A DYNAMIC INPUT OUTPUT MODEL FOR THE MEXICAN ECONOMY[1].

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SUMMARY.

This article intends to present, in a very compact form, the principal aspects of a dynamic mathematical model designed for the mexican economy, with the purpose of simulate the outcome of different economic policies.

Essentially, the model is a variation of Leontief dynamic input output scheme, [4], in witch it is described, in some detail, the role of different kinds of labor, so much as inputs in production as consumers of the different goods and services produced. Besides, a similar description is made of the role of capitalists.

Demand functions are estimated, with the Linear Expenditure System, (LES), [2] and [3], for different levels of income and allowing for variations in household's size, and number of them employed.

Once the demand function are known, as their rates of growth, a method is proposed for calculation of the rates of growth of the different productions which are compatibles with the former ones.

Necessary investments by origin and destination are endogenously calculated in a manner compatible with the rates of growth of production.

The theoretical objective of this work is elucidate the relationships that exist among the growth rates of the gross productions, in an dynamical input product system, and the

[1] The author acknowledges the valuable comments of Rafael Bouchain, Luis Kato and Abelardo Mariña to an earlier version of this paper, but, of course, the responsibility of the text is entirely his own.

growth different rates, of the final consumption demands, supposing that these last are known of a previous analysis of demand functions of the different types of consumers.

In a previous work of the author [1995], this problem was outlined in a general way, but it was to make necessary to explicit the relationships among both growth rates, and to indicate a solution method, to apply it in a dynamical model of economic policy for Mexico. This allow to depart the models type Von Neumann, in which all the sectors grow to the same rate, what is clearly not realistic neither useful in the reality.

The proposed solution, generalize the Pasinetti concept of vertical integration, that he applies only to the static case, where the investments are considered in the final demand, to a dynamical context, where they are endogenous calculated and where the growth rates of each one of the final demands are known.

ANTECEDENTS

Following the author's development [1995], the sectorial investment demands are calculated in a endogenous way, through a simple variable transformation, what leads to the need of invert a matrix that depends on the forward growth rates, for the gross productions, (known as the dynamic input output matrix), in which remain explicit the relationships among the two types of mentioned growth rates are explicit.

It is attempted to demonstrate that the growth rates of the gross productions depends totally on the growth rates on the final consumption demands, and is given a calculation method for the inversion of the dynamic matrix.

The use of the dynamical system proposed by Leontief, [1953] as well as of the model of Von Neumann, [1945] has been strongly criticized by Pasinetti, [1981; 1993], as solution to the real problems of the modern industrial economies, because of the simplification of their hypothesis --- like the constancy of the technical coefficients, in a world that is characterized by the constant technological change as by the inflexibility of the mathematics solutions in the differential equation systems or in finite differences, of first order, that impose solutions of equal uniform growth for the different sectors of the economy; when one of the principal characteristics of the modern economies is the great disparity in the growth of demands of the different goods and services ---; as well as the constant appearance of new consumption products.

The author hopes that the plan that he has proposed [Ibarra, 1995], in many coincident aspects with the ideas of Pasinetti (and using some of his forward steps), could surpass some of those critiques, with those which he coincides. In other work, forthcoming, designated “Comparison between two dynamical economic theory proposals", I detail some similarities and differences among my point of view and Pasinetti's one, in [5]. The principal similarity is the dynamical approach.

A first difference lies in the fact that in my study exists the need of consider, at least, two social groups: salaried and capitalistic, Pasinetti worked with just one, which demerit the richness of analysis in his last books (those which approaches in other of his works).

The second difference is referred to the treatment of the cost of using fixed capital in production. In a first part, called “Fixed capital with a simplifying hypothesis", Pasinetti arrives to a formulation to calculate the *consumption of fixed capital*, that it is formally very similar to one I propose; even though he considers that the length of the fixed capital goods depends only on its use in different sectors. In my approach, the replacement of fixed capital, depends on the useful life period of them, determined by the sector that produces them, and of the growth rates of the productions of the sectors that use them.

