THE TOTAL LINKAGES FOR MEXICO IN 1993.

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1. Introduction.

The analysis of Input-Output (I-O) offers certain advantages in relation to a study in which macroeconomic variables can only be included, that is because the I-O incorporates a description of the intra-and interindustrial structure that predominates in an economy, and that as well allows to find an explanation on the existence of these industrial magnitudes associated with the both global supply and global demand.

An integrated system of national accounting, includes the calculation of tables of I-O, because this elaboration is based on information that comes from censuses that are not made of periodic way and which require a great investment: money and time. The statistic offices elaborate the tables of I-O every ten years with estimations at national or regional level. The supply of an industry is related to the purchases of intermediate and factorial inputs each one of them requires generate its total production. On the other hand the demand shows the value of the output, same of the sales of destiny of the output for its use, intermediate inputs or other industries that final demand.

The I-O analysis proposed by Leontief has captivated many economists, in particular in the literature on industrial linkages. This represents a valuable tool that allows classify the industries of an economy, in agreement with the quality and magnitude of the influences of each industry towards the rest of the economy and vice versa.

2. The backward linkages.

The original proposal by Hirschman (1958) is the calculation of the backward and forward industrial linkages, and identify key sectors. Hirschman focus on demand pressures, the forward linkage acts as important and powerful reinforcement to backward linkage, Miller and Lahr (2001). This perspective remarks the attributes of the Leontief quantity model, in this sense the Leontief inverse contains the output multipliers, which capture both, the direct and indirect industrial linkages. One major drawback of the traditional Leontief model is the strictly demand-driven character, Oosterhaven (1988).

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The proposition of Rasmussen (1956), remarks the average backward linkage index, namely the ‘power of dispersion index’, or influence of that the final demand an one industry, for the others production.

First, we start from the following input-output matrix that describes a National Economic System:

	n Production accounts	Final Demand	Total
n production accounts	Z	y	q
All Other accounts	v'	0	η
Total	q'	η	

Where:

Z = Input matrix (intra-and intersectorial flows of goods and services),

y = final demand vector (includes exports),

v' = Row vector of added value,

q = sectorial gross production vector.

All other accounts included intermediate imported products and, added value or rent.

This scheme satisfies the macroeconomic condition of the equality in the output and the rent. $v'\iota = \iota' y = \eta$

This table of I-O describes the economic transactions from the point of view of the origin and destiny of some ones, inputs and outputs. From the point of view of the columns this, reflects the value of both, the intermediate and factorial inputs (added value), which are required for generation of gross production for each industry. And by the rows, the output or sales of goods and services are purchased by each industry (intermediate demand) and the rest are used as final goods.

The demand system can be expressed in matrix terms by the following equation:

(1) $$q = Z\iota + y$$

Where ι represents the unitary vector.

The demand model based on the introduction of Leontief’s hypothesis, cradle in the existence of production functions with fixed coefficients for each industry, which means that the proportions of inputs contained in a production unit, remain more or less constant through the time.

In this way, with algebra of matrices, we can define a matrix of technical coefficients that are proportional to the gross value of production:

(2) $$A = Z\hat{q}^{-1}$$

Where A is a technical coefficients matrix, and $\hat{q}^{-1}$ is the diagonal and inverse vector of gross production.

We can obtain a solution for the gross value of production with the following way:

(3) $$q = Aq + y$$

(4) $$q = (I - A)^{-1} y = Ly$$

Where $(I - A)^{-1} = L$ is the Leontief's inverse matrix, whose components represent the direct and indirect requirements of the total production by unit of final demand for any industry. If we incorporate the components of each column of Leontief's inverse, we obtain the called backwards linkages BL for each industry that we represent with the vector row:

(5) $$\alpha' = \iota' L$$

The Rasmussen's 'power of dispersion index' it's identical to:

(6) $$U_d = \iota' L \frac{n}{\iota' L \iota} = \alpha' \frac{n}{\alpha' \iota}$$

The value of U_d oscillates around the one, and represents the rate of two averages, Bouchain (2002), Bouchain & Schinca (2003).

3. The forward linkages

In this way, based in the Leontief inverse, Rasmussen proposed the calculus of the particular version of the "forward linkages", die in the average sum of rows of the Leontief inverse. The namely 'sensitivity of dispersion index' (U_s), reflects the power of the average increment demand in all industries, over the direct and indirect average requirements of production of each industry. If $\beta = L\iota$, then:

(7) $$U_s = L\iota \frac{n}{\iota' L \iota} = \beta\iota \frac{n}{\iota' \beta\iota}$$

The Rasmussen's Index (power of dispersion and sensitivity of dispersion) have been viewed with skepticism, and has no convincing economic or statistical interpretation, because this "average" multiplier have to be derived using weights denoting sector size, Miller and Lahr (2001).

