Inter-relationship of Total Labour Requirements and Value Added Productivity of Labour in the Phases of Economic Fluctuations: Case of USA for years 1987 to 1999

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1 INTRODUCTION and FRAMEWORK of the ANALYSIS

Final demands are functions of prices of all sectors given the income levels. That is, $Fi = Fi(P_1, ..., P_n)$ for all sector i, where final demands are sum of private consumption , gross investment , government expenditure and export demand minus imports for each sector i.

When final demands are determined, then gross outputs for each sector are determined following the material balance equations as such: $X = AX + F$.

When gross outputs are determined, this will determine the direct labour requirements as $L_i = l_i X_i$ for each sector i and this will determine factor prices. Factor prices in turn determine the prices of outputs of all sectors, that is,

$P_i = P_i(X_{1,}....., X_n)$ for all sector i.

Prices of all sectors determine the value added productivity of labour of all sectors according to the price equations.

In reality, price of outputs of all sectors and value added productivities are determined simultaneously.

$P = PA + L\widehat{Y}$

On the other hand, total labour requirements are defined as

$T = TA + L$

Then, endogeneous variables are F_1 to F_n, X_1 to X_n, L_1 to L_n, Y_1 to Y_n, P_1 to P_n, and T_1 to $T_n$where exogeneous variables are A and labour coefficients l.

Both prices and output levels, total labour requirements and value added productivities are determined simultaneously according to the GENERAL EQUILIBRIUM framework. However, in the following, prices and output levels are taken as if exogenously given, and we concentrate to analyse the relationship between the value added productivity of labour and total labour requirements. Movements of prices,factor prices and outputs will be considered at the end for the explanation of the empirical results.

Theory tells us that the inverse of total labour requirements are weighted sum of value added productivity of all industries producing intermediate inputs. However when inverse of value added productivity of labour is regressed by total labour requirements (which is equivalent to total labour value, employmenet inducement coefficient), it indicated significant relationship with high goodness of fit in the past analysis.

With dataset supplied by US BEA , we calculated total labour requirements and value added productivity of labour for integrated 43 industries for Benchmark IO of years1987,1992 and 1997, and Annual IO tables of 1996,1997,1998 and 1999.

When inverse of value added productivity of labour of each sectors were regressed by total labour requirements, the value of regression coefficients were 2.02, 1.68 and 2.23 respectively.

Regression coefficients take least value at the trough of an economic fluctuation, which indicates greater value of value added productivity of labour ralative to the variance of total labour requirements at the trough of an economic fluctuation.

Empirical results indicated that log linear approximation of the two variables were more adequate.

These results indicate that the value added productivity of labour are closely related to total labour requirements (total labour value, employment inducement coefficients) and that the former can be expressed in the form of Cobb-Douglas Function of the latter.

To make the relationship with economic fluctuation much clearer, study will were extended to US Annual table input output analysis. Here, input output tables were recompiled into industry classification, and industries were compiled into 43 sectors. The relation between the value added productivity of labour and total labour requierments (total labour value) will be considered a little more carefully under general equilibrium framework.

The objective of this paper is to clarify the relationship between the total labour value (total labour requirements) and value added productivity of labour both theoretically and empirically. For this purpose, input output tables of US Benchmark 1987, 1992 and 1997 were used. Annual tables of 1996, 1997, 1998 and 1999 were also used for further study.

There are few literature on the empirical research of total labour requirements.

Keywords: Total Labour Requirements , Value Added Productivity of Labour , Total Labour Value, Employment Inducement Coefficients, Economic Fluctuations

2 Model

Total labour requirements of one monetary unit of j sectors output, equivalent to total labour value of output j are calculated as follow: where a_{ij}denotes input output coefficients, l_j denotes labour coefficient of sector j.

$$t_j = \sum_i a_{ij} t_i + l_j$$

In matrix, this can be denoted as

$$T = L(I - A)^{-1}$$

This can be understood as

$$t_j = \sum_i b_{ij} l_i$$

where b_{ij} denotes ij factor of Leontief Inverse. That is, total labour requirements are sum of total commodity requirements (b_{ij}) times direct labour required (labour coefficients).

