ECONOMIC EFFECTS OF TAX REFORM IN AN OVERLAPPING-GENERATIONS MODEL

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Abstract: The purpose of this paper is to propose an overlapping generations model with finite lifetime, which assumes that each individual lives for four periods and there are three capitalists of one young generation and two old generations in one period, and to analyze effects of tax reform using the model. Three tax instruments: labor income tax, consumption tax, and capital income tax, are considered in the tax reform of the model. Specifically this paper's focus is on the numerical simulations are used to discuss economic growth effects of tax reform changes in computable general equilibrium model using the overlapping generations model. In addition, the model facilitates analysis of the transition path toward the steady state. It is shown that decrease in labor income and consumption taxes together with an increase in the capital income tax bring about a preferable effect on the economic growth.
Key Words: Overlapping generations model, Economic growth, Tax reform, and Steady state

1. OVERVIEW

Today governments became more concerned with also designing macroeconomic policies to promote long run objectives, such as economic growth because economic growth is the single most important factor in the success of nations in the long run. The ultimate of research on economic growth is to determine whether there are possibilities for raising overall growth or bringing standards of living in poor countries closer to those in the world leaders. Macroeconomic policies for economic growth include tax policy. For example, tax rates were lowered in most industrial countries in order to improve incentives for saving and production. It means taxing a particular activity can distort behavior of economic agents, changes in tax affect in individual's budget constraint or in capital accumulation of economic growth factor.

Most of tax revenue in Japan is composed by income tax, and individual income tax of labor income occupies large part of income tax. But there is small level of capital income tax depends on interest, dividends earned by individual in the individual income tax. At the same time an aging society is going on with an advance in medical technique, it will be expected that there will be an aged person, who is over 65 years, per four persons of the population. It is possible to disturb labor incentive, because explained most of tax revenue is composed by income tax depends on the individual income tax of labor income under the present system with an aging society. Ultimately, it will affect stability of tax revenue. So it needs to consider the tax reform for the capital income tax affects economic activity.

Economists have long studied of economic growth. Solow (1956) model is the starting point for the almost all analysis of growth because this model serves as the basic tool for understanding the gross process in advanced countries and has been applied in empirical studies of the sources of economic growth. This study is limited to explain the growth overtime by capital accumulation as exogenous and to discuss welfare issue without individuals (Goulder and Summers (1989), and Bovenberg and Goulder (1993)). The Ramsey (1928) model and Diamond (1965) overlapping generation models are built up from the behavior of individuals, and therefore can be used to discuss welfare issues. Theses models have become a standard tool in applied economic modeling. The central difference between the Ramsey model and the Diamond model is that the model of Diamond model assumes that there is continual entry of new households into the economy rather than there being a fixed number of infinitely lived households. Diamond model is used to study many issues in economics; thus this is variable tool. Auerbach, Kotlikoff, and Skinner (1983), Auerbach and Kotlikoff (1987) developed the model of fiscal policy impacts using general equilibrium model in a Diamond type of the overlapping generation framework. Glomm and Ravikumar (1997) analyzed influence of government spending on long-run growth and welfare with overlapping generation model.

These studies are limited to explain of capital supplied by young individuals; actually capital is constructed by not only old individuals but also young individuals, because these studies assume that individuals live for two periods. They failed to identify young capitalist on the economic growth with the overlapping generation model. Blanchard (1985), Heijdra and Ligthart (1998), and Benttendorf (1998) attempted to improve above models with assumption individual lives over two periods and to explore analysis of fiscal policy issues. These models are limited to explain the basic point in the overlapping generation model that individual does not work when he or she is old, because these models assume that individual works until alive at selected time. Hence, the paper tries to overcome these shortcomings by using proposed overlapping generation model assumes that each individual lives for four periods, young for two periods and old for the rest. The paper analyzes tax reform changes affecting a one-country, Japan, closed economy on its steady state using the simulation analysis.

This paper is organized as follows. Following this introduction, the framework of the model is discussed. In this section the overlapping generation model, where individuals live for four periods, of capital accumulation is established and economic behaviors; Firms, individual, and government; adopted in this model are presented and derive the equilibrium conditions. In section 3, the parameterization is briefly discussed. Section 4 illustrates results through numerical simulations on the impact of tax reform changes of the closed economy case for Japan on its steady state of the economy and analyzes the influence have an effect on economic behaviors using the comparing of the changes on utility and capital stock between before and after tax reform. Section 5 concludes the paper.

2. OVERLAPPING GENERATION MODEL

2.1 MODELING STRUCTURE

The basic framework of model is based on the Diamond (1965) model, overlapping generation model, of capital accumulation. Diamond considered that there being a non-fixed number of finitely lived households, new individuals are continually being born, and old individuals are continually dying. With this point, it turn out to be simpler to assume that time is discrete rather than continues; that is, the variables of the model are defined for $t = 0, 1, 2\ldots$ rather than for all values of $t \geq 0$. The paper develops a model in which assumes each individual lives for four periods. Each individual supplies labor for two periods when he or she is young and each period labor income is divided by consumption and saving. In each period of young, he or she carries his or her saving forward to the next period as capital. The individual when he or she is old in the third period divides the saving made by young period between consumption and saving; this saving is also used in the next period as capital. In the last period, the old individual consumes the saving and interest he or she earns. In each period, capital stock equals savings of two young generations plus saving of one old generation. This capital is combined with the labor supplied by the present generation and the next generation of young individuals in production function, and process continues. In this paper government collects taxes on consumption, labor income, and capital income from individual to finance capitation grant and public spending.

2.2 PRODUCER'S BEHAVIOR

Firms are assumed that they produce output, Y_t, in period t according to a Cobb-Douglas function with capital, K_t, and labor, L_t, as homogeneous factor inputs which are rented from the individuals. L_t is constituted by individuals born in period $t-1$, ${}_{t-1}l$, and individuals born in period t, ${}_t l$.

$$Y_t = F(K_t, L_t) = \gamma K_t^{\alpha} L_t^{1-\alpha}, \quad \gamma > 0, \quad 0 < \alpha < 1 \tag{2.1}$$

The paper's critical assumption concerning the production function is that it has constant return to scale in its to arguments, capital and labor. That is, doubling the quantities of capital and labor doubles the amount produced. The paper assumes that capital depreciates at rate δ. The accumulation of capital is described by:

$$K_t = (1-\delta)K_{t-1} + I_{t-1} \tag{2.2}$$

where I_{t-1} is the amount of private investment in period $t-1$ and K_{t-1} is the amount of capital in period $t-1$.

Labors', L_t, are assumed that these variables grow at the exogenous rate n.

$$L_{t+1} = (1+n)L_t \tag{2.3}$$

$$L_t = {}_{t-1}l + {}_t l \tag{2.4}$$

The assumption of constant returns allows us to work with the production function in intensive form. To find the intensive form of the production function, divide both inputs by L_t as:

$$\frac{Y_t}{L_t} = F\left(\frac{K_t}{L_t}, 1\right) = \gamma\left(\frac{K_t}{L_t}\right)^{\alpha}, \quad \gamma > 0, \quad 0 < \alpha < 1 \tag{2.5}$$

It can rewrite (2.1) as:

$$y_t = f(k) = F(k,1) = \gamma k_t^{\alpha} \tag{2.6}$$

The intensive form production function, $f(k)$, is assumed to satisfy as:

$$f(0) = 0 \tag{2.7}$$

$$f'(k) > 0 \tag{2.8}$$

$$f''(k) < 0 \tag{2.9}$$

It implies that marginal product of capital is positive, but that it declines as capital per labor rises. In addition, $f(k)$ is assumed to satisfy the Inada conditions (Inada, 1965):

$$\lim_{k=0} f'(k) = \infty \tag{2.10}$$

$$\lim_{k=\infty} f'(k) = 0 \tag{2.11}$$

where the marginal of capital is large when the capital is small and the marginal of capital is small when the capital is large.

