Application of I/O Model in Seasonal Cycle Analysis of one Czech Company

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Abstract:
The I/O models have been used namely at modelling and simulation of macroeconomic problems. This paper deals with a problem of the company's management in pursuance of economic equilibrium. The dynamic equilibrium is determined as a special situation of sensitivity of operational level.
A possibility to use the weighting function for I/O model creation based on particular company data for relation "total costs – total revenues" is shown within the contribution. The results with minimal errors, which were achieved in the valuated period, denote a utility of these models in analysts´, as well managers´ activities.

1. Introduction

An input-output (I/O) economic model is usually a matrix of product flows based on a summary classification of all the industries in a region. The idea is that each row and column of the I/O matrix represents an industry group. The cells of the matrix represent the flows of products from one industry group to others. The diagonal represents the internal flows within each industry group. The structure of the model is non spatial.
Detailed data on the flows among different economic sectors can be used for building static models or can serve as a benchmark dataset for dynamic models. Modern computerized economic techniques enhance traditional input-output analysis originally developed by Wassily Leontief [Leontief, 6].
The fundamental underlying relationship of input-output analysis is that the amount of a product (goods or services) produced by a given economic sector is determined by the amount of that product which is purchased by all the users of the product. This model has not changed since Leontief [Leontief, 7]. However, present-day input-output analysis has become important to all highly-industrialized countries in economic planning and decision making because of this flow of goods and services that it traces through and between different industries and changing within the time [Farský, 3], [Řehořová, 8].
In our presentation we do not focus on macro-economic problems, but we try to use I/O model from the micro economic point of view.

2. I/O Model for Company as a Microeconomic System

Serious problems of management are characteristic of social-economic systems, which the companies are. Irreversibility [von Bertalanfy, 10], which makes impossible to realize standardized experiments, and great numbers of factors that influence the system as well in its relevant environment (often contradictorily and multiplicatively), belong to them.
Thus, we can use exact mathematic methods at the company's management modelling and simulation of economic processes. Some authors use - for example - models based on system of mathematic equations [Taylor, 9]. Interesting ideas are presented with development economy theory, namely the neoclassic theory of the firm. There the economic optimum is determined as a situation when MC = MR (marginal costs are equal to marginal revenues). Furthermore, optimum of company (as company's equilibrium) is jointed with operational level [Kraft, 4], moreover with its sensitivity /1/.

$$S_{OL} = \frac{\dfrac{EBIT_{t+1} - EBIT_t}{EBIT_t}}{\dfrac{TR_{t+1} - TR_t}{TR_{t+1}}} \qquad /1/$$

Legend of the formula: S_{OL} ….. sensitivity of operational level
EBIT …earnings before interests and taxes
TR…….total revenues
t, t+1 … time period (years, or months)

Assume $TR_{t+1}>TR_t$, *and* $EBIT_t>0$. Then, we can describe 4 situations:

I. $S_{OL} = 1$**,.**

Dynamics of the ***EBIT*** growth rate is equal to ***TR*** growth rate. We can say the company is at the point of its "dynamic equilibrium".

II. $S_{OL} > 1$

The company's operational performance increases very quickly, and of course, it is situated in front of the point of equilibrium.

III S_{OL} ***is in the interval (0,1)***

The company's situation is similar to situation II. But ***EBIT*** growth rate is lower than ***TR*** growth rate. However, $EBIT > 0$, and that means, the company is still situated in front of the point of equilibrium.

IV. $S_{OL} < 0$ ***but simultaneously*** $0<EBIT_{t+1}<EBIT_t$

Dynamics of performance development decreases, MC>MR, EBIT decreases. Company gets over its optimum, and it directs to - so called - break point No 2 (behind it EBIT<0).

Other approach is based on input-output model (I/O model), namely used on the macroeconomic level, as mentioned above. However it is true, Simon Kuznets said [Kuznets, 5]: it is very uncertain whether specific economic model (or model of economic growth) is complete if its relations are exact. This fact evokes needfulness to make modelling and simulation with assumption "ceteris paribus".

