

# Miyazawa meet Christaller: Spatial multipliers within a Triple Decomposition of Input-output Central Place Systems.

**Michael Sonis**

Department of Geography, Bar Ilan University, 52900, Ramat-Gan, Israel and  
Regional Economics Applications Laboratory, University of Illinois, 607 South Mathews, #220, Urbana,  
Illinois, 61801-3671 email: [sonism@mail.biu.ac.il](mailto:sonism@mail.biu.ac.il)

**Geoffrey J. D. Hewings**

Regional Economics Applications Laboratory, University of Illinois, 607 South Mathews, #220, Urbana,  
Illinois, 61801-3671 email: [hewings@uiuc.edu](mailto:hewings@uiuc.edu)

**ABSTRACT:** A new triple *UDL*-factorization (*U* represents an upper-triangular block matrix, *L* represents a lower-triangular block matrix, and *D* represents the diagonal block matrix) of the Leontief inverse is based on the Schur block-inversion of matrices. This factorization is applied to the decomposition of the Leontief inverse for input-output systems within a central place hierarchy of the Christaller-Lösch, Bekmann-McPherson type. Such a factorization reflects the process of gradual complication of the central place hierarchy and the parallel augmentation of backward and forward linkages within it. In this scheme the classical Miyazawa interrelation income multipliers play the role of spatial backward and forward linkages multipliers within developing hierarchical central place systems of towns, cities and central capital.

## 1. Introduction: The problem posed.

The idea to investigate the input-output relationship within the central place system is not a new one. The necessity to combine together the hierarchical structure of central place system with the input-output structure of the transaction flows within one unifying framework was stressed in the programmatic paper of Isard (1960, p.141). The next attempt to mention some ideas of central place theory useful for the description of the regional economic interaction was made by Chalmers *et al.*, 1978; however, no attention was directed to the intricate hierarchical structure of central place systems. A more systematic treatment of this problem was undertaken by Robison and Miller (1991). They used the rudimentary structure of (capital and neighboring cities) an intercommunity central place system, without paying attention to the fine structure of the central place hierarchy. The complexity of mathematical presentation was limited to the level of a simple two-community two-order sub region level with one dominant central place.

In this paper, the theory of central place hierarchies and multi-regional input-output analysis will be integrated. As a result, it will be possible to show in what way the decomposition of the Leontief inverse for input-output central place systems reflects the process of complication of the evolving hierarchy of central places. The main feature of the discussion is the interpretation of

the classical Miyazawa interrelational multipliers as *spatial* multipliers in the developing structure of backward and forward linkages within a three-level hierarchical central place systems of towns, cities and central capital.

In the next section, the description of the central place system will be provided. Section 3 will apply a triangular decomposition of the Leontief inverse for an input-output based central place system. Sections 4 and 5 provide the decomposition for a top-down and a bottom-up system. Section 6 introduced the Miyazawa income-consumption distribution framework that is then integrated into the central place system in section 7. The fine structure of this distribution is revealed in section 8 and the paper concludes with some reflections on challenges for empirical implementation.

## **2. Structure of the Classical Central Place System.**

The spatial description of the original Christaller central place model is based on three generic geometric properties of central places associated this central place system (see Sonis, 1986):

1. The first property is that all hinterland areas of the central places at the same hierarchical level form a hexagonal covering of the plane with the centers on the homogeneous triangular lattice;
2. The second property is that the size of the hinterland areas increases from the smallest (on the lower tier of the central place hierarchy) to the largest (on the highest tier of hierarchy) by a constant nesting factor  $k$ . This nesting factor expresses one of the Christaller's three principles, namely, marketing ( $k=3$ ), transportation ( $k=4$ ) and administrative ( $k=7$ ) principles;
3. The third property is that the center of a hinterland area of a given size is also the center of an hinterland of each smaller size (Christaller, 1933, Sonis, 1985)

The Beckmann-McPherson (1970) central place model differs from the Christaller framework by applying variable nesting factors and by using the Löshian principle of all possible coverings of the plane by hexagons of variable integer sizes. Their centers are the vertices of the initial Christaller triangular lattice (Lösch, 1940).