However, dominated by the prevailing concept that the most important thing is calculate the *depreciation*, Pasinetti, opts, thereinafter, for Von Neumann approach, calculating the value of the old machines, of different ages, considering that each production process that use them, constitutes a different activity, with joint production of the natural merchandise of the process and that of a year older machine. The prices of the older machines are calculated through the solution of the dual problem.

My opinion to this respect is that this snaggy procedure, that furthermore has serious theoretical consequences, specially for the value theory, is wrong, because the relevant concept to calculate is the physical replacement of the fixed capital goods, that finished their role in the production, because of the end of its useful life or by obsolescence; and not the financial amortization or depreciation concept.

1 DEMAND

One the principal concerns of the author, is to be able to apply the theoretical schemes to the tasks of the economic planning, therefore it has been seen in the need of adopting, (with modifications) a specific system of demand equations, known as the ***linear expenditure system***, giving them an appropriate form for their use in the general dynamical system, what implies that the calculation of different demands is expressed in function of its respective growth rates, those which can be written as a diagonal matrix $\mathbf{\hat{G}^f}$.

In the linear expenditure system, the final consumption demand by the merchandise $\boldsymbol{i}$, in the $\boldsymbol{t}$ period, (and whose price is $\mathbf{p_{i(t)}}$ of an earners household, for example, compound by η persons, where δ are those who earn an income, each one with a $\boldsymbol{S}$ salary, and a average propensity to consume c, the spent in the merchandise $\boldsymbol{i}$, in the period $\boldsymbol{t}$, would be equal to:

$$(1.1)\quad \eta p_{i(t)} y^{f}_{i(t)} = \eta p_{i(t)} \bar{y}^{f}_{i} + b_i \left[c\delta S_{(t)} - \eta \sum_j p_{i(t)} \bar{y}^{f}_{i} \right]$$

where : $y^{f}_{i(t)}$, is the demanded quantity, per capita, of merchandise i, in the period t, and, $\bar{y}^{f}_{i}$, a minimal consumption per capita of it.

If both members of the previous expression it divided by δ, and it's defined β = η/ δ which is the number of persons of the household that depend on each earner (included himself), the previous expression can be written as:

$$(1.2)\quad \beta p_{i(t)} y^{f}_{i(t)} = \beta p_{i(t)} \bar{y}^{f}_{i} + b_i \left[cS_{(t)} - \beta \sum_j p_{j(t)} \bar{y}^{f}_{j} \right]$$

Where:

$$(1.3)\quad y^{f}_{i(t)} = \bar{y}^{f}_{i} + \frac{b_i \bar{y}^{f}_{i}}{\beta p_{i(t)} \bar{y}^{f}_{i}} \left[cS_{(t)} - \beta \sum_j p_{j(t)} \bar{y}^{f}_{j} \right]$$

Multiplying and dividing the bracket of the second member of the last expression by $\beta \sum_j p_{j(t)} \bar{y}^{f}_{j}$ and defining $\gamma_i = \dfrac{p_{i(t)} \bar{y}^{f}_{i}}{\sum_j p_{j(t)} \bar{y}^{f}_{j}}$, that is the average propensity to consum the merchandise i; and $g^* = \dfrac{\left[cS_{(t)} - \beta \sum_j p_{j(t)} \bar{y}^{f}_{j} \right]}{\beta \sum_j p_{j(t)} \bar{y}^{f}_{j}}$ that

is the geometric rate of total consumption growth per capita, between the current period of the income and that when this was the same as the ***minimal consumption***, the equation can be written as:

$$(1.4) \qquad y_{i(t)}^{f} = \left[1 + \frac{b_i}{\gamma_i} g^{*}\right] \bar{y}_i^{f}$$