The firms maximize its profit under the production function. The firms rent capital and employ labors in competitive factor markets, and sell their output in a competitive output market. Capital and labor employed are paid their marginal products: before tax prices of capital and labor. It explains that they are owned by the individuals, so any profit they earn accrue to the individuals, firms earn zero profits. Firms earn profits, $\prod$, as:

$$\max \prod = Y_t - w_t L_t - q_t K_t \tag{2.12}$$

Divide both side of (2.12) by L_t:

$$\max \pi = y_t - w_t - q_t k_t \tag{2.13}$$

Plug (2.6) into (2.13) as:

$$\max \pi = \gamma k_t^{\alpha} - w_t - q_t k_t \tag{2.14}$$

where q_t is the rental price on capital. The production function has constant returns to private factors so that firms earn zero profits in equilibrium.

The firms therefore equate the marginal products to the rental price. The marginal product of capital, $\frac{\partial F(K_t, L_t)}{\partial K_t}$, and the marginal product of labor, $\frac{\partial F(K_t, L_t)}{\partial L_t}$, in terms of the intensive form of the production function are given as: the rental price on capital in period t is

$$\begin{aligned}\frac{\partial F(K_t, L_t)}{\partial K_t} = q_t &= \alpha\gamma K_t^{\alpha-1} L_t^{1-\alpha} \\ &= \alpha\gamma\left(\frac{K_t}{L_t}\right)^{\alpha-1} \\ &= \alpha\gamma k_t^{\alpha-1}\end{aligned} \tag{2.15}$$

The labor income in period t is

$$\frac{\partial F(K_t, L_t)}{\partial L_t} = w_t = (1-\alpha)\gamma K_t^{\alpha} L_t^{-\alpha}$$
$$= (1-\alpha)\gamma \left(\frac{K_t}{L_t}\right)^{\alpha} \tag{2.16}$$
$$= (1-\alpha)\gamma k_t^{\alpha}$$

Real interest payment r_t is described:

$$\begin{aligned} r_t &= q_t - \delta \\ &= \alpha\gamma k_t^{\alpha-1} - \delta \end{aligned} \tag{2.17}$$

2.3 CONSUMER'S BEHAVIOR

To simplify the analysis, the model assumes that each individual's incomes are generated by sale of labor to own region industries plus capitation grant from government for two periods in his or her lifetime. For the representative young individual, his or her incomes, has to be split between consumptions and savings for the rest of two periods' consumptions. It is assumed that the individual in the second period does not consume interest payments gained by saving made in the first period. There is no inheritance.

Let ${}_{t-1}c_t$ and ${}_t c_t$ denote the consumption in period t for individual born in period $t-1$ and t. Thus the utility of an individual born in period t, denoted U_t, depends on ${}_t c_t$, ${}_t c_{t+1}$, ${}_t c_{t+2}$, and ${}_t c_{t+3}$. The individual wants to maximize its lifetime utility subject to its budget constraint. Consumptions; ${}_t c_{t+2}$ and ${}_t c_{t+3}$, of old individual for two periods depend on the savings made by he or she was young for two periods. And consumption, ${}_t c_{t+3}$, of old individual of last period depends on the saving made by the old individual in third period. With assumption instantaneous utility is logarithmic, the agent maximizes.

The individual's utility function takes the form:

$$\max \quad U_t = \sum_{v=0}^{3} (1+\rho)^{-v} \ln\left({}_t c_{t+v}\right), \quad \rho > -1 \tag{2.18}$$

$$\text{subject to} \quad w_t(1-\tau_w) + cg = {}_t s_t + {}_t c_t (1+\tau_c) \tag{2.19}$$

$$w_{t+1}(1-\tau_w) + cg = {}_t s_{t+1} + {}_t c_{t+1}(1+\tau_c) \tag{2.20}$$

$$\begin{aligned} &[1+(1-\tau_r)r_{t+1}]\cdot[1+(1-\tau_r)r_{t+2}] \cdot {}_t s_t \\ &+ [1+(1-\tau_r)r_{t+2}]\,{}_t s_{t+1} = {}_t c_{t+2}(1+\tau_c) + {}_t s_{t+2} \end{aligned} \tag{2.21}$$

$$ {}_t s_{t+2} \cdot \left[1+\left(1-\tau_r\right)r_{t+3}\right] = {}_t c_{t+3}\left(1+\tau_c\right) \tag{2.22} $$

Where, ${}_t c_t$ denotes consumption of the individual in the first part of his or her life and ${}_t c_{t+2}$ consumption of the individual in the third part of his or her life. ρ is rate of time preference; if $\rho > 0$, individuals place greater weight on first period than third period consumption; if $\rho < 0$, the situation is reversed. The assumption $\rho > -1$ ensures that the weight on third period consumption is positive. ${}_t s_t$ is saving , w_t labor income per labor, cg capitation grant per labor, r_t interest payment on saving. τ_w, τ_c, and τ_r are tax of labor income, consumption, and interest payment. The budget constraint (2.19) and (2.20) characterize the behavior of wealth over time: these can be rearranged by:

$$ w_t\left(1-\tau_w\right)+cg = {}_t c_t\left(1+\tau_c\right)+\frac{\left[1+\left(1-\tau_r\right)r_{t+1}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+2}\right]\ {}_t s_t}{\left[1+\left(1-\tau_r\right)r_{t+1}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+2}\right]} \tag{2.23} $$

$$ w_{t+1}\left(1-\tau_w\right)+cg = {}_t c_{t+1}\left(1+\tau_c\right)+\frac{\left[1+\left(1-\tau_r\right)r_{t+2}\right]\ {}_t s_{t+1}}{\left[1+\left(1-\tau_r\right)r_{t+2}\right]} \tag{2.24} $$

Equation (2.21) is shifted by:

$$ \begin{aligned} &\left[1+\left(1-\tau_r\right)r_{t+1}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+2}\right]\ \cdot {}_t s_t + \left[1+\left(1-\tau_r\right)r_{t+2}\right] {}_t s_{t+1} = {}_t c_{t+2}\left(1+\tau_c\right) \\ &+\frac{\left[1+\left(1-\tau_r\right)r_{t+3}\right] {}_t s_{t+2}}{\left[1+\left(1-\tau_r\right)r_{t+3}\right]} \end{aligned} \tag{2.25} $$

Dividing (2.25) by $\left[1+\left(1-\tau_r\right)r_{t+1}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+2}\right]$:

$$ \begin{aligned} &\frac{\left[1+\left(1-\tau_r\right)r_{t+1}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+2}\right]\ \cdot {}_t S_t}{\left[1+\left(1-\tau_r\right)r_{t+1}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+2}\right]}+\frac{\left[1+\left(1-\tau_r\right)r_{t+2}\right] {}_t S_{t+1}}{\left[1+\left(1-\tau_r\right)r_{t+1}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+2}\right]} \\ &=\frac{{}_t C_{t+2}\left(1+\tau_c\right)}{\left[1+\left(1-\tau_r\right)r_{t+1}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+2}\right]}+\frac{\left[1+\left(1-\tau_r\right)r_{t+3}\right] {}_t S_{t+2}}{\left[1+\left(1-\tau_r\right)r_{t+1}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+2}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+3}\right]} \end{aligned} \tag{2.26} $$

It can thus plug (2.23) and (2.24) into (2.26) and rearranged to get: ${}_t s_{t+2} \cdot \left[1+\left(1-\tau_r\right)r_{t+3}\right] = {}_t c_{t+3}\left(1+\tau_c\right)$:

$$ \begin{aligned} &w_t\left(1-\tau_w\right)+cg+\frac{w_{t+1}\left(1-\tau_w\right)+cg}{\left[1+\left(1-\tau_r\right)r_{t+1}\right]} = {}_t c_t\left(1+\tau_c\right)+\frac{{}_t c_{t+1}\left(1+\tau_c\right)}{\left[1+\left(1-\tau_r\right)r_{t+1}\right]} \\ &+\frac{{}_t c_{t+2}\left(1+\tau_c\right)}{\left[1+\left(1-\tau_r\right)r_{t+1}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+2}\right]}+\frac{{}_t c_{t+3}\left(1+\tau_c\right)}{\left[1+\left(1-\tau_r\right)r_{t+1}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+2}\right]\cdot\left[1+\left(1-\tau_r\right)r_{t+3}\right]} \end{aligned} \tag{2.27} $$

Equation (2.27) shows discounted future wealth. It needs to determine the amount of consumption on each period in lifetime.

