Aggregation of industry-level TFP-measures

Pirkko Aulin-Ahmavaara*

Statistics Finland and University of Helsinki

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Abstract

In the present paper two similar but in fact different methods, characterised as the deliveries to final demand approach and the value-added approach, for measuring the economy level rate of TFP-growth and aggregating the KLEMS type of industry level measures to the economy level are discussed. The aggregation rules for both approaches are derived starting from the accounting identity for an industry. The results are compared with those obtained by Gollop (1987) with the production possibility frontier and the theory of producer behaviour as the starting point. Like in Gollop (1987) an open economy with nonzero product taxes and subsidies on intermediate inputs is assumed. Unlike Gollop the possibility of nonzero aggregate value for product taxes and subsidies on intermediate inputs as well as the possibility of different industries facing different prices for identical products used as intermediate inputs are allowed for.

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Correspondence to Pirkko Aulin-Ahmavaara, Oulunkyläntori 2 C 16, 00640 Helsinki, Finland, aulin@nettilinja.fi

1. Introduction

The measurement of total factor productivity growth, in its present form, has its roots, mainly, in Solow's (1957) seminal article, in which he demonstrated the equivalence of the economy level TFP-growth and the shift of the production function starting from the theory of production and producer behaviour. Jorgenson and Griliches (1967) on the other hand started from the accounting identity equalising the total value of outputs with the total value of inputs. They aimed at homogeneous inputs and outputs and increased both the number of outputs and the number of inputs to obtain this. Their measure was, in principle, applicable to any producing unit. They gave it an economic interpretation as the shift of production function by introducing the constant returns to scale production function and adding the necessary conditions of producer equilibrium in perfectly functioning markets.

In order to study the contribution of different industries to the economy level productivity growth it is necessary to have an aggregation rule showing in which way the economy level rate can be obtained from the industry level rates or alternatively decomposed into the industry level. Domar (1961) derived the aggregation rule in which each industry-level rate of TFP growth is "weighted by the ratio of the output of its industry to the value of the final product of the sector." Hulten (1978) proved the Domar aggregation rule in the case, in which prices paid by the users of the products are equal to those received by the producers and all industries pay identical prices for their primary inputs. Jorgenson, Gollop and Fraumeni (1987) in their seminal contribution to productivity measurement developed an aggregation rule in which neither of these assumptions are needed. But their system requires assumptions about the existence of both industry and economy level value added functions. Their aggregation rule was similar to, but not identical with, the Domar aggregation, as will be shown in this paper.

Frank Gollop (1987) made a systematic study of the two different approaches to aggregation of industry level productivity measure to the economy level and, in fact, to the very definition of the economy level measure. He derived the aggregation/decomposition rules for TFP measures in an economy maximising the aggregate value of the deliveries to final demand on the one hand and for an economy maximising the aggregate value added on the other, with the respective production possibilities frontiers and the theory of producer behaviour as starting points. Unlike Hulten (1978) Gollop (1987) did not assume either the equality between the prices received and paid for products used as intermediate inputs or all the industries paying identical prices for their primary inputs. Also Aulin-Ahmavaara (2003) discussed the need to take into account the product taxes and subsidies on intermediate inputs in productivity measurement based on national accounts.

In his paper Gollop (1987) did however assume that 1) at the economy level product taxes less subsidies on intermediate inputs cancel out and 2) different industries pay identical prices for products used as intermediate inputs.[1] In the present paper also both of these assumptions are relaxed. Unlike Gollop (1987) we do not start from the production possibilities frontier and market equilibrium conditions. We are rather following the lead of Jorgenson and Griliches (1967) and start with the accounting identities. The economy level and industry level TFP-measures as well as the aggregation rules corresponding to the two models introduced by Gollop are derived in section 2. In section 3 we give our results an interpretation in terms of production theory, appropriately modifying Gollop's models, and compare our results with those obtained by him.

2. Productivity accounting: Two different approaches

2.1. Deriving the rate of TFP-growth from the accounting identity

The derivation of TFP or MFP measures can, following Jorgenson and Griliches (1967) start from the following accounting identity:

(1) $$\mathbf{q}'\mathbf{z} = \mathbf{p}'\mathbf{v}$$

[1] This is obvious for instance from his equation (19).

where $\mathbf{q}$ is the price vector of outputs

$\mathbf{z}$ is the vector of output quantities

$\mathbf{p}$ is the price vector of inputs and

$\mathbf{v}$ is the vector of input quantities.