Relation between total labour requirements and price, and value added productivity of labour (y_j) are as follows.

$$p_j = \sum_i a_{ij} p_i + l_j y_j$$
$$= \sum_i b_{ij} l_i y_i$$

On the other hand,

$$t_j = \sum_i a_{ij} t_i + l_j$$

$$= \sum_i b_{ij} l_i$$

Therefore,

$$\frac{p_j}{t_j} = \sum_i b_{ij} l_i y_i / \sum_i b_{ij} l_i$$

$$= \frac{1}{t_j} \sum_i b_{ij} l_i y_i = \sum_i \frac{\tau_{ij}}{t_j} \cdot y_i$$

where τ_{ij} denotes $b_{ij} l_i$ which is direct and indirect labour required in sector i to produce one unit of j product.

That is, inverse of total labour requirements are weighted sum of labour productivities of all industries supplying intermediate products.

While relation of total labour requirement and value added productivity of labour can be written in the following form as well.

$$t_{j=} \sum b_{ij} v_i \frac{1}{y_i}$$

Value added coefficienet of sector i is denoted by v_i and $v_i = y_i \; l_i$ holds for any sector i.

That is, total labour requirements are weighted sum of inverse of value added productivities of labour of all sectors while the weight is direct and indirect value added inducements.

3 Actual Detailed model

The U table and V tables of US Input output tables were united to industry classification. Commodities column which appear in use tables were pre-multiplied by W matrix, and were reclassified into industries. W is a notation in Math IO note annexed to BEA tables.

3.1 Depreciation of fixed capital is added to intermediate inputs as cost.

For this calculation, depreciation allowances are assumed to equal to actual depreciation costs. Then, total depreciation costs are allocated among sectors according to the size of their capital stock in the capital formation table.

Let Z_j be depreciation allowance of sector j,
then capital depreciation cost of sector j are apportionmented into sector i in the following manner.

$$d_{ij} = \frac{Z_j}{X_j} \frac{k_{ij}}{\sum_i k_{ij}}$$

where k_{ij} denotes capital installment of i output in sector j.

3.2 Imported inputs are substituted by domestic labour according to the industry's share in export

Labour embodied in import inputs are substituted by domestic labour embodied in exports assuming that the import value are equivalent to export value.

Let E' denote the column vector of which each factor denotes export composition ratio of each sector's output, and M the import coefficients' vector

T are calculated as

$$T = L(I - A^d - D - E'M)^{-1} \quad .$$

A^d is a matrix of domestically produced inputs i used in sector j where a^d_{ij} are obtained as

$$\frac{a^d_{ij}}{a_{ij}} = \frac{X_i}{X_i + M_i} = \frac{1}{1 + m_i}$$

and

$\frac{a_{ij}^m}{a_{ij}} = \frac{m_i}{1+m_i}$
Import coecfficient of sector j is $m_j = \sum_i a_{ij}^m$

Let t^m denote the total labour embodied in average import,

total labour requirements in sector j will fulfill the following equation.

$t_j = \sum (a_{ij}^d + d_{ij}) t_i + m_j t^m + l_j$
In vector form,

$T = T(A^d + D) + Mt^m + L$

while $t^m = TE'$ or $t^m = \sum t_i e_i$

where e_i denotes export ratio of sector i in total export.

Then, the above equation can be transformed as
$T = L(I - A^d - D - E'M)^{-1}$.

4 Empirical Result (Linear Regression)

We calculated total labour requirements and value added productivity of labour. In this process, industry technology assumption was employed. Data on labour coefficients were obtained from data on persons engaged in production in each industry at BEA's site.

Net value added was calculated as $P - P(A^d + D + E'M)$. Inverse of net value added per labour were regressed by total labour requirements.

4.1 Regression coefficient

Low value of coefficients imply less value of covariance relative to variance of explanatory variable as regression coefficients equal to the following formula.

$$\widehat{\beta} = \frac{\sum_i \left(t_i - \bar{t}\right)\left(\left(\frac{1}{y_i}\right) - \left(\overline{\frac{1}{y}}\right)\right)}{\sum_i \left(t_i - \bar{t}\right)^2}$$

4.2 Constant Term

Actual regression result on constant terms are negative. Negative constant terms are expected as follow.

For simplicity, we assume 2 sector model.