The young individual, who was born in period t, in the first part of his or her life in period t, has to solve the following problem:

$$\max_{{}_t c_t, {}_t s_t} \quad U_t = \ln({}_t c_t) + \frac{\ln({}_t c_{t+2})}{(1+\rho)^2}, \quad \rho > -1 \tag{2.28}$$

$$\text{subject to} \quad {}_t s_t = w_t(1-\tau_w) + cg - {}_t c_t(1+\tau_c) \tag{2.29}$$

$${}_t c_{t+2}(1+\tau_c) = [1+(1-\tau_r)r_{t+1}] \cdot [1+(1-\tau_r)r_{t+2}] \cdot {}_t s_t \tag{2.30}$$

Where, ${}_t c_t$ denotes consumption of the individual in the first part of his or her life and ${}_t c_{t+2}$ consumption of the individual in the third part of his or her life.

Equation (2.29) and (2.30) can be rearranged by:

$$w_t(1-\tau_w) + cg = {}_t c_t(1+\tau_c) + \frac{{}_t c_{t+2}(1+\tau_c)}{[1+(1-\tau_r)r_{t+1}] \cdot [1+(1-\tau_r)r_{t+2}]} \tag{2.31}$$

Equation (2.31) shows new budget constraint. The individual maximizes utility, (2.28), subject to the budget constraint, (2.31), the way to solve the individual's maximization problem is to set up to the Lagrangian:

$$\begin{aligned} L &= \ln({}_t c_t) + \frac{\ln({}_t c_{t+2})}{(1+\rho)^2} \\ &+ \lambda\left[w_t(1-\tau_w) + cg - {}_t c_t(1+\tau_c) - \frac{{}_t c_{t+2}(1+\tau_c)}{[1+(1-\tau_r)r_{t+1}] \cdot [1+(1-\tau_r)r_{t+2}]} \right] \end{aligned} \tag{2.32}$$

It take the derivate of equation (2.32) with respect to ${}_t c_t$, ${}_t c_{t+2}$, and λ:

$$\frac{\partial L}{\partial_t c_t} = \frac{1}{{}_t c_t} - \lambda(1+\tau_c) \tag{2.33}$$

$$\frac{\partial L}{\partial_t c_{t+2}} = \frac{1}{{}_t c_{t+2}(1+\rho)^2} - \lambda\left[\frac{(1+\tau_c)}{[1+(1-\tau_r)r_{t+1}] \cdot [1+(1-\tau_r)r_{t+2}]} \right] \tag{2.34}$$

$$\frac{\partial L}{\partial \lambda} = w_t(1-\tau_w) + cg - {}_t c_t(1+\tau_c) - \frac{{}_t c_{t+2}(1+\tau_c)}{[1+(1-\tau_r)r_{t+1}] \cdot [1+(1-\tau_r)r_{t+2}]} \tag{2.35}$$

Plug (2.33) into (2.34) yields

$${}_t c_{t+2} = \frac{[1+(1-\tau_r)r_{t+1}] \cdot [1+(1-\tau_r)r_{t+2}]}{(1+\rho)^2} \cdot {}_t c_t \tag{2.36}$$

Plug (2.36) into the new budget constraint (2.31) to express ${}_t c_t$:

$${}_t c_t = \frac{[w_t(1-\tau_w) + cg](1+\rho)^2}{(1+\tau_c)(\rho^2 + 2\rho + 2)} \tag{2.37}$$

Then the saving function is given by substituting equation (2.37) into (2.29) to show the relation between saving and income in first part of his life.

$$ {}_t s_t = \frac{w_t(1-\tau_w)+cg}{(\rho^2+2\rho+2)} \tag{2.38} $$

The young individual in the second part of his or her life in period $t+1$ has to solve the following problem:

$$ \max_{{}_t c_{t+1},\, {}_t s_{t+1}} \quad U_t = \frac{\ln({}_t c_{t+1})}{(1+\rho)} + \frac{\ln({}_t c_{t+2})}{(1+\rho)^2}, \quad \rho > -1 \tag{2.39} $$

$$ \text{subject to} \quad {}_t s_{t+1} = w_{t+1}(1-\tau_w) + cg - {}_t c_{t+1}(1+\tau_c) \tag{2.40} $$

$$ {}_t c_{t+2}(1+\tau_c) = [1+(1-\tau_r)r_{t+2}] \cdot {}_t s_{t+1} \tag{2.41} $$

The consumption and saving function from the above maximization problem is given as:

$$ {}_t c_{t+1} = \frac{[w_{t+1}(1-\tau_w)+cg](1+\rho)}{(1+\tau_c)(2+\rho)} \tag{2.42} $$

$$ {}_t s_{t+1} = \frac{w_{t+1}(1-\tau_w)+cg}{(2+\rho)} \tag{2.43} $$

Savings (2.38) and (2.43) are increasing with increasing income and decreasing discount rate.

In the third part of his or her life, the individual consumes the savings and interest payments and he or she saves for the last period of his or her life. The old individual in the third part of his or her life in period $t+2$ has to solve the following problem to choose ${}_t s_{t+2}$ optimally:

$$ \max_{{}_t c_{t+2},\, {}_t s_{t+2}} \quad U_t = \frac{\ln({}_t c_{t+2})}{(1+\rho)^2} + \frac{\ln({}_t c_{t+3})}{(1+\rho)^3}, \quad \rho > -1 \tag{2.44} $$

$$ \begin{aligned} \text{subject to} \quad {}_t s_{t+2} &= [1+(1-\tau_r)r_{t+1}] \cdot [1+(1-\tau_r)r_{t+2}] \cdot {}_t s_t \\ &\quad + [1+(1-\tau_r)r_{t+2}] \cdot {}_t s_{t+1} - {}_t c_{t+2}(1+\tau_c) \end{aligned} \tag{2.45} $$

$$ {}_t c_{t+3}(1+\tau_c) = [1+(1-\tau_r)r_{t+3}] \cdot {}_t s_{t+2} \tag{2.46} $$

The consumption and saving function from the above maximization problem is given as:

$$ {}_t c_{t+2} = \frac{(1+\rho)[1+(1-\tau_r)r_{t+1}] \cdot [1+(1-\tau_r)r_{t+2}]\, {}_t s_t + (1+\rho)[1+(1-\tau_r)r_{t+2}]\ {}_t s_{t+1}}{(1+\tau_c)(2+\rho)} \tag{2.47} $$

$$ {}_t s_{t+2} = \frac{[1+(1-\tau_r)r_{t+1}] \cdot [1+(1-\tau_r)r_{t+2}]\, {}_t s_t + [1+(1-\tau_r)r_{t+2}] \cdot {}_t s_{t+1}}{(2+\rho)} \tag{2.48} $$

In the equations (2.47) and (2.48), ${}_t s_t$ and ${}_t s_{t+1}$ are substituted by (2.38) and (2.43) as:

$$
{}_t c_{t+2} = \frac{(1+\rho)[1+(1-\tau_r)r_{t+1}]\cdot[1+(1-\tau_r)r_{t+2}]\cdot[w_t(1-\tau_w)+cg]}{(1+\tau_c)(\rho^3+4\rho^2+6\rho+4)} + \frac{(1+\rho)[1+(1-\tau_r)r_{t+2}]\cdot[w_{t+1}(1-\tau_w)+cg]}{(1+\tau_c)(\rho^2+4\rho+4)} \tag{2.49}
$$

$$
{}_t s_{t+2} = \frac{[1+(1-\tau_r)r_{t+1}]\cdot[1+(1-\tau_r)r_{t+2}]\cdot[w_t(1-\tau_w)+cg]}{(\rho^3+4\rho^2+6\rho+4)} + \frac{[1+(1-\tau_r)r_{t+2}]\cdot[w_{t+1}(1-\tau_w)+cg]}{(\rho^2+4\rho+4)} \tag{2.50}
$$

Plug (2.50) into (2.46) and rearrange to get the fourth part of his or her life in period $t+3$, ${}_t c_{t+3}$:

$$
{}_t c_{t+3} = \frac{[1+(1-\tau_r)r_{t+1}]\cdot[1+(1-\tau_r)r_{t+2}]\cdot[1+(1-\tau_r)r_{t+3}]\cdot A_t[w_t(1-\tau_w)+cg]}{(1+\tau_c)(\rho^3+4\rho^2+6\rho+4)} + \frac{[1+(1-\tau_r)r_{t+2}]\cdot[1+(1-\tau_r)r_{t+3}]\cdot A_{t+1}[w_{t+1}(1-\tau_w)+cg]}{(1+\tau_c)(\rho^2+4\rho+4)} \tag{2.51}
$$

Plug (2.37), (2.42), (2.49), and (2.51) into (2.27) to show that lifetime consumption is planed by the income in this optimal way.