Further, the input-output partial central place system, including only two hierarchical levels of central city  $K$  and intermediate cities  $C$  and not including the level of towns  $T$ , corresponds to the block matrix of direct inputs

$$A(T) = \begin{bmatrix} A_{CC} & A_{CK} \\ A_{KC} & A_{KK} \end{bmatrix} \quad (4)$$

Its Leontief inverse

$$B(T) = [I - A(T)]^{-1} = \begin{bmatrix} B_{CC}^2(T) & B_{CC}^2(T)A_{CK}B_K \\ B_{KK}^2(T)A_{KC}B_C & B_{KK}^2(T) \end{bmatrix} \quad (5)$$

includes the components

$$B_{CC}^2(T) = [I - A_{CC} - A_{CK}B_KA_{KC}]^{-1}; B_{KK}^2(T) = [I - A_{KK} - A_{KC}B_C A_{CK}]^{-1} \quad (6)$$

The triangular decomposition (see Sonis and Hewings, 2000, p.571)

$$\begin{aligned} B(T) &= \begin{bmatrix} I & \\ B_K A_{KC} & I \end{bmatrix} \begin{bmatrix} B_{CC}^2(T) & \\ & B_K \end{bmatrix} \begin{bmatrix} I & A_{CK} B_K \\ & I \end{bmatrix} = \\ &= \begin{bmatrix} I & \\ L_{KC} & I \end{bmatrix} \begin{bmatrix} B_{CC}^2(T) & \\ & B_K \end{bmatrix} \begin{bmatrix} I & U_{CK} \\ & I \end{bmatrix} \end{aligned} \quad (7)$$

allows interpretation of the component  $B_{CC}^2(T)$  as an extended Leontief inverse of the hierarchical level of intermediate cities  $C$  under the influence of the central city  $K$ . The backward and forward linkages together generate the feedback loop of the form:

$$K \rightarrow C \rightarrow K$$

*< insert figure 1 here >*

The augmented inputs representing the coinfluence of different hierarchical levels (see Sonis and Hewings, 1998) have a form:

$$\begin{aligned} A_{TC}^3 &= A_{TC} + A_{TK}B_KA_{KC}; A_{TK}^3 = A_{TK} + A_{TC}B_C A_{CK} \\ A_{CT}^3 &= A_{CT} + A_{CK}B_KA_{KT}; A_{CK}^3 = A_{CK} + A_{CT}B_T A_{TK} \\ A_{TC}^3 &= A_{KT} + A_{KC}B_C A_{CT}; A_{KC}^3 = A_{KC} + A_{KT}B_T A_{TC} \end{aligned} \quad (8)$$

Moreover, the component

$$B_{TT}^3 = \left[ I - A_{TT} - A_{TC} B_{CC}^2(T) A_{CT}^3 - A_{TK} B_{KK}^2(T) A_{KT}^3 \right]^{-1} \quad (9)$$

represents the extended Leontief inverse of the hierarchical level of towns,  $T$ , under the influence of the central city  $K$  and the intermediate cities  $C$ .

In the triangular decomposition (2) the lower triangular matrix

$$L = \begin{bmatrix} I & & & \\ B_{CC}^2(T) A_{CT}^3 & I & & \\ B_{KK}^2(T) A_{KT}^3 & B_K A_{KC} & I & \\ & & & I \end{bmatrix} = \begin{bmatrix} I & & & \\ L_{CT} & I & & \\ L_{KT} & L_{KC} & I & \\ & & & I \end{bmatrix} \quad (10)$$

represents the backward linkages of all three hierarchical levels such that the second block column of  $L$ :

$$\begin{bmatrix} 0 \\ I \\ B_K A_{KC} \end{bmatrix} = \begin{bmatrix} 0 \\ I \\ L_{KC} \end{bmatrix}$$

represents the new backward linkages of the subsystem of two hierarchical levels of central city  $K$  and intermediate cities  $C$  appearing as a result of adding to the economics of central city  $K$  the economics of intermediate cities  $C$ ; the first column of the matrix  $L$ :

$$\begin{bmatrix} I \\ B_{CC}^2(T) A_{CT}^3 \\ B_{KK}^2(T) A_{KT}^3 \end{bmatrix} = \begin{bmatrix} I \\ L_{CT} \\ L_{KT} \end{bmatrix}$$

represents the extension of the backward linkages within the system  $C$  appearing as a result of further extension of to the economics of central city  $K$  and intermediate cities  $C$  with the help of adding the economy of hierarchical level of the towns  $T$ .