$\frac{p_1}{t_1} = \frac{\tau_{11}}{t_1} y_1 + \frac{\tau_{21}}{t_1} y_2$

$\frac{p_2}{t_2} = \frac{\tau_{12}}{t_2} y_1 + \frac{\tau_{22}}{t_2} y_2$

By replacing $V\widehat{Y}^{-1}B = T$

$t_j = \sum b_{ij} v_i \frac{1}{y_i}$

therefore,

$t_1 = b_{11} v_1 \frac{1}{y_1} + b_{21} v_2 \frac{1}{y_2}$

$t_2 = b_{12} v_1 \frac{1}{y_1} + b_{22} v_2 \frac{1}{y_2}$

By replacing $\theta_{ij} = b_{ij} v_i$

$t_1 = \theta_{11} \frac{1}{y_1} + \theta_{21} \frac{1}{y_2}$

$t_2 = \theta_{12} \frac{1}{y_1} + \theta_{22} \frac{1}{y_2}$

$\frac{1}{y_1} = \frac{1}{\theta_{11}} (t_1 - \theta_{21} \frac{1}{y_2})$

$\frac{1}{y_2} = \frac{1}{\theta_{22}}(t_2 - \theta_{12}\frac{1}{y_1})$

By this transformation, regression coefficients are $\frac{1}{\theta_{11}}$and $\frac{1}{\theta_{22}}$ respectively. Constant terms are expected to be negative.

4.2.1 Regression coefficient

Result of regression analysis are given in the attached table. Approximation give better adjusted $\overline{R}^2$ when the 43rd industry, i.e., not else where included are deleted from regression.

Such empirical regression results are compared to the actual economic fluctuation, and economic growth.

US Economic Fluctuations	Peak	Trough
	April 1960	February 1961
	December 1969	November 1970
	November 1973	March 1975
	January 1980	July 1980
	July 1981	November 1982
	July 1990	March 1991
	March 2001	November 2001

Movements of regression coefficients $\widehat{\beta}$ in the past research are

1.81	for 1960		
1.90	1965		
1.35	1970	periods of	trough
1.41	1975		trough
1.39	1980		trough
1.49	1985.		

Regression coefficients with current research results as indicated in the attached table are,

2.02	for	1987	n=42
1.68		1992	n=42

periods of trough

2.23	1997.	n=42
1.97	1997	n=43
2.01	1998	n=43
1.96	1998	n=42
1.08	1999	n=43
2.31	1999	n=42

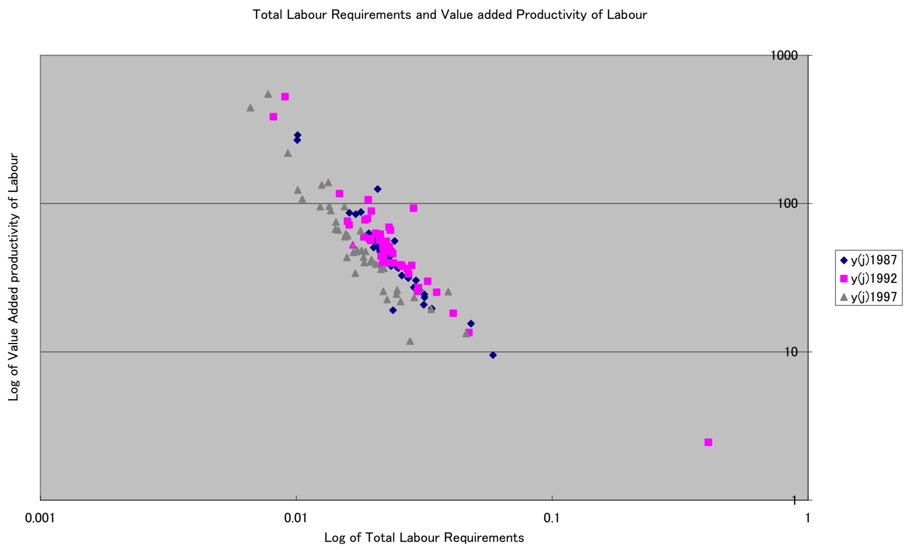
Compared to the past analysis, series of regression coefficients of current research may be taking slighty high value for all years. Such result might have resulted from, different data sourse (past analysis used labour statistics from BLS), and treatment of depreciation allowances (fixed tangible reproducible assets of US data are no longer available and depreciation allowances are obtained from GPO87SIC.xls data for current reserch, which might be over estimating Z_j that will result in low value added productivity of labour, which will result in high value of regression coefficient.