2.4 GOVERNMENT

The paper assumes that government finances its spending consists of capitation grant, $cg \cdot L_t$, and public spending, PS, by levy taxes of labor income, capital income, and consumption on the individuals. Thus the budget constraint of government is given by:

$$
\begin{aligned}
GB_t = {} & \tau_w \cdot_{t-1} w_t \cdot_{t-1} l + \tau_w \cdot_t w_t \cdot_t l + \tau_c \cdot_{t-3} c_t \cdot_{t-3} l + \tau_c \cdot_{t-2} c_t \cdot_{t-2} l + \tau_c \cdot_{t-1} c_t \cdot_{t-1} l \\
& + \tau_c \cdot_t c_t \cdot_t l + \tau_r r_t \cdot_{t-3} s_{t-1} \cdot_{t-3} l + \tau_r r_t \cdot_{t-2} s_{t-1} \cdot_{t-2} l + \tau_r r_t \cdot_{t-1} s_{t-1} \cdot_{t-1} l
\end{aligned} \tag{2.52}
$$

Government expenditure is obtained by:

$$
GB_t = cg \cdot L_t + PS_t \tag{2.53}
$$

Dividing both sides of (2.52) by ${}_t l$ and plugging (2.16) and (2.17) into (2.52) to show the government budget per one individual.

$$
\begin{aligned}
gb_t &= \tau_w w_t \frac{1}{(1+n)} + \tau_w w_t \\
&+ \tau_c \left\{ \frac{[1+(1-\tau_r)r_{t-2}]\cdot[1+(1-\tau_r)r_{t-1}]\cdot[1+(1-\tau_r)r_t]\cdot[w_{t-3}(1-\tau_w)+cg]}{(1+\tau_c)(\rho^3+4\rho^2+6\rho+4)} \right. \\
&+ \left. \frac{[1+(1-\tau_r)r_{t-1}]\cdot[1+(1-\tau_r)r_t]\cdot[w_{t-2}(1-\tau_w)+cg]}{(1+\tau_c)(\rho^2+4\rho+4)} \right\} \frac{1}{(1+n)^3} \\
&+ \tau_c \left\{ \frac{(1+\rho)[1+(1-\tau_r)r_{t-1}]\cdot[1+(1-\tau_r)r_t]\cdot[w_{t-2}(1-\tau_w)+cg]}{(1+\tau_c)(\rho^3+4\rho^2+6\rho+4)} \right. \\
&+ \left. \frac{(1+\rho)[1+(1-\tau_r)r_t]\cdot[w_{t-1}(1-\tau_w)+cg]}{(1+\tau_c)(\rho^2+4\rho+4)} \right\} \frac{1}{(1+n)^2} \\
&+ \tau_c \left(\frac{(1+\rho)[w_t(1-\tau_w)+cg]}{(1+\tau_c)(2+\rho)} \right) \frac{1}{(1+n)} + \tau_c \frac{(1+\rho)^2[w_t(1-\tau_w)+cg]}{(1+\tau_c)(\rho^2+2\rho+2)} \\
&+ \tau_r r_t \left\{ \frac{[1+(1-\tau_r)r_{t-2}]\cdot[1+(1-\tau_r)r_{t-1}]\cdot[w_{t-3}(1-\tau_w)+cg]}{(\rho^3+4\rho^2+6\rho+4)} \right. \\
&+ \left. \frac{[1+(1-\tau_r)r_{t-1}]\cdot[w_{t-2}(1-\tau_w)+cg]}{(\rho^2+4\rho+4)} \right\} \frac{1}{(1+n)^3} \\
&+ \tau_r r_t \left\{ \frac{[1+(1-\tau_r)r_{t-1}]\cdot[w_{t-2}(1-\tau_w)+cg]}{(\rho^2+2\rho+2)} + \frac{[w_{t-1}(1-\tau_w)+cg]}{(\rho+2)} \right\} \frac{1}{(1+n)^2} \qquad (2.54) \\
&+ \tau_r r_t \left(\frac{[w_{t-1}(1-\tau_w)+cg]}{(\rho^2+2\rho+2)} \right) \frac{1}{(1+n)}
\end{aligned}
$$

With the above assumption of balanced government budget, capitation grant and public spending occupy 40% and 60% of government expenditure on the steady state.

2.5 MARKET EQUILIBRIUM

In the paper, behavior of individual, firms, and government today depend on future economic conditions. Equilibrium therefore requires that agent's behavior be consistent not only with current prices; labor income, interest payment and tax rates, but also with the entire path of future prices. It set up the price equals to 1 to simplify the model.

At each period, factor and goods markets clear instantaneously. Investment is an essential economic activity because it increases the capital stock available for future production. One of the most important points about national accounting is the identity between saving and investment. In this closed country economy, individuals can only accumulate domestic assets. As a result, financial market equilibrium in period t as:

$$I_t = {}_{t-2}s_t \cdot {}_{t-2}l + {}_{t-1}s_t \cdot {}_{t-1}l + {}_t s_t \cdot {}_t l \tag{2.55}$$

Labor market requires that the supply of labor by individuals matches labor demand by firms. Goods market equilibrium is satisfied when the supply of goods equals aggregate demand, which consists of private consumption, government expenditure and investment:

$$Y_t = C_t + PS_t + I_t \tag{2.56}$$

Equation (2.56) of goods market equilibrium is rearranged for one individual as:

$$\begin{aligned} y_t + \frac{y_{t+1}}{[1+(1-\tau_r)r_{t+1}]} &= {}_t c_t + \frac{{}_t c_{t+1}}{[1+(1-\tau_r)r_{t+1}]} + \frac{{}_t c_{t+2}}{[1+(1-\tau_r)r_{t+1}]\cdot[1+(1-\tau_r)r_{t+2}]} \\ &+ \frac{{}_t c_{t+3}}{[1+(1-\tau_r)r_{t+1}]\cdot[1+(1-\tau_r)r_{t+2}]\cdot[1+(1-\tau_r)r_{t+3}]} + {}_t ps_t + \frac{{}_t ps_{t+1}}{[1+(1-\tau_r)r_{t+1}]} \\ &+ \frac{{}_t ps_{t+2}}{[1+(1-\tau_r)r_{t+1}]\cdot[1+(1-\tau_r)r_{t+2}]} + \frac{{}_t ps_{t+3}}{[1+(1-\tau_r)r_{t+1}]\cdot[1+(1-\tau_r)r_{t+2}]\cdot[1+(1-\tau_r)r_{t+3}]} \\ &+ {}_t s_t + \frac{{}_t s_{t+1}}{[1+(1-\tau_r)r_{t+1}]} + \frac{{}_t s_{t+2}}{[1+(1-\tau_r)r_{t+1}]\cdot[1+(1-\tau_r)r_{t+2}]} \\ &+ \frac{{}_t s_{t+3}}{[1+(1-\tau_r)r_{t+1}]\cdot[1+(1-\tau_r)r_{t+2}]\cdot[1+(1-\tau_r)r_{t+3}]} \end{aligned} \tag{2.57}$$

2.6 CAPITAL ACCUMULATION

Firms want to invest intersects the saving schedule of what individuals want to save. It is general assumption in the equilibrium level of GDP, national output must be at the intersection where planned saving and investment are equal.

As described above, the capital stock in period $t+1$ is constituted by:

$$K_{t+1} = (1-\delta)K_t + I_t \tag{2.58}$$

where, an amount of private investment in period $t+1$, I_t, equal the amount saved by old individuals born in period $t-2$, ${}_{t-2}l$, young individuals born in period $t-1$, ${}_{t-1}l$, and young individuals born in period t, ${}_t l$. K_t is the amount of capital in period t and δ is the capital depreciation rate.