The rate of total factor productivity growth is defined as the difference between the growth rates of outputs and inputs:

$$(2) \qquad d\log t = \sum_i \alpha_i d\log z_i - \sum_j \beta_j d\log v_j = \sum_j \beta_j d\log q_i - \sum_i \alpha_i d\log p_i \,,$$

where α_i is the share of the ith output in total revenue and β_j the share of the jth input in total cost and $d\log y$ is the logarithmic time derivative of the variable y. This measure can be given an economic interpretation as the shift of the production function when a production function with constant returns to scale is assumed and all the relevant assumptions concerning markets and producer behaviour are made. The economic interpretation will be discussed in more detail in section 3.

Following the SNA93 (ISWGNA, 1993) the accounting identity for an industry with only one type of output is defined as follows:

$$(3) \qquad \begin{aligned} q_j Q_j &= \sum_i q_i M_{ij} + (\sum_i p_{ij} M_{ij} - \sum_i q_i M_{ij}) \\ &+ \sum_i q_i^M M_{ij}^M + (\sum_i p_{ij}^M M_{ij}^M - \sum_i q_i^M M_{ij}^M) + \sum_k p_k^K K_{kj} + \sum_l p_l^L L_{lj} \end{aligned} ,$$

Here the variables are

Q_j quantity of the output of the jth industry

q_j basic price of the output of the jth industry

M_{ij} quantity of the output of the ith industry used as intermediate input by the jth industry

p_{ij} purchaser's price (without trade and transport margins) paid by the j th industry for a unit of the output of the i th industry it uses as intermediate input

M_{ij}^{M} quantity of the i th imported product used as intermediate input by the j th industry

q_i^M c.i.f. price of i th imported product

p_{ij}^{M} purchaser's price (without trade and transport margins) paid by the j th industry for a unit of the i th imported product it uses as intermediate input

M_{ij}^{M} quantity of the i th imported product used by the j th industry as intermediate input

p_{kj}^{K} price paid by the j th industry for the capital input of category k

K_{kj} quantity of the capital input of category k used by the j th industry

p_{lj}^{L} price paid by the j th industry for the labour input of category l

L_{lj} quantity of the labour input of category l used by the j th industry.

When trade and transport margins are treated as separate inputs then the only difference between basic prices and purchasers' prices are taxes and subsidies on products.

Applying the formula in equation (2) to the accounting identity in equation (3) gives the rate of industry level TFP change:

(4)
$$\begin{aligned} d\log t_j = (q_j Q_j)^{-1}[q_j Q_j d\log Q_j - \sum_i q_i M_{ij} d\log M_{ij} - \sum_i (p_{ij} - q_i) M_{ij} d\log M_{ij} \\ - \sum_i q_i^M M_{ij}^M d\log M_{ij}^M - \sum_i (p_{ij}^M - q_i^M) M_{ij}^M d\log M_{ij}^M - \sum_k p_{kj}^K K_{kj} d\log K_{kj} - \sum_l p_{lj}^L L_{lj} d\log L_{ij})] \end{aligned}.$$

2.2. Deliveries to final demand approach

Deliveries to final demand consist of different products valued using some specified price concept. The options are basic prices, producers' prices and purchasers' prices (for the definitions, see ISWGNA, 1993). Here we have chosen basic prices since they represent the prices received by the producers. In order to calculate the value of the deliveries to final output using a specified price concept it is necessary to value also the output as well as the interindustry deliveries using the same price concept.[2] The accounting identity for an industry/ product in this case is

$$(5)\qquad q_j Y_j = q_j Q_j - \sum_i q_j M_{ji}\,.$$

If we wish to consider the economy as one producing unit, then we obviously have to assume that all the industries face identical prices for their inputs. The quantity Z and the price p^Z of an input at the economy level can, in line with JGF (1987), then be defined, on the basis of the industry level quantities and prices, as follows:

$$(6)\qquad \sum_j p_j^Z Z_j = p^Z \sum_j Z_j = p^Z Z\,.$$

Summing over industries in equation (5) and (3) and substituting the former sum into the latter one results, in view of equation (6), in the following economy level accounting equations:

$$(7)\qquad \begin{aligned} \sum_j q_j Y_j &= \sum_j q_j Q_j - \sum_j \sum_i q_j M_{ji} \\ &= \sum_i (p_i - q_i) M_i + \sum_i q_i^M M^M{}_i + \sum_i (p_i^M - q_i^M) M_i^M + \sum_k p_k^K K_k + \sum_l p_l^L L_l. \end{aligned}$$

The economy level rate of TFP change is now obtained, from equation (7), by applying the formula in equation (2):

[2] For more on this, see Aulin-Ahmavaara 2003.