An alternative form, the forward linkages live in the Gosh model, so called supply-driven input-output model, there is a radical alternative to the Leontief model, it is practically in all the complete reverse of the traditional demand-driven model, both are extreme cases of multisectorial general equilibrium models, Oosterhaven (1988).

In a similar form, as to the demand model, Gosh made a generalization in the definition of ,as the supply model, nevertheless, the interpretation of the same one must be made with reserve, we cannot assume the same rigor of the permanence of "fixed coefficients of delivery ", in which is based this model.

The supply model is sustained in the calculation of the delivery coefficients (or sales), this is for each row, exists the function of demand with fixed coefficients in proportion to the gross output, can be calculated. We have:

(8) $$C = \hat{q}^{-1} Z$$

Where C is the matrix of delivery coefficients.

From the same form we can obtain a solution for the gross production in the supply model:

(9) $$q' = q'C + v'$$

(10) $$q' = v'(I - C)^{-1} = v'G$$

On the other hand the matrix $(I - C)^{-1}$ has a particular meaning, it shows to us, they are direct and indirect requirements of deliveries of each sector by unit of supply using intermediate inputs. If now we added the components of the rows of the matrix G we obtain a vector β whose components are the linking towards ahead (FL) for each industry, so:

(11) $$\gamma = G\iota$$

It is possible to indicate that the matrices A and C are from individual linear transformations (by the right and the left) of the matrix inputs *Z*, therefore they can be considered like similar matrices, reason why we can express C in terms of A, in the following form, $C = \hat{q}^{-1}Z$, and $Z = A\hat{q}$. we have:

(12) $$C = \hat{q}^{-1} A\hat{q}$$

Where $\hat{q}$ is the diagonal vector of gross production.

4. The total linkages.

Proposed by Cella (1984), the total linkages allows in a simultaneous way, a measurement of the backwards and the forwards linkages. And they can separate ahead. The idea it's based on the hypothetical extraction method, that consisting in removing the row and the column corresponding to the industry from which they wanted to obtain the corresponding linkages. In agreement with this tradition, this operation allows the more precise measurement of the linkages in each industry.

> "It seemed to us that it would be possible to create a several categorical boxes through alternative ways of "hypothetical extracting" one or more sectors in an interindustry framework in order to assess their "linkage" whit or "importance" to the economy from which they were extracted." Miller and Lahr (2001, p. 407).

The hypothesis that guides the hypothetical extraction procedure indicates that the industry at issue does not send products, and no receives inputs for other industries. The total linkages (TL) can be formulated of following way:

(13) $$TL = \iota'\left(q - \bar{q}\right)$$

Where q is the vector of gross production considering the totality of the industries and $\bar{q}$ is the gross production vector after coming to the hypothetical extraction. Now let us consider one sector of m_1 industries and the remaining m_2. Where the total industries of the economy are $n=m_1+m_2$.

In this exposition we must begin with I-O divide table in several sub matrices and their corresponding vectors of gross production, the final demand and other accounts (included added value and imports of intermediate), also divide, can represent like this:

	Sector 1 (***m₁*** industries)	Sector 2 (***m₂*** industries)	Final Demand	Totals
Sector 1 (***m₁*** industries)	Z_{11}	Z_{12}	y_1	q_1
Sector 1 (***m₂*** industries)	Z_{21}	Z_{22}	y_2	q_2
All other accounts	v_1'	v_2'	0	η
Totals	q_1'	q_2'	η	

Immediately, a set of square matrices with the same dimension is generated like original, but putting zeros in the places where the extraction has taken place, in the proposition of Cella (1984), we obtain the following systems, first, for the gross production:

(14)
$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

and the other, the production after the extraction procedure:

(15)
$$\begin{bmatrix} \bar{q}_1 \\ \bar{q}_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & 0 \\ 0 & Z_{22} \end{bmatrix} + \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$$

Solving both we obtain the gross production:

(16)
$$\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} H & HA_{12}L_{22} \\ L_{22}A_{21}H & L_{22}(I + A_{21}HA_{12}L_{22}) \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

where $H = \left(I - A_{11} - A_{12}B_{22}A_{21}\right)^{-1}$, and on the production after extracting procedure:

(17)
$$\begin{bmatrix} \bar{q}_1 \\ \bar{q}_2 \end{bmatrix} = \begin{bmatrix} L_{11} & 0 \\ 0 & L_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

The total linkages are the difference of the two systems (16) minus (17):