The equation (2.58) is rewritten with (2.55) as:

$$K_{t+1} = (1-\delta)K_t + {}_{t-2}s_t \cdot {}_{t-2}l + {}_{t-1}s_t \cdot {}_{t-1}l + {}_t s_t \cdot {}_t l \tag{2.59}$$

In above equation (2.59), saving function is substituted by:

$$K_{t+1} = \left\{ \frac{\left[1+(1-\tau_r)r_{t-1}\right]\cdot\left[1+(1-\tau_r)r_t\right]\cdot\left[w_{t-2}(1-\tau_w)+cg\right]}{\left(\rho^3+4\rho^2+6\rho+4\right)} \right\} \frac{(1+n)L_{t-2}}{(2+n)}$$
$$+ \left\{ \frac{\left[1+(1-\tau_r)r_t\right]\cdot\left[w_{t-1}(1-\tau_w)+cg\right]}{\left(\rho^2+4\rho+4\right)} \right\} \frac{L_{t-1}}{(2+n)}$$
$$+ \left\{ \frac{\left[1+(1-\tau_r)r_t\right]\cdot\left[w_{t-1}(1-\tau_w)+cg\right]}{\left(\rho^2+2\rho+2\right)} \right\} \frac{(1+n)L_{t-1}}{(2+n)} \quad (2.60)$$
$$+ \left\{ \frac{\left[w_t(1-\tau_w)+cg\right]}{(\rho+2)} \right\} \frac{L_t}{(2+n)} + \left\{ \frac{\left[w_t(1-\tau_w)+cg\right]}{\left(\rho^2+2\rho+2\right)} \right\} \frac{(1+n)L_t}{(2+n)}$$
$$+ (1-\delta)K_t$$

Divide both sides of (2.60) by L_{t+1} to show the capital per labor, $\frac{K_{t+1}}{L_{t+1}}$, and plug (2.16) and (2.17) into (2.58) to substitute r and w as:

$$k_{t+1} = \frac{\left[1+(1-\tau_r)\left(\alpha\gamma k_{t-1}{}^{\alpha-1}-\delta\right)\right]\cdot\left[1+(1-\tau_r)\left(\alpha\gamma k_t{}^{\alpha-1}-\delta\right)\right]\cdot\left[\left((1-\alpha)\gamma k_{t-2}{}^{\alpha}\right)(1-\tau_w)+cg\right]}{(2+n)(1+n)^2\left(\rho^3+4\rho^2+6\rho+4\right)}$$
$$+ \frac{\left[1+(1-\tau_r)\left(\alpha\gamma k_t{}^{\alpha-1}-\delta\right)\right]\cdot\left[\left((1-\alpha)\gamma k_{t-1}{}^{\alpha}\right)(1-\tau_w)+cg\right]}{(2+n)(1+n)^2\left(\rho^2+4\rho+4\right)}$$
$$+ \frac{\left[1+(1-\tau_r)\left(\alpha\gamma k_t{}^{\alpha-1}-\delta\right)\right]\cdot\left[\left((1-\alpha)\gamma k_{t-1}{}^{\alpha}\right)(1-\tau_w)+cg\right]}{(2+n)(1+n)\left(\rho^2+2\rho+2\right)} \quad (2.61)$$
$$+ \frac{\left[\left((1-\alpha)\gamma k_t{}^{\alpha}\right)(1-\tau_w)+cg\right]}{(2+n)(1+n)(\rho+2)} + \frac{\left[\left((1-\alpha)\gamma k_t{}^{\alpha}\right)(1-\tau_w)+cg\right]}{(2+n)\left(\rho^2+2\rho+2\right)} + \frac{(1-\delta)}{(1+n)}k_t$$

The paper sets up an equation (2.61) of capital accumulation in period $t+1$, k_{t+1}, as the functions of k_{t-2}, k_{t-1}, and k_t. It therefore determines how k evolves over time given its initial values.

3. BENCHMARK PARAMETRIZATION

The paper depends on results through numerical simulations to show economic effect of tax reforms. Simulations need parameter values and the policy variables are considered as exogenous and that a model is proposed. The parameters and policy variables are as follows:

Table 1: Parameters and policy variables of benchmark

Notation	Explanation	Equation	Japan
δ	Rate of capital depreciation	2.2	0.76
γ	Scaling constant	2.1	2.053
ρ	Rate of time preference	2.18	1.00
α	Distribution parameter for production	2.1	0.42
n	Rate of labor growth	2.4	0.01
τ_w	Labor income tax rate	2.19	0.09
cg	Capitation grant per labor	2.19	0.071
τ_c	Consumption tax rate	2.19	0.05
τ_r	Capital income tax rate	2.21	0.20

The paper used annual data for Japan from 1970 to 2000 and found parameters and policy variables. α is set equal to 0.42, it means that production is presumed to more intensive in the use of human capital. δ is set equal to 0.76, because a yearly depreciation rate is set equal to 0.09. The tax rates on the labor income, consumption, and capital income employed in the production function, τ_w, τ_c, and τ_r are set equal to 0.09, 0.05, and 0.20. The scaling constant, γ, is chosen that the pre-tax labor income is equal to one (γ =2.053). Capitation grant, cg, and rate of time preference, ρ, are set equal to 0.071 and 1.00 in the paper. Although the replication of an actual economy is not the purpose, it is self-evident that outcomes must be lie in a realistic range. The paper assumes that active life stars at the age of 20 and that the average age of death is 80 and that there are four periods for the lifetime of individuals, it implies that a period spans 15 years. In the benchmark parameterization, policy variables of labor income, τ_w, and capital income tax rate, τ_r, are changed to show results effects of tax reform and parameters concerning distribution parameter for production function, α, and rate of time preference, ρ, of the model are adjusted for sensitivity analysis in section 4.3.

4. SIMULATION

4.1 CONDITION OF STEADY STATE

As described above, the paper illustrated individuals' behavior to characterize the dynamics of the economy was expressed by the capital stock. The paper focuses on the steady state where three values of k_{t-2}, k_{t-1}, and k_t such that $k_{t+1} = k_t = k_{t-1} = k_{t-2}$ satisfies are the equilibrium values of k: once k reaches that value, it remains there. It needs to show whether there is a steady state value of k, and whether k converges to such a value if it does not begin at one. The paper discusses how capital per labor k evolves over time given its initial value. The paper shows that the economy converges to the steady state using the equation of capital accumulation (2.61) and benchmark parameters are given in table 1. Before demonstrating the economy converges to the steady state, an initial capital stock in period t, k_t, is set equal to 0.001 and put into (2.61) to get the capital stock in period $t+1$, k_{t+1}. Once and again the result k_{t+1} is put into the equation of capital accumulation to get the capital stock in period $t+2$, k_{t+2}. This process is repeatedly done to show the steady state equilibrium: when both sides are equal in (2.61). The benchmark parameterization of the economy generates the steady state equilibrium with a capital per labor ratio of 0.65968. The paper chooses two initial values of k to explain that these values converge to the value of generated steady state: one is smaller then the origin steady state, k =0.0001, the other is bigger than the origin steady state, $k = 5$, to show that these values converge to origin steady state. If theses values converge to the origin steady state, it will be considered that the steady state has stability. Table 2. shows two initial values converge to the origin steady state in period 25 and 22, so the steady state generated with the benchmark parameters in the model can be demonstrated that this steady state has stability.

Table 2: Stability of steady state

Initial value k	0.0001	5.0
1	0.00934	1.42206
2	0.16047	0.89970
5	0.56934	0.70601
10	0.65616	0.66139
15	0.65955	0.65974
16	0.65961	0.65971
19	0.65967	0.65968
20	0.65967	0.65968
21	0.65967	0.65968
22	0.65968	0.65968
23	0.65968	0.65968
24	0.65968	0.65968
25	0.65968	0.65968
∞	0.65968	0.65968

4.2 EFFECTS OF TAX RATE CHANGE

The paper analyzes how the steady state is moved from the initial value by the changes of policy variables in this section. The following figures show the process of steady state's changing for labor income and capital income tax rates.