(8)
$$d\log T = (\sum_j q_j Y_j)^{-1}[\sum_j q_j Y_j d\log Y_j - \sum_i (p_i - q_i) M_i d\log M_i - \sum_i q_i^M M^M_i d\log M_i - \sum_i (p^M_i - q_i^M) M_i^M d\log M_i^M - \sum_k p_k^K K_k d\log K_k - \sum_l p_l^L L_l d\log L_l].$$

The second term in the square brackets disappears regardless of the rates of growth of individual intermediate inputs if $p_i = q_i$ for all values of i i.e. if there are no taxes or subsidies on products used as intermediate inputs. On the other hand if $p_i \neq q_i$ for some domestic intermediate inputs the value of the term depends on the rates of growth of individual intermediate inputs. Likewise the fourth term in the brackets disappears if there are no product taxes (e.g. import duties) or subsidies on imported intermediate inputs. We have now obtained:

Result 1. *When the output of an economy is represented by the deliveries to final demand valued at basic prices the economy level rate of TFP growth depends, besides the rates of growth of these deliveries as well as those of labour and capital inputs, also on the rates of growth of imported intermediate inputs. Unless taxes and subsidies on intermediate inputs are non-existent, it depends also on the rates of growth of individual domestic intermediate inputs.*

To establish the relation between the industry level measures and the economy level measure we multiply each industry level rate of TFP growth in equation (4) by the value of the industry's output and sum over industries to obtain:

(9)
$$\begin{aligned}\sum_j q_j Q_j d\log t_j &= \sum_j q_j Q_j d\log Q_j - \sum_j \sum_i q_i M_{ij} d\log M_{ij} + \sum_j \sum_i (p_{ij} - q_i) M_{ij} d\log M_{ij} \\ &- \sum_j \sum_i q_i^M M^M_{ij} d\log M_{ij}^M + \sum_j \sum_i (p_{ij}^M - q_i^M) M_{ij}^M d\log M_{ij}^M \\ &- \sum_j \sum_k p_{kj}^K K_{kj} d\log K_{kj} - \sum_j \sum_l p_{lj}^L L_{lj} d\log L_{lj}.\end{aligned}$$

Dividing both sides of (9) by the aggregate value of the deliveries to final demand, $\sum q_j Y_j$, and deducting resulting expression from both sides of equation (8) gives, in view of the first expression in equation (7):

$$
\begin{aligned}
(10) \qquad d\log T = {} & (\sum_j q_j Y_j)^{-1} [\sum_j q_j Q_j d\log t_j \\
& - (\sum_i (p_i - q_i) M_i d\log M_i - \sum_j \sum_i (p_{ij} - q_i) M_{ij} d\log M_{ij}) \\
& - (\sum_i (p^M_i - q^M_i) M^M_i d\log M^M_i - \sum_j \sum_i (p^M_{ij} - q^M_i) M^M_{ij} d\log M^M_{ij}) \\
& - (\sum_k p^K_k K_k d\log K_k - \sum_j \sum_k p^K_{kj} K_{kj} d\log K_{kj}) \\
& - (\sum_l p^L_l L_l d\log L_l - \sum_j \sum_l p^L_{lj} L_{lj} d\log L_{lj})]
\end{aligned}
$$

The contribution of the industry level rates of TFP growth is represented by the first term in square brackets. The rest of the terms represent the contribution of the reallocation of the individual inputs by industry. If the price of an input Z is identical for all the industries, i.e. if $p^Z_j = p^Z$ for all values of j, then it follows directly from the definition (6) that:

$$
(11) \qquad \sum_j p^Z_j Z_j d\log Z_j = \sum_j p^Z dZ_j = p^Z dZ = p^Z Z \; d\log Z \; .
$$

Substituting this result into equation (10) shows that in this case the overall rate of TFP growth does not depend on the reallocation of the input by industry. This leads to our:

Result 2. *In the deliveries to final demand approach the rate of economy level TFP growth consists of 1) the weighted sum of the industry-level rates of TFP-growth with the ratios of the industries' outputs to the total value of deliveries to final demand as weights and 2) terms that reflect reallocation of capital, labour and intermediate inputs, both domestic and foreign, by industry. However, if all the industries pay* *identical price for an individual input, the rate of the economy level TFP-growth does not depend on the reallocation of that input by industry.*