As is known, the quotient among the marginal and middle propensities to consume the merchandise ***i***, is equal to the income elasticity ε_i, of such merchandise, so that:

$$(1.5) \qquad y_{i(t)}^{f} = \left[1 + g_i^{f}\right] \bar{y}_i^{f}$$

Where $g_i^f = \varepsilon_i g^*$

The **g*** rate, that governs for a long period of years, **(n_p)**, can be expressed in annual rate terms of growth: $\mathbf{g^f}$, **(g^f= (1+g*) 1/n^p -1)** , so that the growth of the final consumption demand can be written as a column vector, of ***n x 1*** dimensions

$$(1.6) \qquad Y_{(t-1)}^{f} = Y_{(t)}^{f}\left(I + \hat{G}^{f}\right)$$

and also as:

$$(1.6.1) \qquad Y_{(t)}^{f} = Y_{(t-1)}^{f}\left(I + \hat{G}^{f}\right)$$

So, as demand of the ***j*** good, witch belongs to the known vector, grows according to the following equation:

$$(1.7) \qquad y_{j(t+1)}^{f} = y_{j(t)}^{f}\left(1 + g_j^{f}\right)$$

Which can be reordered as:

$$(1.8) \qquad y_{j(t+1)}^{f} - y_{j(t)}^{f} = y_{j(t)}^{f} g_j^{f}$$

Where:

$$g_i^f = \varepsilon_i g^f \qquad \textit{(1.9)}$$

In the matter of fact, in the linear expenditure system, the income elasticities of demand, are not constant and tend to the unit when the income tends to infinite, what is opposite to the empirical evidence. To remedy this defect, in the model for Mexico, the estimates of the expenditure functions will be made for various successive sections of income levels, for each social group; and will be assumed that the elasticities remain constant for each one of those sections.

2 THE DYNAMICAL INPUT OUTPUT SYSTEM

As to the input-output relationships, the proposed scheme use the traditional matrix of technical coefficients of current input (designated here, with **M**), as well as those of incremental coefficients capital - production (designated here with **K**), used by Leontief in his dynamical model.

The author believes that the dynamics of the technological change is expressed, in practice, in a much more accelerated change in the coefficients of the **K** matrix, that in those of the **M**. For practical effects, this means that the **M** matrix calculated for a given year, can be used for several years without meaningful changes; on the other hand, the **K** matrix, would be calculated each year, to incorporate the current technical changes .

The Leontief model equation, in finite differences, is:

$$Q_{(t)} = MQ_{(t)} + K\left(Q_{(t+1)} - Q_{(t)}\right) + Y_{(t)}^f \qquad \textit{(2.1.1)}$$

or also, according to (1.6.1):

(2.1.2) $$Q_{(t)} = MQ_{(t)} + K\left(Q_{(t+1)} - Q_{(t)}\right) + Y^{f}_{(t-1)}\left(I + \hat{G}^{f}\right)$$

In this model, is added an investments matrix in necessary variation of stocks, that depends from the production periods in different sectors expressed in a diagonal

matrix $\hat{T}$ *:*

$$M\hat{T}\left(Q_{(t+1)} - Q_{(t)}\right)$$

Furthermore, a replacement matrix of the fixed capital goods is added:

$$K \oplus RQ_{(t)}$$

the ⊕ symbol indicate a special matricial multiplication, where the elements of the product matrix result from multiplying term to term, the elements of the multiplied matrix .

The elements of the **R** matrix, are calculated through the expression:

$$r_{ij} = g^{*}_{ij} / \left[\left(1 + g^{*}_{ij}\right)^{ni} - 1\right]$$

Where: $\mathbf{n_i}$, is the useful life span of the fixed capital goods produced by the sector **i**, and $\mathbf{g^*_{ij}}$, the growth rate of the gross production of the ***j*** sector, in the past, during the last $\mathbf{n_i}$ years.