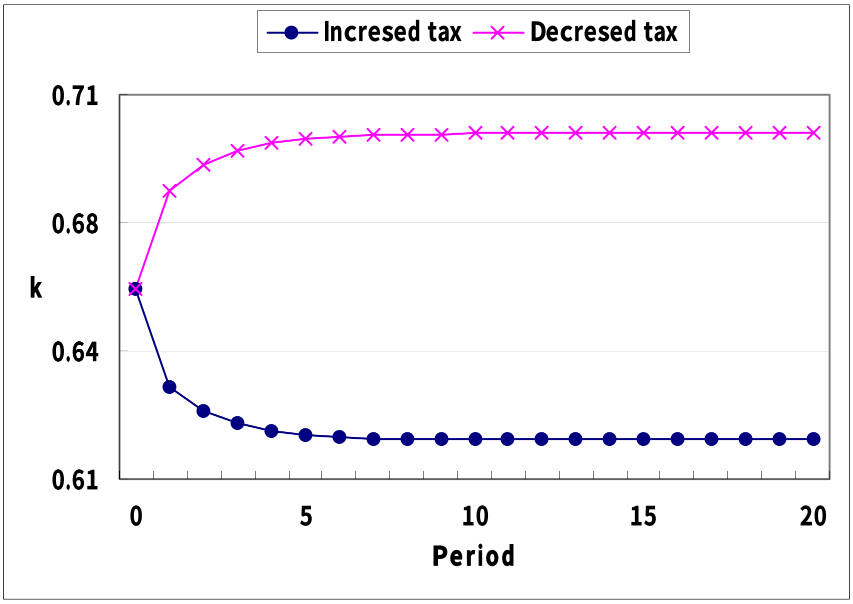

Figure 1. Effect of labor income tax rate

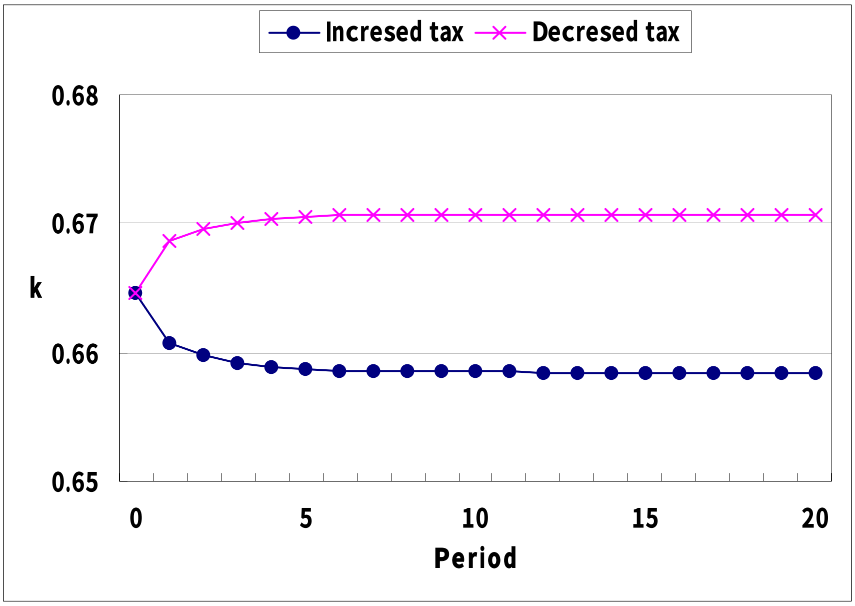

Figure 2. Effect of capital income tax rate

Figure 1 and 2 show the steady state changed by labor income and capital income tax rate moved between +5% and –5% respectively. From these figures, Investments are stimulated and capital stock evolves gradually to its higher steady state level by decrease of labor and capital income tax rate, and its lower steady state level by increase of these tax rates. It needs 12 periods to show moving to new steady state after tax from the initial steady state level and shows that an almost 7 periods are needed to converge to new steady state of economy after tax in these figures.

4.3 SENSITIVITY ANALYSIS

The paper shows that steady state is existed under the benchmark parameters and demonstrates that the steady state is stable in the model. Section 4.2 analyzed the effecting of the changes of policy variables. In this section, sensitivity analysis is confined on the origin steady state to parameters concerning distribution parameter for production function, α , and rate of time preference, ρ , and compared with the results of section 4.2.

Changing of distribution parameter for production function, α : this parameter is changed 0.42 to 0.52, and 0.42 to 0.32 with tax reform for labor income and capital payment.

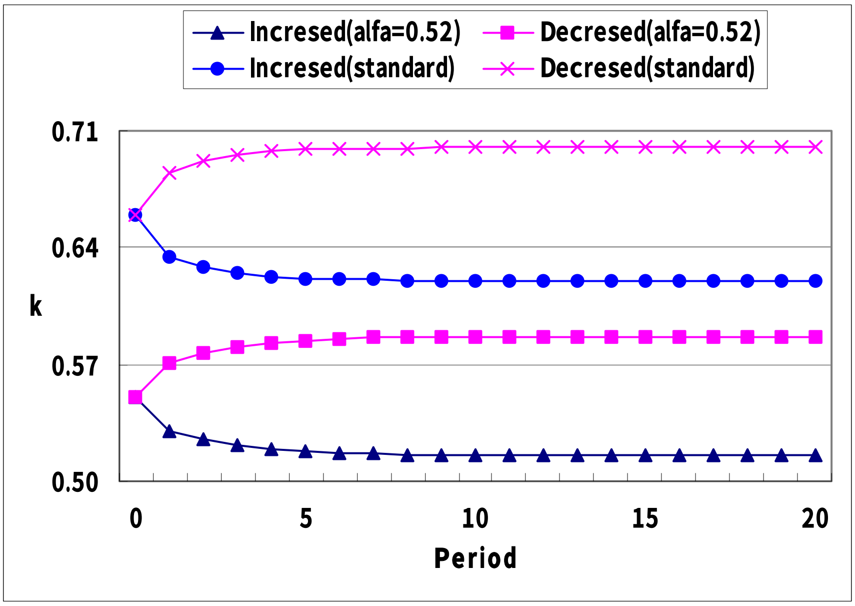

Figure 3. Effect of labor income tax rate (α =0.52)

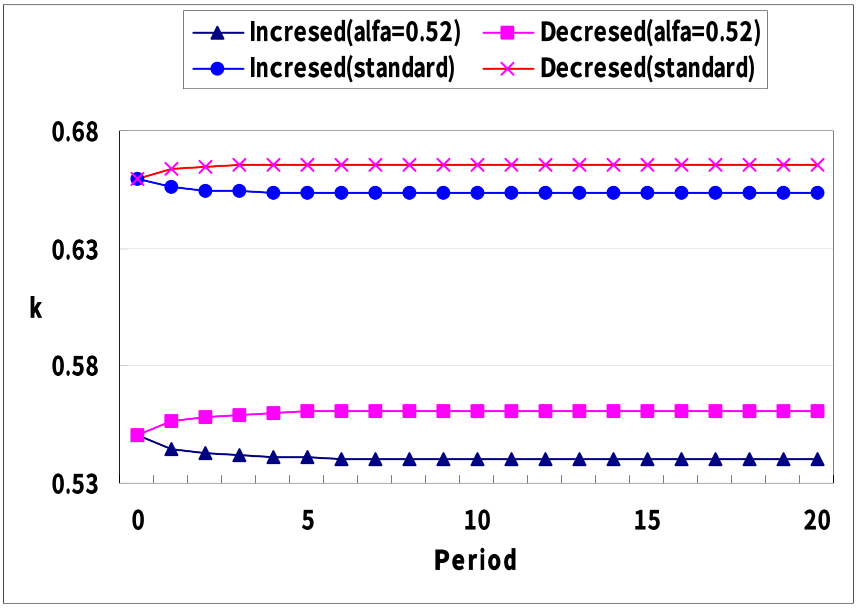

Figure 4. Effect of capital income tax rate (α =0.52)

Figure 3 and 4 show that increased value of α have effecting to decrease origin steady state of economy based on the benchmark parameters. The process of steady state's changing with tax rate changed is same with the case of the section 4.2. When α =0.52, steady state is decreased by 16.78% and 16.27% respectively for the labor income tax rate changes, and decreased by 17.28% and 15.81% respectively for the capital income tax rate changes.

