,369	33,877	34,467	5,397	2,496
5	Germany	**2,791,212**	1,954,766	164,784	100,329	82,419	58,515	36,011
6	Japan	**3,983,587**	10,654	157,003	132,524	114,771	6,294	4,144
7	Mexico	**1,006,439**	5,012	126,033	25,315	16,341	3,207	2,395
8	United Kingdom	**2,504,962**	25,336	111,463	65,765	24,399	17,107	11,350
9	India	**1,815,477**	7,514	86,681	85,780	20,899	4,021	3,499
10	South Korea	**1,245,712**	6,512	73,022	76,953	99,590	3,341	2,961

Source: own calculations based on FIGARO-E3 2015 (EC, JRC)

Column totals: domestic final use of each country

Linking Chains - **DE** Fabricated metal products for **US** Motor vehicles

Supply chain activity: D domestic E export for FD S simple GVC C complex GVC



2015

m €

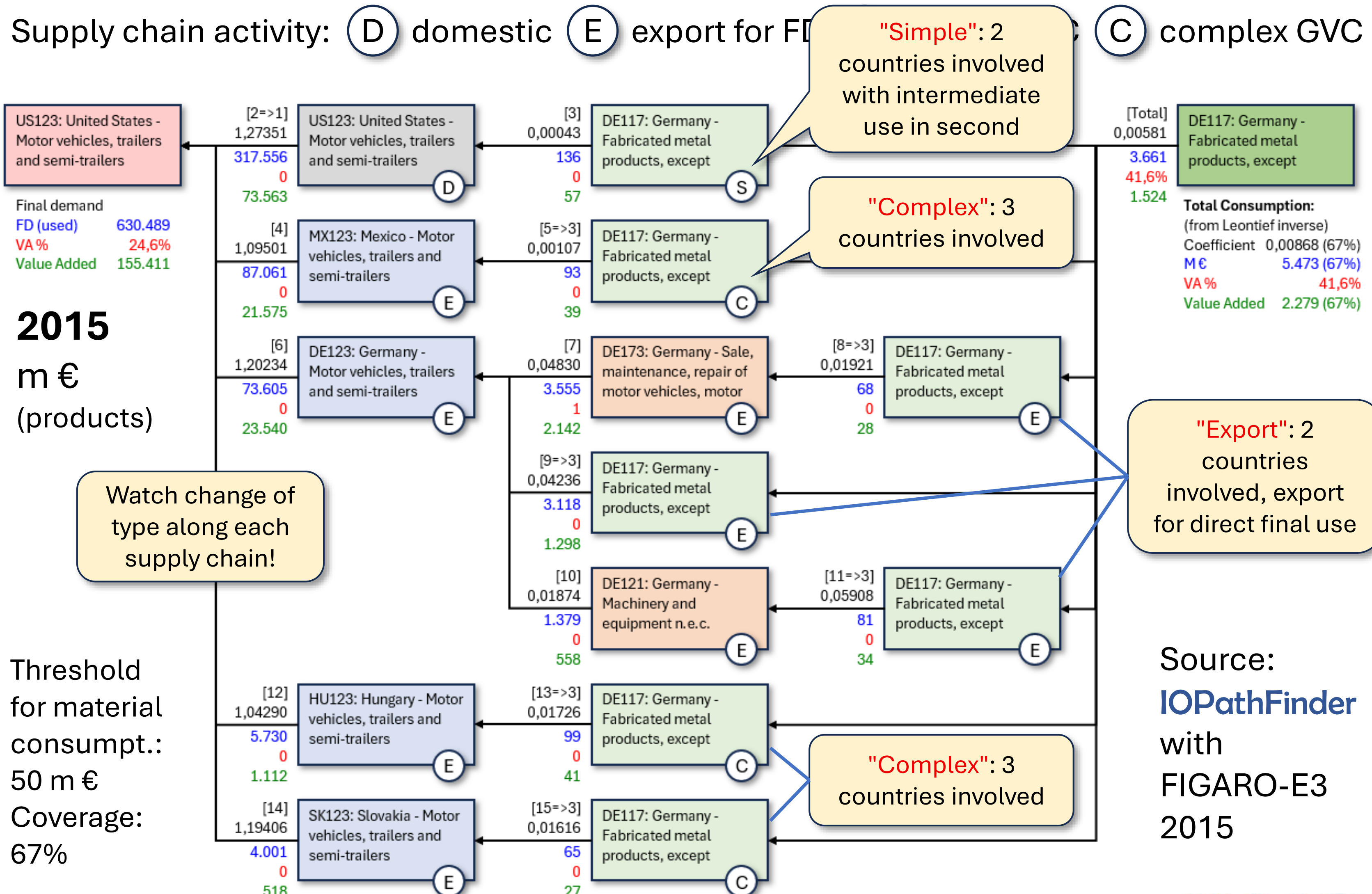
(products)