Whether or not all the industries pay identical prices for their capital and labour inputs is, more or less, on empirical question. With perfect markets one could expect this to be the case, but markets hardly are perfect enough to produce exactly identical prices. Besides the classification of these inputs to different categories is not likely to be dense enough to produce identical prices even with perfect markets. This means that part of the differences in the distribution of these inputs by type of the input, e.g. by type of labour, can actually appear as a price differences. As to differences in the prices of intermediate inputs caused by taxes and subsidies on products, they often, but not always, can be expressed as a percentage of the value of the input. If this were the case there would be no price differences in intermediate inputs caused by taxes or subsidies on products, if all industries were facing the same taxes and subsidies. However, for instance in countries with VAT-system, some of the industries may have to pay VAT on their inputs, while others are exempted, depending whether or not their outputs are liable to VAT. Besides, the classification of intermediate inputs is not likely to be dense enough to make the categories homogenous with respect to possible rates of taxes/ subsidies.

2.3. Value added approach

Value added at the industry level equals the value of industry output valued at basic prices less the value of intermediate inputs valued at purchasers' prices:[3]

$$(12) \qquad v_j V_j = q_j Q_j - \sum_i p_{ij} M_{ij} - \sum_i M_{ij}^M = \sum_k p_{kj}^K K_{kj} + \sum_l p_{lj}^L L_{lj} \, ,$$

[3] In fact value added also includes other (than product) taxes less subsidies on production, that have to be allocated to the capital and labour inputs. We are here, in line with JGF (1987), also assuming here that the entire operating surplus, (added by taxes on production relating to capital input) can be interpreted as capital compensation. This is interpretation is problematic. For on more on this, see e.g. Diewert (2003) and Aulin-Ahmavaara (2003).

Summing over industries and assuming that all the industries pay identical prices for their capital and labour inputs gives:

(13) $$\sum_j v_j V_j = \sum_j q_j Q_j - \sum_j \sum_i p_{ij} M_{ij} - \sum_j \sum_i p_{ij}^M M_{ij}^M = \sum_k p_k^K K_k + \sum_l p_l^L L_l .$$

Substituting this in equations (7) produces

(14) $$\sum_j q_j Y_j = \sum_j v_j V_j + \sum_i (p_i - q_i) M_i + \sum_i q_i^M M_i^M + \sum_i (p_i^M - q_i^M) M_i^M \ .$$

Accordingly, the sum of industry value added and the sum of the values of the deliveries to final demand are equal if and only if there are no imported intermediate inputs and the aggregate value of taxes or subsidies on products in intermediate uses equals zero.

Applying the definition of the rate of TFP-change in equation (2) to the industry level accounting identity in (12) and to the economy level accounting identity in (13) produces the industry level rate of TFP-growth

(15) $$d\log t_j^v = (v_j V_j)^{-1} [v_j V_j d\log V_j - \sum_k p_{kj}^K K_{kj} d\log K_{kj} - \sum_l p_{lj}^L L_{lj} d\log L_{lj}] ,$$

and the aggregate rate of TFP growth based on the value-added approach

(16) $$d\log T^v = (\sum_j v_j V_j)^{-1} [(\sum_j v_j V_j d\log V_j - \sum_k p_k^K K_k d\log K_k - \sum_l p_l^L L_l d\log L_l)] \cdot$$

Multiplying both sides of (15) by the ratio $(v_j V_j)(\sum_j v_j V_j)^{-1}$, summing over industries and subtracting the result from both sides of equation (16) produces:

$$
(17)\qquad
\begin{aligned}
d\log T^{v} &= (\sum_{j} v_j V_j)^{-1}[\sum_{j} v_j V_j d\log t_j^{v} \\
&\quad - (\sum_{k} p_k^K K_k d\log K_k - \sum_{j}\sum_{k} p_{kj}^K K_{kj} d\log K_{kj}) \,. \\
&\quad - (\sum_{l} p_l^L L_l d\log L_l - \sum_{l}\sum_{l} p_{lj}^L L_{lj} d\log L_{ij})]
\end{aligned}
$$

This gives the economy level measure based on the value added approach in terms of the industry level measures based on the value added.