(18)
$$\begin{bmatrix} q_1 - \bar{q}_1 \\ q_2 - \bar{q}_2 \end{bmatrix} = \begin{bmatrix} H - L_{11} & HA_{12}L_{22} \\ L_{22}A_{21}H & L_{22}A_{21}HA_{12}L_{22} \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

Total linkages are:

(19)
$$TL = \iota'\left[\left(H - L_{11}\right) + L_{22}A_{21}H\right]f_1 + \iota'\left[HA_{12}L_{22} + L_{22}A_{21}HA_{12}L_{22}\right]y_2$$

(19’)
$$TL = BL + FL$$

The equation (19) defines TL and discomposes it in two components which depend on the technical coefficients of *A* and the vector of final outputs *y*, Cella (1984). The *H* and *L* are non singular matrices, and y_1 and y_2 are non-cero vectors, clearly $BL=0$ if $A_{21}=0$, and $FL=0$ only if $A_{12}=0$.

The first term of (19) contains the measure currently used (the total direct and indirect inputs required to support the final output y_1 of sector *1*, $\iota'(H+L_{22}A_{21}H)f_1$, and subtracts the scalar whit contains the transactions purely internal to the sector, $\iota' L_{11}y_1$. But this procedure is not fully correct because Hy_1 includes both indirect effects due to intra-sector transactions and the feedback of sector *1* on itself due to intermediate purchases from sector *2*. Cella (1984).

In the second component of (19), the first term is the gross output of sector 1 required to support the final output of sector 2, while the second is the feedback of this gross output on to sector 2.

5. Discussion

We first discuss the importance of the correct selection on boxes that represents the backward and forward linkages, in the extraction procedure. Meller and Marfan for example, define the $TL^{**}=TL+\iota' L_{11}y_1$, and incorporates the feedback entirely internal to sector *1*. If we be suppressing entirely the sector *1*, for $T^*=TL-\iota'(H-L_{11})y_1-\iota' HA_{12}L_{22}y_2$, and underestimates the total linkage. Cella (1984).

In opinion of Miller and Lahr (2001) the correctly BL measurement is when $A_{21}=0$ in the extracting procedure, and if we want extract intersectorial connections then $A_{11}=A_{21}=0$.

Clements (1990) has critiqued to Cella, in his decomposition of total linkages into backward and forward linkage components. His problem is the overestimation of the FL then the second component of the last term of (19) is part of the BL .

Finally, the more criticism for the Cella's method is related whit the use of the Gosh model. First, Oosterhaven has critiqued the plausibility of this model, because the production function is abandoned and it's based on the incorrect casual interpretation.

> The Ghosian model takes demand for granted, demand is supposed to be perfectly elastic. For final demand this means that local consumption or

investment reacts perfectly to any change in supply, and the purchases are made, of cars without gas and factories without machines. Oosterhaven (1998. P. 207)

In the case of intermediate demand this means that inputs vary arbitrarily, this is a general alternative to modeling centrally planned or resource-oriented economies. Reviewing the literature we concluded that the model may only used, if carefully interpreted, in descriptive analyses. Any causal interpretation or application leads to a best meaningless, probably non essential results.

The equation (12) expresses the matrix *C* (coefficients of sales) in terms of the matrix *A* (to technical coefficients), and they are similar matrices. In this sense Cella completely avoiding the Gosh price model and the economic assumptions driving it, but the row of matrix *A* don't have an appealing economic interpretation. Miller and Lahr (2001).

Other authors insisting that only backward linkages are to be found from the Leontief model. Forward linkages must be come from elements of the Gosh model.

6. Application to Mexico in 1993

This is a numerical illustration of the calculus of total, backward and forward linkages, proposed by Cella, use I-O table of 1993, at current and constant prices (of 1980). (Graphs 1 to 4).

We have established the order of the key sectors for nine gross divisions in the Mexican economy. And it's important the results by the normalized or pondered scheme (by the gross production structure), in which changes the order of the key sectors are be produced.

The key sector is the manufacturing, and this importance in terms of linkages is little. In the other place the key sector of total linkages (Graph 1) are electricity, mining, livestock and construction, while in the pondered indexes they up to down.

These are, the many services sectors can’t be important in terms of these linkages.

The sectors included are:

1	Agriculture and livestock
2	Mining and oil
3	Manufacturing
4	Construction
5	Electricity and water
6	Commerce and hotel and restaurants
7	Transport and communications
8	Financial services
9	Other services

7. Conclusions

Evaluations of a sector's total linkage seem to us to be the appropriate measure believes the original hypothetical extraction approach.

The Goshian model is a correct base of the forward linkages, but needs a further discussion and empirical study.

The interpretation of the forward linkages in the demand-driven model it's not clearly, and we need a main power tools box.