Regression coefficient of 1992 take low value compared to the other years, which may be interpretted as such that value added productivity of labour indicated relatively high value due to reduced employment, resulted in low deviation of inverse of value added productivity of labour, which resulted in low regression coefficient.

These results tell us, that this regression coefficients have tendency to take relatively high value either at the upward swing of an economic fluctuation, or during economic boom. On the other hand, this value takes low value at the trough of an economic fluctuation.

5 Empirical Result: (Log Linear Regression)

Log linear regression gave significant goodness fit. The results are shown in attached table. Scatter diagrammes are attached. From these regressions,



elasticity of value added productivity of labour with respect to total labour requirements are around -1.34 (1992) to -1.92(1987) -1.93(1997).

Movements along with log linear regression gave following results:

Movement from 1987 to 1992 can be seen as upward shift of plotted values of scatter diagramme. Plotted values of scatter diagramme shift upward from 1987 to 1992. This indicates that higher value added productivity were realized due to reduced employment

Movements from 1992 to 1997 can be seen in the scater diagramme as leftward shift of plotted values. Shift toward left are interpretted as lowered total labour requirements due to technical progress at the upward swing of an economic fluctuation, and economic boom.

6 Theoretical Backgrounds Behind Regression Coefficients

6.1 Property of Regression Coefficient

The price equilibrium equation implies

$$P\left(I-A\right)=L\widehat{Y}$$

and the total labour value equation implies

$$L=T\left(I-A\right).$$

Substituting the latter into the former,

$$P\left(I-A\right)=T\left(I-A\right)\widehat{Y}$$

or equivalently,

$$PB^{-1}\widehat{Y}^{-1}B=T \tag{1}$$

where $B=\left(I-A\right)^{-1}$.

Since $PB^{-1}=P\left(I-A\right)=V$,

$$t_j=\sum_i b_{ij}v_i\frac{1}{y_i} \tag{2}$$

or equivalently

$$t_j=\sum_i b_{ij}l_i \tag{3}$$

where b_{ij} is the (i,j) component of B, the Leontief's inverse matrix. The increase in the final demand for the j-th sector by one unit induces the production of the i-th sector by b_{ij} units and the increase of the demand for the direct labour input of the i-th sector by $b_{ij}l_i$ units. Hence t_j implies how much additional employment is induced by the increase of the final demand of the j-th sector.

In the model where $1/y_j$'s are regressed on t_j's such that

$$\frac{1}{y_j}=\alpha+\beta t_j+e_j$$

(where e_j is the residual), the least square regression parameters (α,β) are

$$\beta=\frac{Cov\left(t,\frac{1}{y}\right)}{Var\left(t\right)},\ \alpha=E\left(\frac{1}{y}\right)-\beta E\left(t\right)$$

where Cov, Var, and E are sample covariance, variance, and mean respectively.

6.2 The Implication of β (1)

Given $Var(t)$, the β sign of β is the same as that of $Cov\left(t, \frac{1}{y}\right)$. Positive β implies that those sectors with the lower value added productivity of labour tend to induce larger aggregate employment enhance if the final demand for those sector increases. In another words, larger the employment inducement foefficient is, lower the value added productivity of same sector.

6.3 The Implication of β (2)

t_j implies the aggregate employment inducement created by the final demand increase of sector j. On the other hand, by definition,

$$\frac{1}{y_j} = \frac{l_j}{v_j} = \frac{b_{jj} l_j}{b_{jj} v_j}$$

that is, $\frac{1}{y_j}$ is the direct employment requirement of sector j induced by the final demand increase of its own sector per value added increase of sector j. Therefore, β coefficient reflects the covariance of total employment inducement req and the direct (sectoral) employment requirement induced by the final demand increase of each sector.

The Value of β and α With straightforward calculation,

$$\begin{aligned} & Var(t) \\ = \; & n^2 \{ Var_j \left[Cov_i \left(\theta_{ij}, \frac{1}{y_i} \right) \right] + 2E\left(\frac{1}{y}\right) Cov_j \left[Cov_i \left(\theta_{ij}, \frac{1}{y_i} \right), E_i(\theta_{ij}) \right] \\ & + \left\{ E\left(\frac{1}{y}\right) \right\}^2 Var_j \left[E_i(\theta_{ij}) \right] \} \end{aligned}$$

where $\theta_{ij} \equiv b_{ij} v_i$, $E_i(\theta_{ij}) = \frac{1}{n} \sum_i \theta_{ij}$, $E\left(\frac{1}{y}\right)$ is the mean of $1/y_j$, $Cov_i(\cdot, \cdot)$ is the covariance of each argument with respect to the subscripts i given j, $Var_j(\cdot)$ is the variance of the argument with respect to j.