Taking the logarithmic derivative of the first expression in equation (12) produces:

$$
(18)\qquad v_j V_j d\log V_j = q_j Q_j d\log Q_j - \sum_{i} p_{ij} M_{ij} d\log M_{ij} - \sum_{i} p_{ij}^M M_{ij}^M d\log M_{ij}^M \;.
$$

Substituting this in equation (15) gives in view of equation (4) the following relation between the two industry level measures:

$$
(19)\qquad d\log t_j^{v} = (v_j V_j)^{-1}(q_j Q_j) d\log t_j \,.
$$

Substituting (19) into (17) produces an expression for the relationship between the aggregate value-added based rate of TFP-growth and the industry level rates expressed in terms of total output:

$$
(20)\qquad
\begin{aligned}
d\log T^{v} &= (\sum_{j} v_j V_j)^{-1}[\sum_{j} q_j Q_j d\log t_j \\
&\quad - (\sum_{k} p_k^K K_k d\log K_k - \sum_{j}\sum_{k} p_{kj}^K K_{kj} d\log K_{kj}) \,. \\
&\quad - (\sum_{l} p_l^L L_l d\log L_l - \sum_{j}\sum_{l} p_{lj}^L L_{lj} d\log L_{ij})]
\end{aligned}
$$

This together with equation (11) provides us with our

Result 3. *In the value added approach the rate of economy level TFP growth consists of 1) the weighted sum of the industry level rates of TFP growth with the rates of industries' outputs to the aggregate value added as weights and 2) terms that reflect the reallocation of capital and labour inputs by industry. However, if all the industries pay the identical price for an individual input, the rate of the economy level TFP-growth does not depend on the reallocation of that input by industry. Taxes and subsidies on products do not appear in equation (17) and therefore the rate of the aggregate TFP growth does not, in this case, depend on the reallocation of intermediate inputs by industries.*

Multiplying both sides of equation (20) by the ratio $\sum v_j V_j / \sum q_j Y_j$ and substituting the result into (10) gives an expression to the rate of aggregate TFP-growth based on final demand approach in terms of the rate of aggregate TFP-growth based on the value added approach:

(21)
$$\begin{aligned} d\log T &= (\sum_j q_j Y_j)^{-1}[(\sum_j v_j V_j) d\log T^v \\ &\quad - (\sum_i (p_i - q_i) M_i d\log M_i - \sum_j \sum_i (p_{ij} - q_i) M_{ij} d\log M_{ij}) \\ &\quad - (\sum_i (p^M{}_i - q_i^M) M_i^M d\log M_i^M - \sum_j \sum_i (p_{ij}^M - q_i^M) M_{ij}^M d\log M_{ij}^M)] \end{aligned}$$

The second and third lines disappear if industries pay identical prices for their domestically produced intermediate inputs as well as for their imported intermediate inputs. This can be concluded from equation (11). On the other hand it is obvious from equation (14) that

(22) $$\sum_j q_j Y_j = \sum_j v_j V_j \text{ iff } \sum_i (p_i - q_i) M_i + \sum_j \sum_i q_i^M M_{ij}^M + \sum_i (p_i^M - q_i^M) M_i^M = 0 .$$

From (21) and (22) we obtain:

Result 4. *If all the industries pay identical prices for their intermediate inputs, both domestic and imported, the difference between the two economy level rates of growth depends only on the ratio of value of deliveries to final demand to total value added. This ratio depends on the aggregate value of imported intermediate inputs and of the aggregate value of product taxes less subsidies on intermediate inputs. If the prices paid by different industries for intermediate inputs are not identical, then the difference between the two economy level rates of TFP growth depends also on the reallocation of these inputs by industry.*

Until now we have, at the economy level, only been dealing with the sum of the industries' value added. Next we shall define the price and the quantity of the economy level value added by the following expression:

(23) $$vV = v\sum_j V_j = \sum_j v_j V_j$$

Then the economy level accounting identity can be written as follows:

(24) $$vV = \sum_k p_k^K K_k + \sum_l p_l^L L_l \ .$$

Again applying the formula of equation (2) to this expression gives the economy level rate of TFP-growth:

(25) $$d\log T^W = (vV)^{-1}(vVd\log V - \sum_k p_k^K K_k d\log K_k - \sum_l p_l^L L_l d\log L_l)$$