8. References

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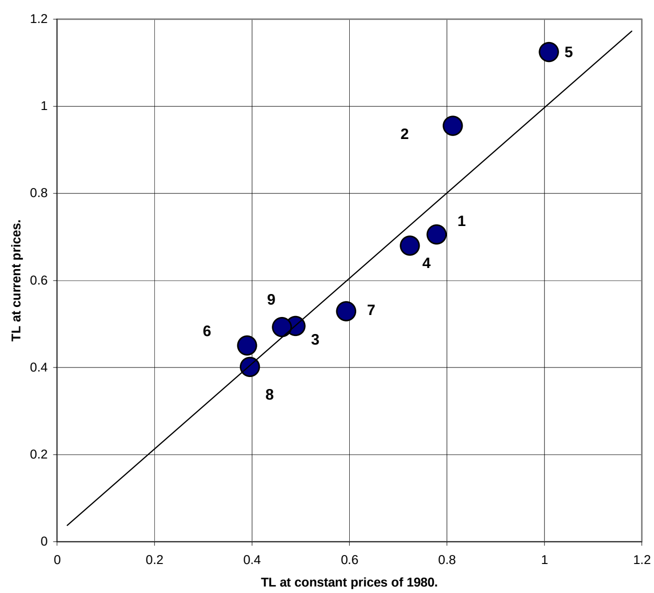
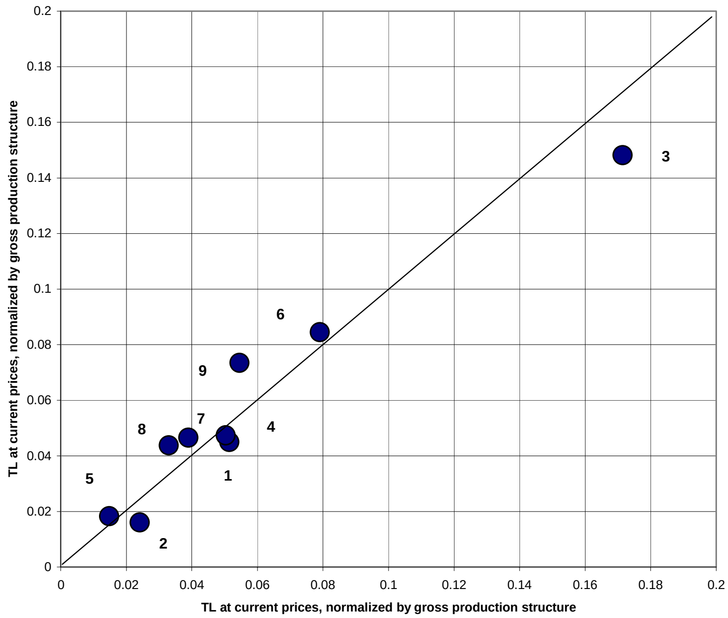
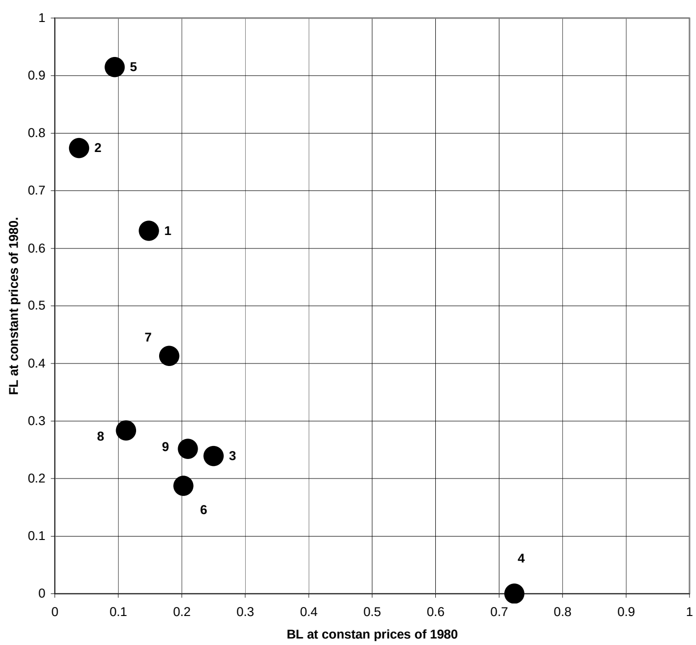
Graph 1. Mexico, 1993: Total linkages in percent of gross production.



Graph 2, Mexico, 1993: Total linkages whit percent of gross production, normalized by the gross production structure.



Graph 3. Mexico, 1993. Backward and forward linkages, at constan prices of 1980.



Graph 4. Mexico, 1993. Backward and Forward linkages with percent of groos production, at constant prices of 1980, normalized by gross production structure.