On the other hand,

$$\begin{aligned} & Cov\left(t_j, \frac{1}{y_j}\right) \\ = \; & n \left\{ Cov_j \left[Cov_i \left(\theta_{ij}, \frac{1}{y_i} \right), \frac{1}{y_j} \right] + E\left(\frac{1}{y}\right) Cov_j \left[E_i(\theta_{ij}), \frac{1}{y_j} \right] \right\} \end{aligned}$$

and

$$E_j\left(t_j\right)=n\left\{E_j\left[Cov_i\left(\theta_{ij},\frac{1}{y_i}\right)\right]+\overline{\theta}E\left(\frac{1}{y}\right)\right\}$$

where $\overline{\theta}=\frac{1}{n^2}\sum_j\sum_i\theta_{ij}$.

Then

$$\begin{aligned}&\beta\\ =\;&\frac{1}{n}\frac{Cov_j\left[Cov_i\left(\theta_{ij},\frac{1}{y_i}\right),\frac{1}{y_j}\right]+E\left(\frac{1}{y}\right)Cov_j\left[E_i\left(\theta_{ij}\right),\frac{1}{y_j}\right]}{\left(\begin{array}{c}Var_j\left[Cov_i\left(\theta_{ij},\frac{1}{y_i}\right)\right]+2E\left(\frac{1}{y}\right)Cov_j\left[Cov_i\left(\theta_{ij},\frac{1}{y_i}\right),E_i\left(\theta_{ij}\right)\right]\\ +\left\{E\left(\frac{1}{y}\right)\right\}^2Var_j\left[E_i\left(\theta_{ij}\right)\right]\end{array}\right)}\end{aligned}$$

and

$$\begin{aligned}&\alpha\\ =\;&E\left(\frac{1}{y}\right)-\beta E_j\left(t_j\right)\\ =\;&E\left(\frac{1}{y}\right)\\ &-\frac{\left(\begin{array}{c}\left\{Cov_j\left[Cov_i\left(\theta_{ij},\frac{1}{y_i}\right),\frac{1}{y_j}\right]+E\left(\frac{1}{y}\right)Cov_j\left[E_i\left(\theta_{ij}\right),\frac{1}{y_j}\right]\right\}\\ *\left\{E_j\left[Cov_i\left(\theta_{ij},\frac{1}{y_i}\right)\right]+\overline{\theta}E\left(\frac{1}{y}\right)\right\}\end{array}\right)}{\left(\begin{array}{c}Var_j\left[Cov_i\left(\theta_{ij},\frac{1}{y_i}\right)\right]+2E\left(\frac{1}{y}\right)Cov_j\left[Cov_i\left(\theta_{ij},\frac{1}{y_i}\right),E_i\left(\theta_{ij}\right)\right]\\ +\left\{E\left(\frac{1}{y}\right)\right\}^2Var_j\left[E_i\left(\theta_{ij}\right)\right]\end{array}\right)}\end{aligned}$$

6.3.1 An Example

For simplicity, suppose that $b_{jj}v_j=\theta$ for all j and $b_{ij}v_i=\theta-\delta$ $(\delta>0)$ for all $i\neq j$. Then

$$t_j=(\theta-\delta)\,nE\left(\frac{1}{y}\right)+\delta\frac{1}{y_j},\;\;E\left(t_j\right)=\left[n\left(\theta-\delta\right)+\delta\right]E\left(\frac{1}{y}\right)$$

and

$$Cov\left(t,\frac{1}{y}\right)=\delta Var\left(\frac{1}{y}\right),\;\;Var\left(t\right)=\delta^2 Var\left(\frac{1}{y}\right).$$

This implies

$$\beta = \frac{1}{\delta}$$

and

$$\alpha = -\frac{n(\theta-\delta)}{\delta}E\left(\frac{1}{y}\right).$$

β increases with the decrease in δ, the difference of value added increase between sector j and the other sectors induced by final demand for the sector j.