The relationship between the industry level rates and the economy level rate is obtained following the familiar procedure by forming the weighted average of the industry level value added based measures in equation (15), substituting equation (19) into the result and subtracting it from both sides of equation (25):

$$
(26)\quad \begin{aligned}
d\log T^{W} &= (vV)^{-1}[\sum_{j} q_j Q_j d\log t_j \\
&- (vVd\log V - \sum_{j} v_j V_j d\log V_j) \\
&- (\sum_{k} p_k^K K_k d\log K_k - \sum_{j}\sum_{k} p_{kj}^K K_{kj} d\log K_{kj}) \\
&- (\sum_{l} p_l^L L_l d\log L_l - \sum_{j}\sum_{l} p_{lj}^L L_{lj} d\log L_{lj})]
\end{aligned}.
$$

The second term in the square brackets now represent the effects of reallocation of value added. On the basis of equation (11) it is again obvious that the reallocation terms disappear if the prices of value added are identical for all of the industries or if the rates of growth of value added are identical in all of the industries. In the latter case, of course, no reallocation takes place. Also the two last rows disappear if all the industries pay the same prices for their capital and labour inputs or if the rates of growth of the quantities of these inputs are identical in all the industries, i.e. if no reallocation takes place. Thus equations (26) and (11) together provide us with:

Result 5. *If the economy level value added is used as the output variable in the value added approach the economy level rate of TFP consists of 1) the weighted sum of the industry level rates of TFP growth with the ratios of industries' outputs to the aggregate value added as weights and 2) terms representing reallocation of the value added as well as of capital and labour inputs by industry. If prices of the value added or of an input are identical for all industries then the economy rate of TFP growth does not depend on the reallocation of value added/that of the respective input.*

3. The economic interpretation of the different approaches

3.1. Deliveries to final demand approach

In the deliveries to final demand approach the economy's production problem is to maximise the value of aggregate deliveries to final demand. Assuming that at the economy level taxes and subsidies on products used as intermediate inputs cancel out we can directly use Gollop's (1987) formulation of the problem. The maximum value of deliveries to final demand (μ) is expressed as a function of industries' deliveries to final demand (Y_j), primary inputs (K, L), imported intermediate inputs (M_i^M), and time (t):

(27) $$\mu = H(Y_1, Y_2, \ldots Y_n, L, K, M_1^M, M_2^M, \ldots, M_u^M, t).$$

The value of deliveries to final demand is maximised subject to fixed supplies of domestic capital and labour inputs, market equilibrium and linearly homogeneous industry level production functions:

(28) $$Q_j = h^j(L_j, K_j, M_{1j}, M_{2j}, \ldots M_{nj}, M_{1j}^M, M_{2j}^M, \ldots M_{uj}^M, t).$$

The function H is homogeneous of degree minus one in industry deliveries to final demand and degree one in capital, labour and imported inputs and accordingly of degree zero in deliveries to final demand and capital, labour and imported inputs. Setting μ equal to unity transforms the function H into a production-possibilities frontier. The rate of aggregate TFP growth, as a shift of the production possibilities frontier, is obtained by taking the total logarithmic derivative of H with respect to time and substituting the producer equilibrium conditions into the result. The producer equilibrium conditions require the price ratios of inputs and outputs to be equal to the respective marginal rates of transformation. The problem in this formulation is that the aggregate value of the taxes and subsidies on products used as intermediate inputs is assumed to equal zero, which is not necessarily true.

Jorgenson and Stiroh (2000) have a similar approach writing the aggregate frontier in the following form:

(29) $Y(I_t, C_t) = A_t \cdot X(K_t, L_t)$

The problem in this approach is, from our point of view, that the fact that imported inputs and taxes and subsidies on products used as intermediate inputs are not taken into account.

In order to take the product taxes and subsidies on intermediate inputs into account we try to, somewhat, modify Gollop's (1987) approach and rewrite the production-possibilities frontier in the following form:

(30) $\gamma = F(Q_1, Q_2, \ldots Q_n, L, K, M_1, M_2, \ldots M_n, M_1^M, M_2^M, \ldots M_u^M, t)$.

The economy is now assumed to be maximising the value of total output γ. The function F is homogenous of degree minus one in industry gross output and of degree one in the rest of the variables (the input variables), except t.