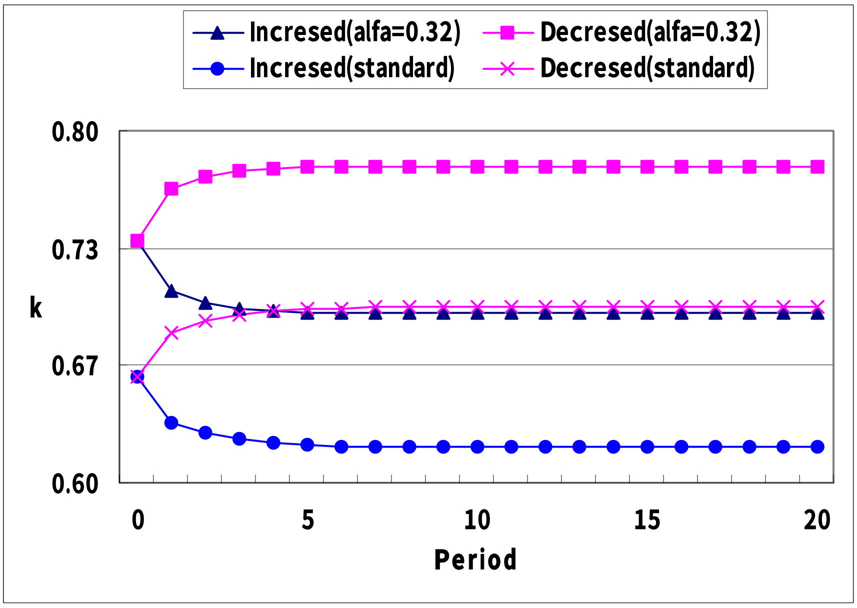

Figure 5. Effect of labor income tax rate (α =0.32)

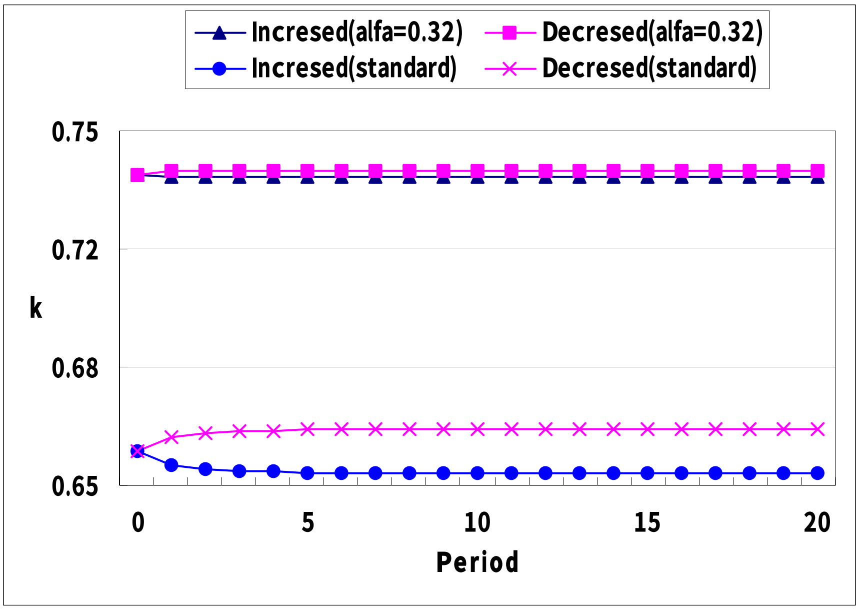

Figure 6. Effect of capital income tax rate (α =0.32)

Figure 5 and 6 show that there is possibility the fact that the origin steady state of economy increases when the paper decreases value of α compare with benchmark parameters. When α =0.32, steady state is increased by 12.14% and 11.53% respectively for the labor income tax rate changes, and increased by 12.68% and 11.03% respectively for the capital income tax rate changes. It explains that an increasing (decreasing) value of α decreases (increases) the labor income means decreases (increases) the capital with the Cobb-Douglas function.

Changing of rate of time preference, ρ: this parameter is changed 1.0 to 1.20, and 1.0 to 0.80 with tax reform for labor income and capital income.

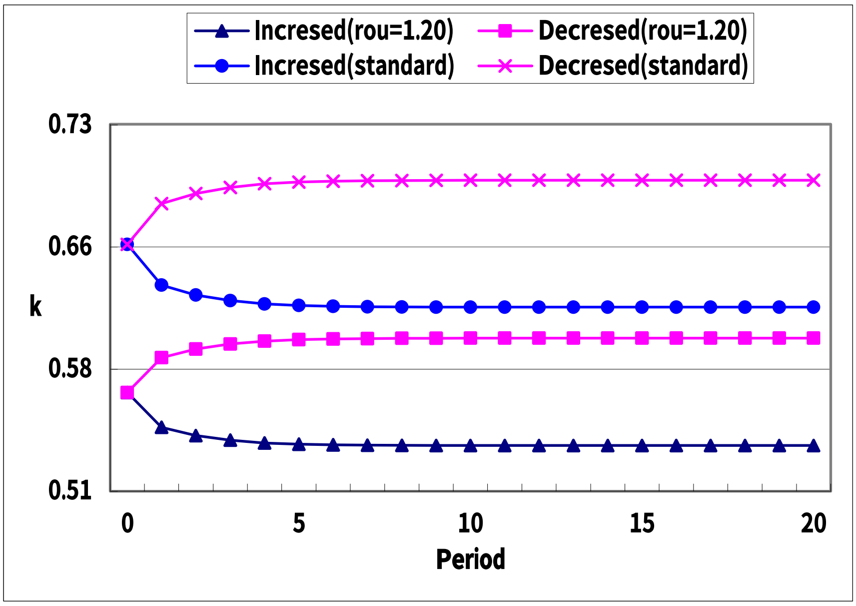

Figure 7. Effect of labor income tax rate (ρ =1.20)

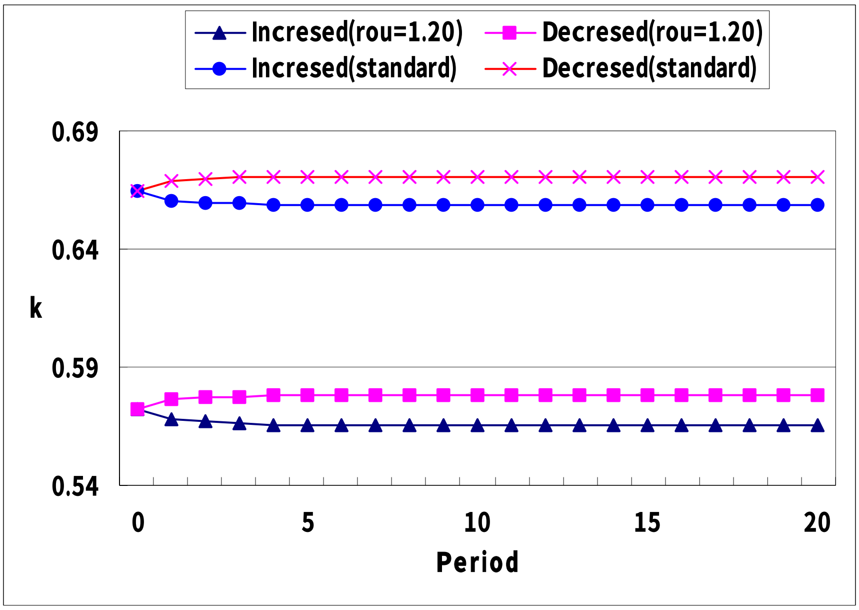

Figure 8. Effect of capital income tax rate (ρ =1.20)

Figure 7 and 8 show that increased of ρ decreases the origin steady state of economy. When ρ =1.20, steady state is decreased by 13.99% and 14.13% respectively for the labor income tax rate changes, and decreased by 14.22% and 13.90% respectively for the capital income tax rate changes.