This example verifies for avery simplified case. However, this example implies the following. Great value of regression coefficient β implies that it is the period that increased demand in j sector's output is inducing income=value added of other sectors. Economic boom are period that are associated with great value of regression coefficients and that suggets us that these are years that interindustial linkages are high to increase the income of other sectors.

6.4 Technological Background of β Coefficient

The β coefficients in our regression reflect technology of production and the structural difference between industries, if we assume that the production function of each industry has constant return to scale property.

Suppose that the market structure of intermediate goods is the same as that of final goods and the demand function for the product of industry j is

$$X_j = P_j^{-\gamma_j}$$

where X_j is the demand for commodity j and γ_j is the elasticity of demand, which is assumed to be common among the intermediate and the final goods of industry j. We assume that $\gamma_j > 1$ for any sector j. As is well known, the inverse of γ_j reflects the monopolistic power of the seller of the commodities of industry j.

Production function of the industry j is the Cobb-Douglas form as follows.

$$X_j = F(X_1, X_2, ..., X_n, L_j, K_j) = \left(\prod_{i=1}^{n} X_{ij}^{a_{ij}}\right) L_j^{a_{Lj}} K_j^{a_{Kj}}$$

where X_{ij} is intermediate inputs of industry i by industry j, L_j and K_j are direct labour input and capital input of industry j respectively. We assume that $\sum_{i=1}^{n} a_{ij} + (a_{Lj} + a_{Kj}) = 1$ for all j. That is, the productions of all the industries have constant to scale property with respect to all the inputs of intermediate goods and labour and capital.

By the first order conditions for the firms' profit maximization, the followings hold.

$$\left(1 - \frac{1}{\gamma_j}\right) a_{Lj} P_j X_j = W_j L_j \tag{4}$$

$$\left(1 - \frac{1}{\gamma_j}\right) a_{Kj} P_j X_j = R_j K_j \tag{5}$$

$$\left(1 - \frac{1}{\gamma_j}\right) a_{ij} P_j X_j = P_i X_i \tag{6}$$

where W_j and R_j are nominal wage rate and user's cost of capital for the industry j respectively.

Let A^* be the input coefficient matrix whose ij component a^*_{ij} is intermediate input of industry i per one unit money outcome of industry j. Therefore,

$$a^*_{ij} \equiv \frac{P_i X_i}{P_j X_j} = \left(1 - \frac{1}{\gamma_j}\right) a_{ij} \tag{7}$$

Implication of $1/y^*_j$ Value added per one unit of money outcome of industry j, $v^*_j$is

$$\begin{aligned} v^*_j &= \frac{P_j X_j - \sum_{i=1}^{n} P_i X_i}{P_j X_j} \\ &= 1 - \left(1 - \frac{1}{\gamma_j}\right) \sum_{i=1}^{n} a_{ij} \\ &= 1 - \left(1 - \frac{1}{\gamma_j}\right) \{1 - (a_{Lj} + a_{Kj})\} \\ &= \left(1 - \frac{1}{\gamma_j}\right) a_{Lj} + \left(1 - \frac{1}{\gamma_j}\right) a_{Kj} + \frac{1}{\gamma_j} \end{aligned}$$

The first and the second term of the last row of the equation above are labours' and the capital owners' shares per industry j gross production. The residual $\frac{1}{\gamma_j}$ is the firms' profit per industry j gross production. Since the direct labour requirement per one unit money output of industry j, l_j^* is

$$l_j^* \equiv \frac{1}{W_j}\frac{W_j L_j}{P_j X_j} = \frac{1}{W_j}\left(1 - \frac{1}{\gamma_j}\right) a_{Lj}$$

then

$$\frac{1}{y_j^*} = \frac{l_j^*}{v_j^*} = \frac{1}{W_j}\frac{\left(1 - \frac{1}{\gamma_j}\right) a_{Lj}}{\left(1 - \frac{1}{\gamma_j}\right)(a_{Lj} + a_{Kj}) + \frac{1}{\gamma_j}}$$

or

$$\frac{W_j}{y_j^*} = \frac{\left(1 - \frac{1}{\gamma_j}\right) a_{Lj}}{\left(1 - \frac{1}{\gamma_j}\right)(a_{Lj} + a_{Kj}) + \frac{1}{\gamma_j}}. \tag{8}$$

That is, the inverse of value added productivity times nominal wage rate is the workers' share of each industry. Let s_j denote this value.

s_j value is determined by the technological parameters a_{Lj}, a_{Kj} and the monopolistic power of the firms within the industry j, $1/\gamma_j$. The more labour intensive the industry j is, the larger s_j is. The less competitive the industry j, the smaller s_j is.