2. Methodology

The dynamic equilibrium might be specified as one of the biggest and the most important problems of company's management. Company's equilibrium was determined by neoclassic economic scientists as a situation when MC=MR (see above). Sensitivity of operational level can point where the analyzed company is situated. And processes which a company comes in the point of equilibrium by are the dynamical side of the term "dynamic equilibrium".

Use of the I/O model might help to manage economic processes in a scope financial management [Čapek, Kraftová, 2].

To describe changes between inputs and outputs of cycle data in this article, we have decided to point at development of the seasonal cycle of company's performance and its position in connection with its equilibrium.

2.1 Description of the Real Data

The modelling of company's seasonal cycle is based on data of a medium Czech company, which operates in spa branch. The company has around 370 workers; it is visited by more then 150 000 clients per year. The data of the years 1999 and 2000 are calculated.

Input data present all the resources, which are means used for the production of services:

a) Assets (i.e. property, fortune) which respond in value of total capital (total financial sources),
b) Costs that present all the resources, which are used for achievement of revenues of given year.
Output data present total revenues, extraordinary items are minimal, i. e. the total revenues match with the operating incomes in this case.
The data are picked up from financial statements, especially from balance sheets and profit and loss statements. The quarter data are picked up from monthly accounts.

2.2 Approach to the I/O Model Creation

2.2.1 Background

It is supposed that the given model can be imaged as linear or close linear system of the ***n**-th* order from the relation input - output point of view /2/.

$$a_0\, y(k) + a_1\, y(k-1) + \ldots + a_n\, y(k-n) = b_0\, u(k) + b_1\, u(k-1) + \ldots + b_m\, u(k-m) \qquad /2/$$

Furthermore, it is assumed here that for the given model ***u*** denotes input and ***y*** denotes output of the model in an I/O relation and that the following time series of goes in and out of the object respectively.

$$\left.\begin{aligned} &u: u(k), u(k-1), u(k-2), \ldots, u(k-m) \\ &y: y(k), y(k-1, y(k-2), \ldots, y(k-n) \end{aligned}\right\} \qquad /3/$$

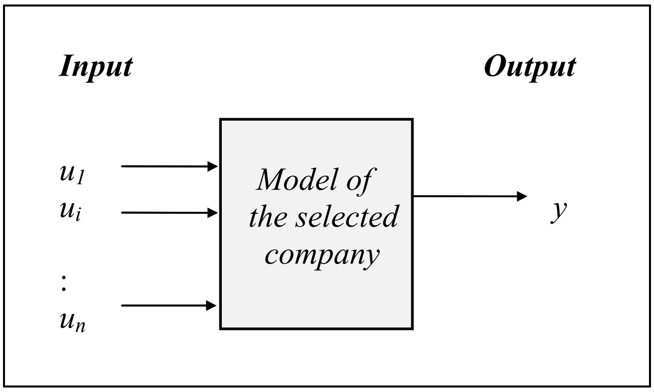

Figure No 1: ***Relation I/O of the Object***

The number of past samples of inputs an outputs respectively depend on the order of systems and generally do not depend on the samples period. It is yields to the following model:

$$y(k) = \sum_{i=1}^{n} \left[b_i u(k-i) - a_i y(k-i) \right] \qquad /4/$$

2.2.2 Weighting function

It is supposed is possible to find a new time series following manner:

$$\left.\begin{aligned}
&y(0) = u(0)W(0)\Delta t\\
&y(1) = u(0)W(1)\Delta t + u(1)W(0)\Delta t\\
&y(2) = u(0)W(2)\Delta t + u(1)W(1)\Delta t + u(2)W(0)\Delta t\\
&y(3) = u(0)W(3)\Delta t + u(1)W(2)\Delta t + u(2)W(1)\Delta t + u(3)W(0)\Delta t
\end{aligned}\right\} \qquad /5/$$

$$y(k) = \Delta t \sum_{j=0}^{k} u(j)W(k-j) \qquad /6/$$

The coefficients a_1 ... a_n , b_0 .... b_n for equation /2/ are solved by matrixes /7/ and /8/ respectively.