Finally, replaced the expressions ($Q_{(t+1)}$ -$Q_{(t)}$) for $\hat{G}Q_{(t)}$

With these changes, the equation (2.1.2), becomes:

$$(2.2) \qquad Q_{(t)} = MQ_{(t)} + M\hat{T}\hat{G}Q_{(t)} + K \oplus RQ_{(t)} + K\hat{G}Q_{(t)} + Y^{f}_{(t-1)}\left(I + \hat{G}^{f}\right)$$

It is defined:

$$(2.3) \qquad B = \left(M + K \oplus R\right) \quad y \quad C = \left(M\hat{T} + K\right)$$

So (2.2) can be written as:

$$(2.4) \qquad Q_{(t)} = BQ_{(t)} + C\left(Q_{(t+1)} - Q_{(t)}\right) + Y^{f}_{(t-1)}\left(I + \hat{G}^{f}\right)$$

Alternatively this equation can be written as:

$$(2.5) \qquad Q_{(t)} = BQ_{(t)} + C\left(Q_{(t+1)} - Q_{(t)}\right) + Y^{f}_{(t-1)}\left(I + \hat{G}^{f}\right)$$

In a not planned economy, the growth rates of the sectorial gross productions, contained in the matrix **Ĝ**, are the entrepreneurs decisions , and if they would be known, as well as the parameters of the **M**, **K**, **R** and **T** matrix, the inverse of the matrix of the equation (2.5) could be calculated without greater difficulties.

Calling $\mathbf{W = [\ I - (B + \hat{G}\ C]^{-1}}$, the solution of (2.1.1) is $Q_{(t)} = WY_{(t)}^{f}$

The problem lies in if those investment decisions are arbitraries, and have no relation to the growth rates of final demands, contained in the $\mathbf{\hat{G}^{f}}$ matrix, the production capacities of the period ***t+1***, will be hardly congruent with the demands in that same period.

The following calculation method proposed look to find that congruity, from a theoretical point of view, especially thinking in a planning framework, where it previously estimate the future consumption demands, in function of a general goal of its total growth of consumption per capita.

Supposing, therefore, already known the elements of the vector: $\mathbf{Y_{(t)}}^{\mathbf{f}}$, which is ***n x 1*** dimension, as well as the matrix values $\mathbf{\hat{G}^f}$, an inverting method of the matrix by parts is proposed, that include ***n*** stages, and , in essence, consists in applying ***n*** times what Pasinetti called, the vertical integration, to the case of the ***n*** final demands, that grow at different rates.

The central idea is that all the dynamics of the system results from the growth rates of the final consumption demands, so, therefore, the growth rates of the gross productions are induced by the previous and depend totally on them, on the specified way.

Starting from the (2.4) equation, in each stage is outlined a problem where it is considering only that exists a not negative final demand whose growth rate is known, being equal to zero the other final demands. The aim is the calculation of the corresponding gross productions.

Be $Y^{j}_{(t)}$ corresponding demand vector to the j stage, where the not negative element is $y^{f}_{j(t)}$, that grows to the rate g^{f}_{j}, and be $Q^{j}_{(t)}$ the corresponding gross sectorial productions solutions vector, so the equations (2.4) can be written as :

$$(2.6) \qquad Q^{j}_{(t)} = BQ^{j}_{(t)} + C\left(Q^{j}_{(t+1)} - Q^{j}_{(t)}\right) + Y^{j}_{(t)}$$

$$j = 1,\dots, n$$

As solution of that system the following relationships are proven:

$$(2.7) \qquad Q^{j}_{(t+1)} - Q^{j}_{(t)} = Q^{j}_{(t)} g^{f}_{j}$$

j = 1,…, n

Introducing (2.7) in (2.6), has:

(2.8) $$Q_{(t)}^{j} = BQ_{(t)}^{j} + CQ_{(t)}^{j} g_{j}^{f} + Y_{(t)}^{j}$$

j = 1,…, n

and the solution is:

(2.9) $$Q_{(t)}^{j} = \left[I - \left(B + Cg_{j}^{f}\right)\right]^{-1} Y_{(t)}^{j}$$

j = 1,…, n

Writing this same equation, for the period *t+1*:

(2.10) $$Q_{(t+1)}^{j} = \left[I - \left(B + Cg_{jf}\right)\right]^{-1} Y_{(t+1)}^{j}$$

j = 1,…, n

Subtracting (2.9) of (2.10), and using the relationship (1.8):