Analogously, the upper triangular matrix

$$U = \begin{bmatrix} I & A_{TC}^3 B_{CC}^2(T) & A_{TK}^3 B_{KK}^2(T) & \\ & I & A_{CK} B_K & \\ & & I & \\ & & & I \end{bmatrix} = \begin{bmatrix} I & U_{TC} & U_{TK} & \\ & I & U_{CK} & \\ & & & I \end{bmatrix} \quad (11)$$

represents the growing system of forward linkages within the augmenting system of all three hierarchical levels.

The backward and forward augmentation of linkages can be presented with the help of following spatial scheme:

*<Insert here scheme>*

In this scheme three feedback loops of spatial economic dependencies:

$$\begin{array}{l} C \rightarrow T \rightarrow C \\ K \rightarrow T \rightarrow K \\ K \rightarrow C \rightarrow K \end{array}$$

The diagonal block matrix

$$D = \begin{bmatrix} B_{TT}^3 & & \\ & B_{CC}^2(T) & \\ & & B_K \end{bmatrix} \quad (12)$$

represents the Leontief inverse of the hierarchical level of the central city  $K$  and the extended Leontief inverses corresponding to the augmentation of the level  $K$  to the two hierarchical levels  $K, C$  and to the three hierarchical levels  $K, C, T$ .

The augmentation of the backward and forward linkages in three-tier hierarchical system is presented in figure 2. This figure presents the stages of development of the Central Place system starting from one central place (capital) and gradually expending to two hierarchical levels (capital and neighboring cities) and further, to three hierarchical levels (capital, neighboring cities and surrounding towns)

*< insert figure 2 here >*

In the two following sections, the decomposition will be presented for a top-down and bottom up central place model respectively.

#### **4. Decomposition of Leontief Inverse for Input-Output “top-down” Central Place Model**

If the forward linkages in the system  $K, C, T$  are negligible, i.e. the matrix of direct inputs will have the form

$$A = \begin{bmatrix} A_{TT} & 0 & 0 \\ A_{CT} & A_{CC} & 0 \\ A_{KT} & A_{KC} & A_{KK} \end{bmatrix} \quad (13)$$

then the triangular representation (2) will include only backward linkages:

$$B = \begin{bmatrix} I & & & \\ B_C A_{CT} & I & & \\ B_K (A_{KT} + A_{KC} B_C A_{CT}) & B_K A_{KC} & I & \end{bmatrix} \begin{bmatrix} B_T \\ B_C \\ B_K \end{bmatrix} \quad (14)$$

This decomposition corresponds to the circulation of flows of intermediate flows within the central places and to the case of “top-down” transaction flows where each central place includes the same structure of industries and is sending “transaction flows” of direct inputs to each dependent central place within its own hinterland area and exchanging production with the central places of the same hierarchical tier.

## 5. Decomposition of the Leontief Inverse for Input-Output “bottom-up” Central Place Model

The “bottom-up” system of transaction flows corresponds to the case of negligible backward linkages with the following block-matrix of direct inputs:

$$A = \begin{bmatrix} A_{TT} & A_{TC} & A_{TK} \\ 0 & A_{CC} & A_{CK} \\ 0 & 0 & A_{KK} \end{bmatrix} \quad (15)$$

The corresponding triangular decomposition of the Leontief inverse will include only forward linkages:

$$B = \begin{bmatrix} B_T & & \\ & B_C & \\ & & B_K \end{bmatrix} \begin{bmatrix} I & (A_{TC} + A_{TK} B_K A_{KC}) B_C & A_{TK} B_K \\ & I & A_{CK} B_K \\ & & I \end{bmatrix} \quad (16)$$

The block-structure form of the triangular decomposition (2) allows the incorporation of the fine structure of the spatial economic dependencies between central places within different hierarchical levels. Each such specifications of this fine structure can be incorporated into the triangular decomposition (2). Next, consideration will be given to the income generated from production and the impact of its expenditures on goods and services produced in the system. While exchange in production need not be hierarchical, it will be assumed that consumption of goods is made according to the usual central place principles of seeking the good at the nearest place.