$$\begin{bmatrix} W(n+1) \\ W(n+2) \\ . \\ W(2n) \end{bmatrix} = - \cdot \begin{bmatrix} W(1) & W(2) & . & W(n) \\ W(2) & W(3) & . & W(n+1) \\ . & . & . & . \\ W(n) & W(n+1) & . & W(2n-1) \end{bmatrix} \begin{bmatrix} a_n \\ a_{n-1} \\ . \\ a_1 \end{bmatrix} \qquad /7/$$

$$\begin{bmatrix} b_0 \\ b_1 \\ ... \\ b_n \end{bmatrix} = \Delta t . \begin{bmatrix} 0 & ... & 0 & W(0) \\ ... & 0 & W(0) & W(1) \\ ... & ... & ... & ... \\ W(0) & ... & W(n-1) & W(n) \end{bmatrix} . \begin{bmatrix} a_n \\ ... \\ a_1 \\ 1 \end{bmatrix} \qquad /8/$$

It is possible to determine the unknown weighting function from the I/O samples by following equation obtained from the form /6/ after some manipulations.

$$W(k-1) = \frac{1}{u(1)} \left[\frac{y(k)}{\Delta t} - \sum_{j=2}^{k} u(j)W(k-j) \right] \qquad /9/$$

Note: The weighting function itself fully described the given model, because it is the response on the unit impulse. From the individual members of this function putting into matrix form one can solve the order of the difference equation. This order is given by the degree of the following matrix:

$$W_n = \begin{bmatrix} W(1) & W(2) & . & W(n) \\ W(2) & W(3) & . & W(n+1) \\ . & . & . & . \\ W(n) & W(n+1) & . & W(2n-1) \end{bmatrix} \qquad /10/$$

The order of the difference equation /1/ is an integer $\boldsymbol{n}$ for $\boldsymbol{det\ (W_n) \neq 0}$, but $\boldsymbol{det\ (W_{n+1}) = 0}$.

3. Application of the Model on Particular Company's Data

We used the input cyclic data (according to quarters) of the selected years (1999 and 2000), respectively value of assets, total costs and total revenues – as the table No 1 presents:

Table No 1: ***Input data*** (in thousands of CZK)

	Quarter 1	Quarter 2	Quarter 3	Quarter 4
Assets of 1999	160 819	159 184	158 440	159 044
Total Costs of 1999	35 231	38 446	40 012	49 831
Total Revenues 1999	33 881	47 289	53 971	36 606
Assets of 2000	166 443	164 841	161 829	157 697
Total Costs of 2000	36 757	36 129	38 018	58 267
Total Revenues 2000	33 011	47 314	50 924	50 447

We can see the value of one possible input – the assets in the table No 1 –changed slowly so it is possible to omit one and to take into account total costs, as an input, only. The total revenues play the role of the output.

$$yy_k = -(a_{11}y_{k-1} + a_{22}y_{k-2} + a_{33}y_{k-3}) + b_0u_k + b_1u_{k-1} + b_2u_{k-3} \qquad /11/$$

I. e. after modification the I/O model of the relation total costs – total revenues can be obtained:

$$a_{11} = 1{,}742 \quad a_{22} = 1{,}151 \quad a_{33} = 1{,}319 \quad b_0 = 0{,}962 \quad b_1 = 1{,}229 \quad b_2 = 1{,}512 \quad b_3 = 0{,}938$$

$$1{,}319\ y(k\text{-}3) + 1{,}151\ y(k\text{-}2) + 1{,}742\ y(k\text{-}1) + y(k) =$$

$$= 0{,}962\ u(k) + 1{,}229\ u(k\text{-}1) + 1{,}512\ u(k\text{-}2) + 0{,}938\ u(k\text{-}3) \qquad /12/$$

We get a chart where ***yy*** are the calculated values and ***y*** are the original values of the total revenues (from the table No 1):