The rate of aggregate TFP growth is again obtained by setting $\gamma = 1$, taking the total logarithmic derivative of F with respect to time and substituting the producer equilibrium conditions into the result. Multiplying both sides of the result by the ratio of the value of the aggregate output to the aggregate value of deliveries to final demand and taking into account that $Q_i = Y_i + X_i$, X_i representing the deliveries to intermediate uses, produces:

(31)
$$d\log T = (\sum_j q_j Y_j)^{-1}[\sum_j q_j Y_j d\log Y_j + \sum_j q_j X_j d\log X_j - \sum_i p_i M_i d\log M_i - \sum_i p_i^M M_i^M d\log M_i^M - p^K K \; d\log K \; - p^L L \; d\log L_l].$$

This is exactly the same as the expression for the rate of economy level TFP growth equation (8), apart from the facts that different qualities of labour and capital inputs have been suppressed and that interindustry deliveries and intermediate inputs now carry different symbols. The expression in equation (31) is also identical with

Gollop's (1987) formula for aggregate productivity growth in an "open economy with tax distorted transfer prices" apart from the fact that the second and the third term in the brackets do not appear in his equation (17). This is because he assumes the aggregate value of product taxes less subsidies on intermediate inputs to equal zero. But the possibility of achieving this in two subsequent periods depends on changes in the distribution of intermediate inputs by type of product as can be seen from our equation (31).

The rate of industry level productivity growth is obtained by taking the total logarithmic derivative of the industry level production function in (28) and substituting the conditions of producer equilibrium into the result. This produces an expression identical with the one in our equation (4) apart from the suppression of different types of capital and labour inputs.

Our aggregation formula for the final demand approach, equation (10) is different from the one given by Gollop (33), because he, unlike us, assumes 1) the aggregate value of product taxes and subsidies on intermediate inputs to equal zero and 2) different industries to face identical prices for products used as intermediate inputs. These two assumptions made by Gollop (1987) mean that term representing reallocation of intermediate inputs in his equation (33) in fact should disappear.[4]

3.2. Value added approach

In this case we can directly follow Gollop's (1987) presentation. The maximum value of the aggregate value added (λ) is a function of all quantities of industries' value added (V_j), aggregate labour (L) and capital (K) inputs, and time (t):

(32) $\lambda = G(V_1, V_2, \ldots V_n, L, K, t)$.

[4] The last term representing the reallocation of intermediate inputs is $(\sum_j q_j^y Y_j)^{-1} \sum_j \sum_i (p_i - q_i^x) X_{ij} \frac{d \ln X_{ij}}{dt} = \sum_j q_j^y Y_j)^{-1} [\sum_i (p_i - q_i^x) \sum_j \frac{dX_{ij}}{dt}]$. The term in the square brackets is equal to the change in the aggregate value of product taxes and subsidies on intermediate inputs, which has to be zero, since the aggregate value is always assumed to equal zero.

The economy is maximising λ subject to linearly homogeneous value added functions

(33) $V_j = V^j(L_j, K_j, t)$.

as well as market equilibrium conditions, and aggregate supplies of capital and labour. The existence of industry level value added functions implies that the industry level production functions are value-added separable:

(34) $Q_j = g^j[V^j(L_j, K_j, t), M_{1j}, M_{2j}, \ldots M_{nj}, M_{1j}^M, M_{2j}^M, \ldots M_{uj}^M]$.

Again taking the total logarithmic derivative of (32) and substituting the conditions of market equilibrium into it yields an expression identical with the economy level rate of TFP-growth in equation (16). Likewise expression of the industry level rate of TFP-growth is obtained applying the same procedure to the value added function in (33). As pointed out by Gollop (1987, p. 213) "while the decision to exclude intermediate inputs from the value added model may be based on the disarmingly straightforward assumption that transactions in intermediate products are self-cancelling, that decision effectively implies that all properties of sectoral productivity growth can be analysed isolation from intermediate inputs." It also means that the economy level output (value-added) could not be separated to different product or types of products. This is obvious from our equation (14) and is discussed by Aulin-Ahmavaara (2003).

The relation between aggregate TFP-growth based on the value added approach and the one based on the deliveries to final demand approach given in our equation (21) is somewhat different from the one given by Gollop (1987, equation 21). This, again, is caused by the fact that Gollop assumes the aggregate value of net taxes equal to zero as well as all the industries to pay identical prices for, both imported and domestically produced, intermediate inputs. These two assumptions together also mean that second term in Gollop's equation (21) is equal to zero by definition and taxes and accordingly subsidies on products in intermediate uses are of no consequence in the relation between the two rates of TFP-growth shown in that equation.

If the aggregate value-added function does exists then we can write:

$$(35) \qquad V = V(K, L, t).$$

The economy level rate of TFP-growth identical with the one in (25) can be obtained from (33) by following the usual procedure. The existence of aggregate value-added function requires all the industry value added functions to be identical up to a scalar multiple (see JGF, 1987).