## 6. Miyazawa Income-Consumption Distribution in the Input-Output Model.

Miyazawa (1968, and 1976 for the most complete exposition) introduced the matrix model:

$$\begin{bmatrix} X \\ Y \end{bmatrix} = \begin{bmatrix} A & C \\ V & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} f \\ g \end{bmatrix} \quad (17)$$

with a block matrix  $M = \begin{bmatrix} A & C \\ V & 0 \end{bmatrix}$  for the analysis of the interrelationships among various income groups in the process of income formation. In the Miyazawa interpretation, the matrix  $A$  represents the inter industry direct inputs; vector  $x$  is gross output, vector  $f$  is final demand (excluding consumption expenditures), vector  $Y$  represents the total income, the matrix  $V$  represents the value-added ratios of household sectors; and the matrix  $C$  represents the coefficients of consumption expenditures. Miyazawa considered the usual Leontief interindustry inverse  $B = (I - A)^{-1}$ ; the matrix multiplier  $VB$  showing the induced income earned from production activities among industries; the matrix multiplier  $BC$  showing the induced production due to endogenous consumption (per unit of income) in each household sector; the matrix multiplier  $L = VBC$  showing interrelationships among incomes through the process of propagation from consumption expenditures.

The matrix  $K = (I - L)^{-1} = (I - VBC)^{-1}$  is interpreted as the interrelational income multiplier, and the matrix multiplier  $KVB$  is interpreted as the matrix multiplier of income formation.

Further, the following Frobenius-Schur form:

$$B_e = (I - A - CV)^{-1} \quad (18)$$

may be referred to as the enlarged Leontief inverse; its properties are:

$$KVB = VB_e; \quad BCK = B_e C \quad (19)$$

For the Miyazawa income generation scheme based on block matrix  $M = \begin{bmatrix} A & C \\ V & 0 \end{bmatrix}$  the Leontief block-inverse has the following decomposition that separates the backward and forward linkages effects:

$$B(M) = (I - M)^{-1} = \begin{bmatrix} B_e & B_e C \\ VB_e & K \end{bmatrix} = \begin{bmatrix} B_e & BCK \\ KVB & K \end{bmatrix} \quad (20)$$

The structure of the interconnection between the interrelational income multiplier  $K$  and the enlarged Leontief inverse  $B_e$  can be seen and interpreted from the following formula:

$$K = I + VB_e C \quad (21)$$

where the enlarged matrix multiplier  $L_e = VB_e C$  shows the interrelationships among incomes through the process of enlarged propagation from consumption expenditures.

Next, the fundamental *UDL*-factorization

$$B(M) = (I - M)^{-1} = \begin{bmatrix} B_e & BCK \\ KVB & K \end{bmatrix} = \begin{bmatrix} I & BC \\ 0 & I \end{bmatrix} \begin{bmatrix} B & 0 \\ 0 & K \end{bmatrix} \begin{bmatrix} I & 0 \\ VB & I \end{bmatrix} \quad (22)$$

separates multiplicatively the induced income earned from production activities among industries  $VB$  and the induced production due to endogenous consumption (per unit of income) in each household sector  $BC$  from the industrial production activities. In this factorization, the interrelational income multiplier,  $K$ , appears explicitly and the enlarged Leontief inverse,  $B_e$ , appears implicitly. The structure of the interconnection between the enlarged Leontief inverse,  $B_e$ , and the interrelational income multiplier,  $K$ , can be seen and interpreted from the following formula:

$$B_e = B + BCKVB \quad (23)$$

## 7. Miyazawa Income-Consumption Distribution in Input-Output Central Place Model

The Miyazawa income-consumption distribution in the input-output hierarchical central place model can be presented with the help of the following matrix:

$$M = \begin{bmatrix} A_{TT} & A_{TC} & A_{TK} & C_T \\ A_{CT} & A_{CC} & A_{CK} & C_C \\ A_{KT} & A_{KC} & A_{KK} & C_K \\ V_T & V_C & V_K & \end{bmatrix} = \begin{bmatrix} \tilde{A} & \tilde{C} \\ \tilde{V} & \end{bmatrix} \quad (24)$$

where the matrix  $\tilde{A}$  is the block matrix of direct inputs on different hierarchical levels of towns, cities and capital city,  $\tilde{V} = (V_T \ V_C \ V_K)$  represents the income corresponding to the different

sectors in towns, cities and capital city and  $\tilde{C} = \begin{pmatrix} C_T \\ C_C \\ C_K \end{pmatrix}$  represents the consumption in the central

place hierarchy. Analogously to formula (2), the following formula provides the following triple decomposition:

$$\begin{aligned} B(M) &= \\ &= \begin{bmatrix} I & & & \\ & I & & \\ & & I & \\ V_T & V_C & V_K & I \end{bmatrix} \begin{bmatrix} I & & \\ L_{CT} & I & \\ L_{KT} & L_{KC} & I \end{bmatrix} \begin{bmatrix} D_T & & \\ & D_C & \\ & & D_K \end{bmatrix} \begin{bmatrix} I & U_{TC} & U_{TK} \\ & I & U_{CK} \\ & & I \end{bmatrix} \begin{bmatrix} I & & C_T \\ & I & C_C \\ & & I \end{bmatrix} \begin{bmatrix} C_T \\ C_C \\ C_K \\ I \end{bmatrix} = \\ &= \begin{bmatrix} L & \\ \tilde{V}L & I \end{bmatrix} \begin{bmatrix} D & \\ & I \end{bmatrix} \begin{bmatrix} U & U\tilde{C} \\ & I \end{bmatrix} \end{aligned} \quad (25)$$

where induced income and induced production are

$$\begin{aligned} \tilde{V}L &= (V_T \ V_C \ V_K) \begin{bmatrix} I & & \\ L_{CT} & I & \\ L_{KT} & L_{KC} & I \end{bmatrix} = (V_T + V_C L_{CT} + V_K L_{KT} \ V_C + V_K L_{KT} \ V_K) \\ U\tilde{C} &= \begin{bmatrix} I & & \\ U_{CT} & I & \\ U_{KT} & U_{KC} & I \end{bmatrix} \begin{bmatrix} C_T \\ C_C \\ C_K \end{bmatrix} = \begin{bmatrix} C_T \\ U_{CT}C_T + C_C \\ U_{KT}C_T + U_{KC}C_C + C_K \end{bmatrix} \end{aligned} \quad (26)$$

Therefore, the Miyazawa interrelation income multiplier (see 23) will have the form:

$$\begin{aligned} K &= I + (V_T \ V_C \ V_K) \begin{bmatrix} I & & \\ L_{CT} & I & \\ L_{KT} & L_{KC} & I \end{bmatrix} \begin{bmatrix} D_T & & \\ & D_C & \\ & & D_K \end{bmatrix} \begin{bmatrix} I & & \\ U_{CT} & I & \\ U_{KT} & U_{KC} & I \end{bmatrix} \begin{bmatrix} C_T \\ C_C \\ C_K \end{bmatrix} = \\ &= (V_T + V_C L_{CT} + V_K L_{KT} \ V_C + V_K L_{KT} \ V_K) \begin{bmatrix} D_T & & \\ & D_C & \\ & & D_K \end{bmatrix} \begin{bmatrix} C_T \\ U_{CT}C_T + C_C \\ U_{KT}C_T + U_{KC}C_C + C_K \end{bmatrix} \end{aligned} \quad (27)$$

Analogously to the spatial feedback loop interpretation of the formula (2), the Miyazawa interrelation income multiplier can be interpreted as spatial multiplier in the central place input-output system.