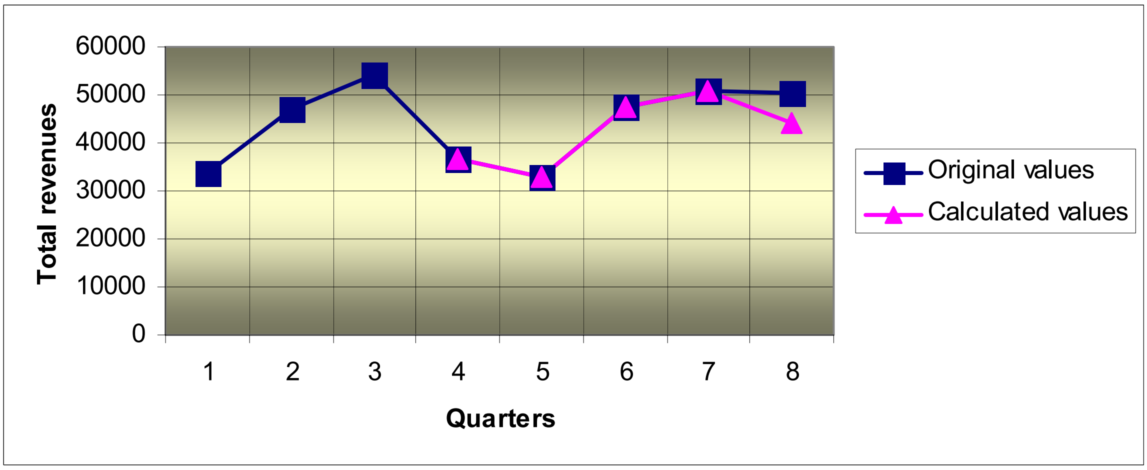

Chart No 1: ***Comparison of Original and Calculated Values***

Errors are calculated in percentages, they are calculated by /13/

$$\boldsymbol{Error_k} = \frac{|yy_k - y_k| * 100}{y_k} \qquad /13/$$

Table No 2: **Expression of the Error computed by /13/**

Q	3	4	5	6	7
Error [%]	$1,988*10^{-14}$	$4,408*10^{-14}$	$3,076*10^{-14}$	0	12,221

4. Conclusion

If we pay our attention to the sensitivity of operational level 2000/1999, we get the value $S_{OL(2000/1999)}$*=9,01*. (Notice: the sensitivity is modified here opposite to the /1/. EBIT is replaced with EAT, i.e. earning after taxes, i.e. difference between total revenues and total costs). This value shows that company is situated in front of economic optimum point; company is not in its equilibrium. It can increase its production markedly. However, the situation is not so easy if we analyse company's seasonal cycle. Numerical and verbal valuation of sensitivity of operational level during the seasonal cycles 1999 and 2000 is shown in the Table No 3.

Table No 3: ***Valuation of S_{OL} and Its Components during Seasonal Cycles 1999 and 2000***

	I Q 99	II Q 99	III Q 99	IV Q 99	I Q 00	II Q 00	III Q 00	IV Q 00
EBIT	***-13 450***	8 846	13 960	***-13 225***	***-3 746***	11 185	12 906	***-7821***
ΔEBIT/EBIT	x	-7,55	0,58	-1,95	-0,72	-3,99	0,15	-1,61
ΔTR/TR	x	0,40	0,14	-0,32	-0,10	0,43	0,08	-0,01
S_{OL}	**x**	**-19,08**	**4,09**	**6,05**	**7,30**	**-9,20**	**2,02**	**171,52**
Situation	in front of BP_1 or behind BP_2	IV	II	in front of BP_1 or behind BP_2	in front of BP_1 or behind BP_2	IV	II	in front of BP_1 or behind BP_2

We can see: the company's complexly stimulating situation is reached in the third quarters only. The seasonal cycle is very important in the performance management of the company that does not reach its equilibrium.

The applied I/O model denotes high approximation to the reality, to the real data (see chart No 1.) This can be considered as applicability of this model in the practice of management. However, the attention must be paid to sufficient number of data. It is denoted that I/O models are usable in the cases if considerable quantum of data is available.

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