6.4.1 Implication of t_j^*

Let T^* be the row vector whose j-th component t_j^* is the total labour input within the one unit money output of industry j. Since $T^* = T^*A^* + L^*$, $T^* = L^*B^*$ where $B^* \equiv (I - A^*)^{-1}$, or

$$\begin{aligned} t_j^* &= \sum_{i=1}^{n} l_i^* b_{ij}^* \\ &= \sum_{i=1}^{n} \frac{1}{W_i}\left(1 - \frac{1}{\gamma_i}\right) a_{Li} b_{ij}^*. \end{aligned}$$

t_j^* implies how much employment is induced by the increase of final demand for the industry j.

6.4.2 The Implication of β value

The regression parameter β is

$$\begin{aligned}
\beta &= \frac{Cov\left(t_j^*, 1/y_j\right)}{Var(t_j^*)} \\
&= \frac{Cov\left(\sum_{i=1}^n \frac{1}{W_i}\left(1-\frac{1}{\gamma_i}\right)a_{Li}b_{ij}^*, \frac{1}{W_j}s_j\right)}{Var\left(\sum_{i=1}^n \frac{1}{W_i}\left(1-\frac{1}{\gamma_i}\right)a_{Li}b_{ij}^*\right)} \\
&= \frac{Cov\left(\sum_{i=1}^n \frac{1}{W_i}s_i v_i^* b_{ij}^*, \frac{1}{W_j}s_j\right)}{Var\left(\sum_{i=1}^n \frac{1}{W_i}\left(1-\frac{1}{\gamma_i}\right)a_{Li}b_{ij}^*\right)}
\end{aligned}$$

If the nominal wage rate is equalized among the industries and $W_i = W \ \ \forall i$,

$$\beta = \frac{\frac{1}{W^2}Cov\left(\sum_{i=1}^n s_i v_i^* b_{ij}^*, s_j\right)}{\frac{1}{W^2}Var\left(\sum_{i=1}^n s_i v_i^* b_{ij}^*\right)} = \frac{Cov\left(\sum_{i=1}^n s_i v_i^* b_{ij}^*, s_j\right)}{Var\left(\sum_{i=1}^n s_i v_i^* b_{ij}^*\right)}.$$

That is, β reflects the relation between the labours' share of industry j and $\sum_{i=1}^n s_i v_i^* b_{ij}^*$, the labours' income increase in the whole economy induced by the increase in the final demand for the industry j. As was seen in the previous subsection, s_j value is larger for relatively more labour intensive, more competitive industry. Positive and larger β implies that final demand increase for labour intensive or competitive industry induces larger labours income increase as a whole.

7 Conclusion

Scatter diagramme and regression of log y by log t also indicated high goodness of fit. Following log linear scatter diagrammes, movements of y and t, can be explained as, (i) increased value added productivity of labour due to reduced employment (1987-1992), and (ii) technological progress at the upward swing of the economic fluctutation for years 1992 to 1997.

Scatter diagramme and regressionof $1/y$ by t indicates significant goodness of fit. Regression coefficients take low value in 1992, indicating economic recession (trough) of economy.

Theoretical explanation, although yet preliminary and simplified, explains that the economic boom are years associated with high regression coefficient β,and these are years that inter industry income generation linkages are high.

7.1 References

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Case of Japanese Economic Fluctuations and Regression Coefficients For reference, Japaneses economic fluctuations are as follows.

Peak	Trough
	June 1958
December 1962	October 1962
October 1964	October 1965
June 1970	December 1971
November 1973	March 1975
January 1977	October 1977
February 1980	February 1983
June 1985	November 1986

	January 1991	October 1993

Result of regression coefficients with past research are as follow..

1.49	for 1960	
1.68	1965	
1.66	1970	
1.72	1975	
2.01	1980	Peak
1.70	1985	

1960 and onwards are years of rapid economic growth for Japanese economy where regression coefficients show relatively high value. It is likely that 1980 was a turning point for the Japanese economy, from the point of view of regression coefficients. However, we need further research to interpret the past research.