Threshold for material consumpt.: 50 m €

Coverage: 67%

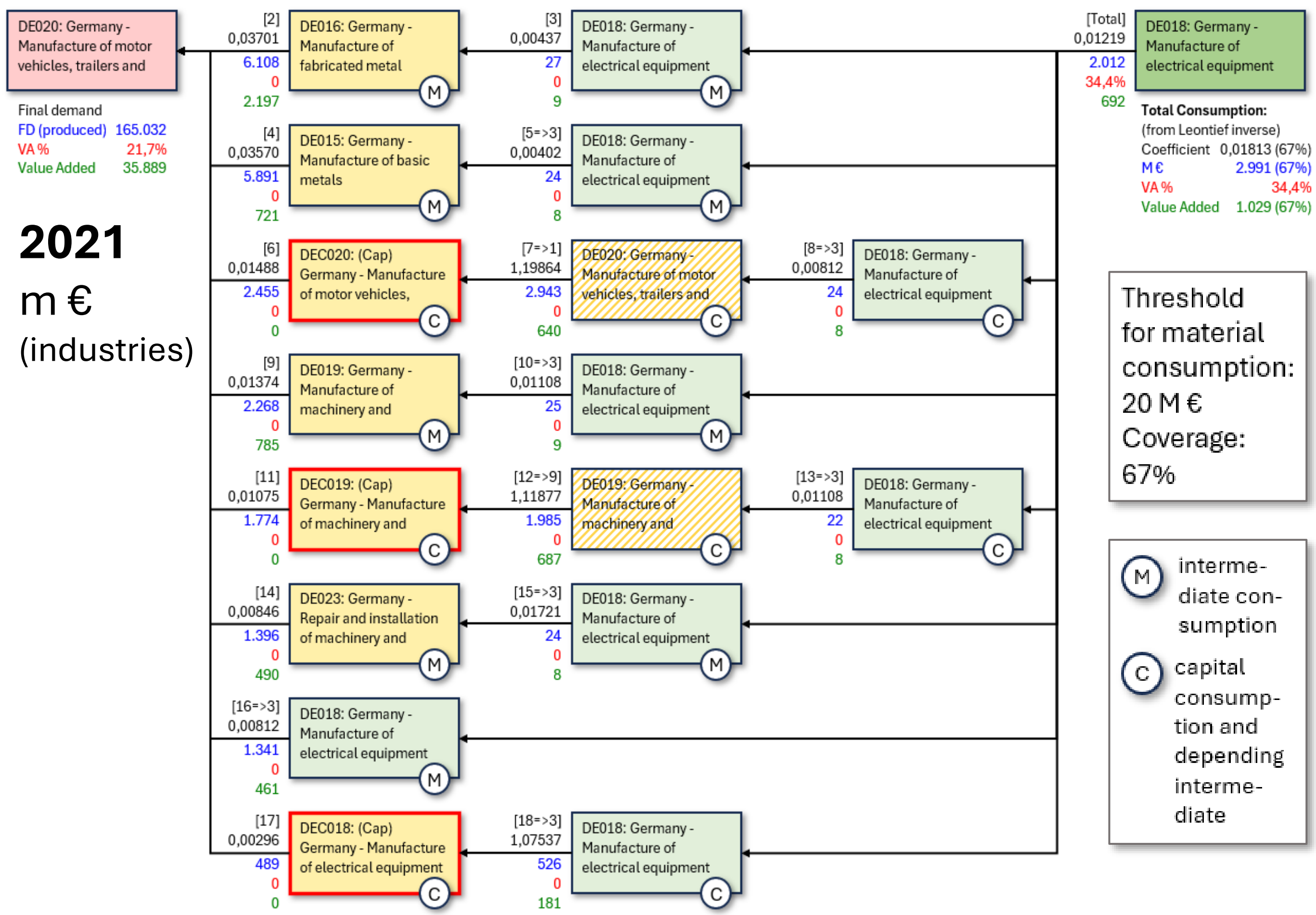
Source: IOPathFinder with FIGARO-E3 2015

Linking Chains - **DE** Fabricated metal products for **US** Motor vehicles



Use case 3 - Endogenizing VA embodied in capital goods

DE Electrical equipment for Motor vehicles and parts - incl. capital goods



Source: IOPathFinder, FIGARO 2021 (2025), Capital flow matrix: J. Blain, La Société Nouvelle (2026)

A few words about the underlying Excel/Python integration

- The frontend is always a Microsoft Excel report (flowchart or table), not a Python IDE.
- Python scripts are called from Excel and can be displayed in each report.
- Typical Python installations are automatically detected, with check for required packages.
- Built-in "Data explorer" displays data objects from annotated list in Excel or Python format.
- ETL processes for data preparation are included in the Excel workbook, starting with the original data files from websites.
- Python syntax and runtime errors are handled via redirection of outputs.

For more information…

Please refer to the full paper available on the conference website

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Königswinter / Germany

Königswinter, May 2026

32nd IIOA Conference, Seville, Spain, June 22-26, 2026

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Contents

Contact the author for a live demonstration and further information

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* Please refer to the full paper on the conference website for data set references.

MS Copilot about Input-Output researchers and stakeholders

Give me a fixed point, and I will move the earth. (Archimedes)

Thank You!

The stakeholder

Politicians, managers, civil society,...

The Input-Output researcher

Image generated with Microsoft Copilot (GPT-4o) on April 30, 2026

Disclaimer

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