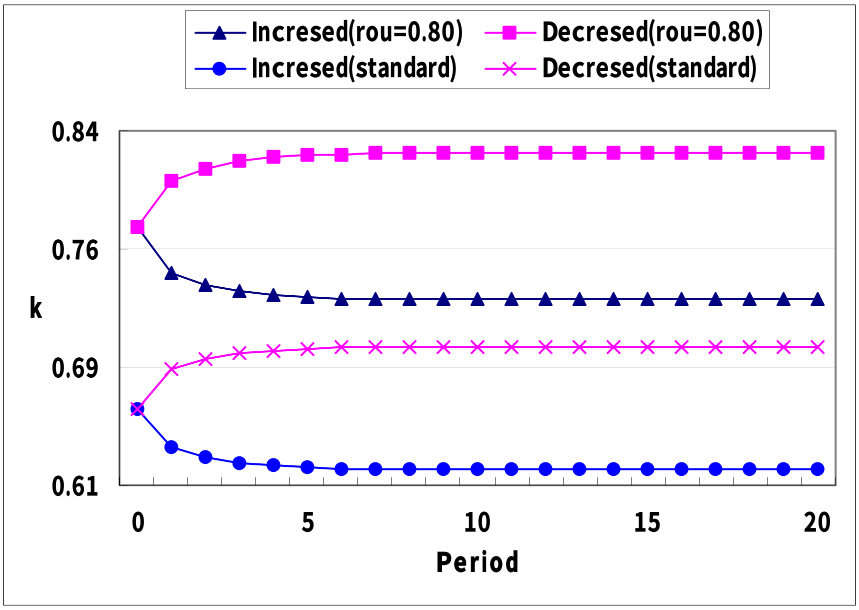

Figure 9. Effect of labor income tax rate (ρ =0.80)

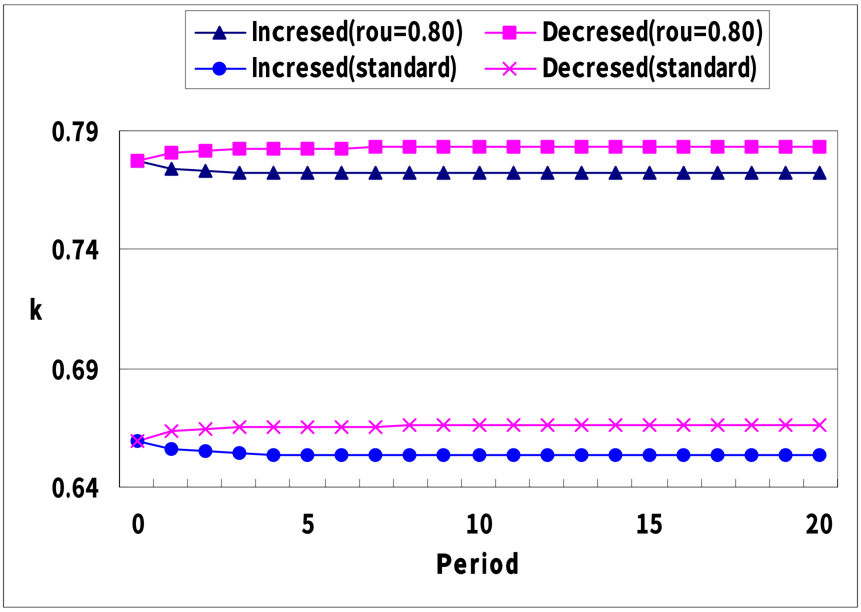

Figure 10. Effect of capital income tax rate (ρ =0.80)

Figure 9 and 10 decreased of ρ increases the origin steady state of economy. When ρ =0.8, steady state is increased by 17.75% and 17.96% respectively for the labor income tax rate changes, and increased by 18.11% and 17.62% respectively for the capital income tax rate changes. In above results, increasing of time preference rate means individuals place greater weight for the consumption on present period than future period.

It explains that decreasing in the future consumption means the decrease of saving as brings the decrease of capital. Case of decreasing of time preference rate is reversed.

4.4 TAX REFORM

With assumption of fixed government expenditure the paper analyzes tax reform in the economy has steady state using 4 cases as follows:

Case 1: Changing of capital income tax rate 20% to 25% and changing of labor income tax rate 9% to 8.15%

Case 2: Changing of capital income tax rate 20% to 15% and changing of labor income tax rate 9% to 9.86%

Case 3: Changing of capital income tax rate 20% to 25% and changing of consumption tax rate 5% to 4.21%

Case 4: Changing of capital income tax rate 20% to 15% and changing of consumption tax rate 5% to 5.78%

The paper sets τ_w=0.09, τ_c=0.05, and τ_r=0.2 as the paper's benchmark. These values are motivated by the fact. Table 3 presents (steady state) values capital, interest payment, labor income, total consumption, and utility for individual of the values τ_w, τ_c, and τ_r.

Table 3: Effects of tax reform based on fixed government expenditure

	Initial state	Case 1	Case 2	Case 3	Case 4
Capital	0.65968	0.66031	0.65896	0.65347	0.66573
Interest rate	0.34059	0.33997	0.34128	0.34662	0.33479
Labor income	1.00000	1.00041	0.99954	0.99604	1.00384
Total income	1.75194	1.77870	1.72547	1.75312	1.75090
Consumption of period 1	0.78480	0.79193	0.77761	0.78191	0.78760
Consumption of period 2	0.51396	0.52586	0.50230	0.51715	0.51093
Consumption of period 3	0.30212	0.30727	0.29704	0.30270	0.30158
Consumption of period 4	0.15106	0.15364	0.14852	0.15135	0.15079
Total consumption	1.75194	1.77870	1.72547	1.75312	1.75090
Saving of period 1	0.19620	0.19798	0.19440	0.19548	0.19690
Saving of period 2	0.45318	0.46091	0.44555	0.45406	0.45237
Saving of period 3	0.15106	0.15364	0.14852	0.15135	0.15079
Total saving	0.80044	0.81253	0.78848	0.80089	0.80005
Utility	-4.19011	-4.12434	-4.25620	-4.15338	-4.22562

The case 1 and 2 show that government levies the labor income tax, which is decreased (increased) as an amount of increased (decreased) capital income tax. In the case 1, labor income tax is decreased by 9.49%, compare with the initial state capital per labor, k, and total saving, ts, are increased by 0.10% and 1.51%, interest payment, r, is decreased by 0.18%, total labor income per labor, tw, and total consumption per labor, tc, are increased by 1.53%, and utility is increased by 1.57%. In the case 2, labor income tax is increased by 9.52%, k, $tw = tc$, ts, and utility are decreased by 0.11%, 1.51%, 1.49%, and 1.58%. r is increased by 0.20%.

The case 3 and 4 show that government levies the consumption tax, which is decreased (increased) as an amount of increased (decreased) capital income tax. In the case of 3, consumption tax is decreased by 15.88%, k is decreased by 0.94%. $tw = tc$, r, ts, and utility are increased by 0.07%, 1.77%, 0.06%, and 0.88%. In the case of 4, consumption tax is increased by 15.58%, k is increased by 0.92%. $tw = tc$, r, ts, and utility are decreased by 0.06%, 1.70%, 0.05%, and 0.85%.

5. CONCLUSION

The paper proposed computable general equilibrium model with overlapping generation framework assumed by each individual lives for four periods. The overlapping generation framework has been clearly defined with economic behaviors: consumer, producer, and government, in the paper. Another feature of the paper is the identification of different tax policies, which affect the economic growth intensities. The structure of the proposed model allows to make four cases of comparisons between the effects of a decrease (increase) in the labor income tax rate with an increase (decrease) in the capital tax rate and of a decrease (increase) in the consumption tax rate with an increase (decrease) in the capital income tax rate.
If there is no change of the labor income tax rate, an increase (decrease) in the capital income tax rate yields decreases (increases) in the capital accumulation and real labor income, and yields increase (decrease) in the real interest payment on the steady state economy compare with the value of the benchmark.

In the case of the labor income tax rate's change is considered, a decrease (increase) in the labor income tax rate with an increase (decrease) in the capital income tax rate yields increase (decreases) in the capital accumulation and real labor income, and yields decreases (increases) in the real interest payment compare with benchmark.

The paper yields a number of results. First, both labor and capital income tax affect savings and have similar effects on the steady state capital accumulation. Consumption tax hits harder the consumption of old generations who consume out of accumulated financial wealth.

Second, because consumptions depend on disposable income, total consumption and utility increased (decreased) by a decrease (increase) in the labor income and consumption tax rate with capital income tax rate. Indeed, with proposed overlapping generation model, a decrease in the labor income and consumption tax rates together with an increase in the capital income tax rate contributed much to the increase in the economic growth in one-country closed model.

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