4. Concluding remarks

We have, starting from the respective accounting identities, derived the industry-level as well as the economy level rates of total factor productivity growth based on the deliveries to final demand approach on the one hand on the value added approach on the other. It appeared that, in the case of the deliveries to final demand approach it was necessary, in addition to the terms representing capital and labour inputs, to include terms representing imported intermediate inputs as well as terms representing reallocation of intermediate inputs by industry of origin/type of product. The latter were needed because of the product taxes and subsidies on intermediate inputs.

We have also derived the aggregation rules from the industry-level to the economy level. The economy-level rates of TFP growth could, in both cases, be represented as weighted sums of the same industry-level rates added by reallocation terms. But the weights were different in different approaches. In the deliveries to final demand approach the weights were equal to the ratios of industries' outputs to the aggregate value of deliveries to final demand. In the value added approach they were equal to the ratios of industries' outputs to the aggregate value added. The terms representing reallocation of labour and capital inputs were needed, in the aggregation equation, in both cases. But in the deliveries to final demand approach also terms representing reallocation of, both imported and domestically produced, intermediate deliveries by industry, were required. In the end the difference in the aggregate TFP growth between these two approaches depends on the total value of imported inputs, the aggregate value of product taxes and subsidies on intermediate inputs, and on the reallocation of intermediate inputs by industry.

We also studied another variation of the value-added approach, in which the economy level output was represented by the economy level value added. In this case the aggregation equation required reallocation terms of value added as well as those of labour and capital inputs by industry.

We compared our results with the ones obtained by Gollop (1987) using the production possibilities frontier and the theory of producer behaviour as starting point. As was to be expected our results mainly confirmed his results. However there are differences, caused by the facts that Gollop (1987) assumed the aggregate value of product taxes and subsidies on intermediate inputs to be equal to zero and all the industries to pay identical prices for products used as intermediate inputs. These two assumptions together actually appeared to nullify the effect of the reallocation of intermediate inputs.

The significance of the difference between the deliveries to final demand approach and the value added approach is, in the end, an empirical question. There is, however, no doubt about the fact that every economy uses imported intermediate inputs. Also there are product taxes and subsidies on intermediate inputs in any country, at least in any developed country. And there are good reasons to believe that there are price differences. These may be partly caused by the aggregation of the intermediate inputs, but nevertheless they do exist.

Which of these two approaches, or actually of these three approaches, remembering that there are two value added based approaches, should then be preferred? As pointed out by Gollop (1987) the value added approach is disarmingly simple. But it is simple because it is based on a rather strong assumption about the value-added separability of the industry-level production functions. That would mean that industry level productivity growth could be analysed in separation of intermediate inputs. Even stronger assumption is required if the economy level output is represented by economy level value-added. In this case the industry level value added functions needn't only exist, but they should be identical up to a scalar multiple. So in this sense the deliveries to final demand approach seems preferable. It also has the advantage of making it possible to separate the different products or different types of products e.g. the investment goods in the economy level output.

But then there is also the question what is the economy is actually maximising? If we are thinking of a national economy as a producing unit it would seem natural to assume that it is maximising the value of the output gross of depreciation that it is able to deliver outside the unit with respect to the inputs it uses. But from the consumers' point of view the economy is assumed to be maximising its welfare. Weitzman (1976) has shown that consumption plus changes in net worth, i.e. final output net of depreciation, can be used as an indicator of the present value of future consumption. This issue is beyond the limits of the present paper. It is discussed e.g. by Hulten (1992 and 2001), who concludes that the appropriate welfare, i.e. NNP, based analysis is separate from and complementary to, the GDP based analysis of productive efficiency. Besides we also have to decide whether the value of the output is seen from the producers' point of view (basic prices) or from the consumers point of view (purchasers' prices). ten Raa and Mohnen (2002) are maximising the deliveries to domestic final demand gross of depreciation, of both domestic output and imported products. The proportions of the actual final demand are preserved. Also the observed proportions during any time period of course depend on the price concept used to measure the components of domestic final demand.

Finally, we have been discussing the symmetric input-output framework instead of supply and use table -framework. This could be easily changed by assuming that every industry can have different products as output. We also have been discussing the economy level instead of e.g. business sector. This is because there is no such thing as business sector in the present SNA. One might think of the distinction between market produces and non-market produces, but the latter can also produce market output used as inputs by the former. So these two groups are really closely intertwined.

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