## 8. The Fine Structure of the Miyazawa Income-Consumption Distribution in Input-Output Central Place Model

Next, a simple case of the structure will be presented that includes, the circulation of flows of intermediate goods within the central places and the “top-down” structure of the transaction flows between the dependent central places only. Such a structure is presented in figure 2.

*<insert figure 2 here>*

This scheme generalizes slightly the usual assumption of classical central place theory demanding that both income earned from production and consumption expenditures from this income will be concentrated in same location. The corresponding block-matrix of direct inputs reflecting the central place hierarchy of such system has a form:



$$B_C = \begin{bmatrix} b_{C_1C_1} & & & & & \\ & b_{C_2C_2} & & & & \\ & & b_{C_3C_3} & & & \\ & & & b_{C_4C_4} & & \\ & & & & b_{C_5C_5} & \\ & & & & & b_{C_6C_6} \end{bmatrix} \quad (30)$$

where

$$b_{T_iT_i} = (I - a_{T_iT_i})^{-1}; b_{C_iC_i} = (I - a_{C_iC_i})^{-1}; i = 1, 2, \dots, 6 \quad (31)$$

and

$$B_C A_{CT} = \begin{bmatrix} b_{C_1C_1} a_{C_1T_1} & & & & & b_{C_1C_1} a_{C_1T_1} \\ b_{C_1C_1} a_{C_1T_1} & b_{C_1C_1} a_{C_1T_1} & & & & \\ & b_{C_1C_1} a_{C_1T_1} & b_{C_1C_1} a_{C_1T_1} & & & \\ & & b_{C_1C_1} a_{C_1T_1} & b_{C_1C_1} a_{C_1T_1} & & \\ & & & b_{C_1C_1} a_{C_1T_1} & b_{C_1C_1} a_{C_1T_1} & \\ & & & & b_{C_1C_1} a_{C_1T_1} & b_{C_1C_1} a_{C_1T_1} \end{bmatrix}, \quad (32)$$

$$B_K A_{KT}^3 = B_K (A_{KT} + A_{KC} B_C A_{KC}) = (b_{KT_1} \quad b_{KT_2} \quad b_{KT_3} \quad b_{KT_4} \quad b_{KT_5} \quad b_{KT_6})$$

$$\text{where } b_{KT_i} = B_K (a_{KT_i} + a_{KC_i} b_{C_iC_i} a_{C_iT_i} + a_{KC_{i+1}} b_{C_{i+1}C_{i+1}} a_{C_{i+1}T_i}), i = 1, 2, \dots, 5, \quad (33)$$

$$\text{and } b_{KT_6} = B_K (a_{KT_6} + a_{KC_1} b_{C_1C_1} a_{C_1T_6} + a_{KC_6} b_{C_6C_6} a_{C_6T_6})$$

$$B_K A_{KC} = (B_K a_{KC_1} \quad B_K a_{KC_2} \quad B_K a_{KC_3} \quad B_K a_{KC_4} \quad B_K a_{KC_5} \quad B_K a_{KC_6}) \quad (34)$$

So the places of non-zero blocks in the matrix  $L$  of the backward linkages can be seen on the following scheme:



$$\tilde{C} = \begin{bmatrix} c_{T_1T_1} & c_{T_1T_2} & & & & & c_{T_1T_6} & c_{T_1C_1} & & & & & c_{T_1C_6} & c_{T_1K} \\ c_{T_2T_1} & c_{T_2T_2} & c_{T_2T_3} & & & & & c_{T_2C_1} & c_{T_2C_2} & & & & & c_{T_2K} \\ & c_{T_3T_2} & c_{T_3T_3} & c_{T_3T_4} & & & & & c_{T_3C_2} & c_{T_3C_3} & & & & c_{T_3K} \\ & & c_{T_4T_3} & c_{T_4T_4} & c_{T_4T_5} & & & & & c_{T_4C_3} & c_{T_4C_4} & & & c_{T_4K} \\ & & & c_{T_5T_4} & c_{T_5T_5} & c_{T_5T_6} & & & & & c_{T_5C_4} & c_{T_5C_5} & & c_{T_5K} \\ c_{T_6T_1} & & & & & c_{T_6T_5} & c_{T_6T_6} & & & & & & c_{T_6C_6} & c_{T_6T_1} & c_{T_6K} \\ c_{C_1T_1} & c_{C_1T_2} & & & & & & c_{C_1C_1} & & & & & & & c_{C_1K} \\ & c_{C_2T_2} & c_{C_2T_3} & & & & & & c_{C_2C_2} & & & & & & c_{C_2K} \\ & & c_{C_3T_3} & c_{C_3T_4} & & & & & & c_{C_3C_3} & & & & & c_{C_3K} \\ & & & c_{C_4T_4} & c_{C_4T_5} & & & & & & c_{C_4C_4} & & & & c_{C_4K} \\ & & & & c_{C_5T_5} & c_{C_5T_6} & & & & & & c_{C_5C_5} & & & c_{C_5K} \\ c_{C_6T_1} & & & & & & c_{C_6T_6} & & & & & & & c_{C_6C_6} & c_{C_6K} \\ c_{KT_1} & c_{KT_2} & c_{KT_3} & c_{KT_4} & c_{KT_5} & c_{KT_6} & c_{KC_1} & c_{KC_2} & c_{KC_3} & c_{KC_4} & c_{KC_5} & c_{KC_6} & & & c_{KK} \end{bmatrix} \quad (37)$$

The formulae (24-37) include all blocks of the Miyazawa interrelational income multiplier (27) and can be used for its analysis.

## 9. Conclusions

In this paper the attempt is described to unify two central theories in the regional science: the classical input-output theory of Leontief and the classical Christaller-Lösch central place theory.

The considerations are presenting the elaboration of complete theoretical framework for such unification. It is expected that the proposed theoretical methodology will be useful for the analysis of organization of the production economics in geographical space.

## References:

- Beckmann, M.J. and J.C. McPherson. 1970."City size distribution in the central place hierarchy: an alternative approach," *Journal of Regional Science*, 10, 243-248.
- Chalmers J A, E J Anderson, T Beckhelm and W Hannigan. 1978."Spatial Interaction in sparsely populated regions: An hierarchical Economic Base Approach", *International Regional Science Review*, vol.3, no. 1, pp. 75-92.
- Christaller, W. 1933. *Die Zentralen Orte in Suddeutschland*, Fischer, Jena (English translation, 1966). *Central places in Southern Germany*, Prentice Hall, Englewood Cliffs, NY.
- Isard W, 1960. "The Scope and Nature of Regional Science: Channels of Synthesis." Reprinted in C. Smith (ed.) 1990. *Practical Methods of Regional Science and Empirical Applications* , Selected Papers of Walter Isard, vol. 2, pp. 137-152.

- Lösch A. 1940. *Die Raeumliche Ordnung der Wirtschaft*, Fischer, Jena. (English translation, 1954 *The Economics of Location*, Yale University Press, New Haven, Connecticut.
- Robison, M.H. and J.R. Miller. 1991. "Central place theory and intercommunity input-output Analysis." *Papers in Regional Science*, 70, 399-417.
- Sonis, M. 1985. "Hierarchical structure of central place System: The barycentric calculus and decomposition principle." *Sistemi Urbani*, 1, 1985, 3-28.
- Sonis, M. 1986. "A Contribution to the central place Theory: Super-imposed Hierarchies, Structural Stability, Structural Changes and Catastrophies in central place Hierarchical Dynamics." In R. Funck and A. Kuklinsky (eds), *Space-Structure- Economy: A Tribute to Alfred Lösch*, Karlsruhe Papers in Economic Policy Research, pp. 159-176.
- Sonis, M. and G.J.D. Hewings. 1998. "Economic Complexity as Network Complication: Multiregional input-output Structural Path Analysis," *Annals of Regional Science*, 32, 407-436.
- Sonis, M. and G.J.D. Hewings. 2000. "LDU-factorization of the Leontief inverse, Miyazawa income multipliers for multiregional systems and extended multiregional Demo-Economic analysis," *Annals of Regional Science*, 34, 569-589.