(2.11) $$\left(Q_{(t+1)}^{j} - Q_{(t)}^{j}\right) = \left[I - \left(B + Cg_{j}^{f}\right)\right]^{-1} Y_{(t)}^{j} g_{j}^{f}$$

j = 1,…, n

Using the relationship (2.9) in the (2.11), has:

(2.12) $$\left(Q_{(t+1)}^{j} - Q_{(t)}^{j}\right) = Q_{(t)}^{j} g_{j}^{f}$$

j = 1,…, n

This equation is identical to the (2.7), so it is demonstrated that the proposed solution is correct.

If the solution vector (2.9) is defined as: $\mathbf{q}^{\mathbf{j}}_{\mathbf{(t)}}$, when the corresponding element of final demand is replaced by 1, on have:

(2.13) $$q^{j}_{(t)} = \left[I - \left(B + Cg^{f}_{j}\right)\right]^{-1} e_{j}$$

j = 1,…, n

Where $\boldsymbol{e_j}$, are the unitary vectors, for j=1,…,n.

The elements of this vector represent the direct and indirect requirements of production, by final demand unit of the ***j*** sector, that is growing at the rate: $\mathbf{g}^{\mathbf{f}}_{\mathbf{(j)}}$

Now it can be built a ***n x n*** dimensions matrix, these is the sought dynamic inverse matrix:

(2.14) $$D = \left[q^{1}_{(t)}, \ldots\ldots, q^{n}_{(t)}\right]$$

So:

(2.15) $$Q_{(t)} = DY^{f}_{(t)}$$

The **D** matrix, ~~plays~~, therefore, the same role that the **W** matrix, and in its calculation, participate the growth rates looked for diagonal matrix $\hat{\mathbf{G}}^{\mathbf{f}}$.

If it want to know the growth rates of the sectorial gross productions, that conform the $\hat{\mathbf{G}}$ matrix, they can be calculated as a weighted average of the growth rates of final demands:

(2.16) $$g = \hat{Q}_{(t)}^{-1} D \hat{Y}_{(t)}^{f} g^{f}$$

Where the sign ^ above the Q and Y variable, means the diagonalization of its respective vectors.

3 AN APPLICATION TO THE STUDY OF THE MEXICAN ECONOMY

It is important to indicate that the theoretical and practical developments here outlined, are intimately bound to a big effort, in the Economic Research Institute of the UNAM, where the author collaborates as consultor, to design a dynamical model of economic policy, for the Mexican economy.

For that study, they are being use, mainly: the National Accounts, the Input Output Matrix, the Income and Expenses surveys of Households, elaborated by INEGI; as well as the Survey of Capital Assets and other studies of the Bank of Mexico.

Amongst other things, the construction of the model requires of the estimate of the Capital - Production matrix, for something which, it is of great value the Survey on Capital Assets and the periodical statistics on banking loans to the various sectors of the economy, of the Bank of Mexico.

Other fundamental aspect of the model is the estimation of demand systems, for the composited goods that are detailed in the input output matrix. Those functions will be calculated for various strata of incomes, of households composed of different profession and qualifications, and of the social groups comprised by: the Salaried and workers by its own Account, the rentists and the Capitalists.

For such estimates it is fundamental the use of the Income and Expenses Surveys of Households, whose analysis has suggested the conceptual framework of the concrete analysis of demand that here is review.

4 CONCLUSIONS

The author believes to have shown the relationship among the growth rates of the final consumption demands and the growth rates of the gross productions, in a dynamical input output model.

On the other hand, the transformation that is proposed of the equations system in finite differences; in one of simultaneous equations, adding the additional variables of the growth rates of the gross productions that, at the same time, depends on the growth rates on the consumption demands, permits to avoid the use of the instrumental of the first, that lead to solutions little realistic, in which all the sectors grow to the same rate.

The calculation algorithm that is proposed to calculate the dynamic inverse matrix, can be a great practical usefulness for those which work the dynamical input output model.

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