A GENERAL EQUILIBRIUM INPUT-OUTPUT TRIAL:

THE EMPIRICAL RELEVANCE FOR NORWEGIAN TRADE FLOWS
OF THE HECSCHER-OHLIN CONJECTURE

by

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Abstract: H&O conjectured that a country will export (import) goods in the production of which its relatively abundant (scarce) factors play a dominant role because the country has a comparative advantage in producing goods that are intensive in its abundant factors. I scrutinize the validity of H&O’s conjecture in a general equilibrium input-output model of international trade, and develop axioms for an empirical analysis in which I try the conjecture’s relevance for Norwegian trade flows. The core of my analysis comprises three parts, two disjoint universes for theory and data and a bridge between them. The theory universe contains variables that satisfy relations on which my general equilibrium input-output model insists. The data universe contains variables that satisfy relations on which my data from Norwegian industrial statistics and input-output tables insist. Finally, the bridge consists of principles that show how variables in one universe are related to variables in the other.

Key Words: Input-output analysis, general equilibrium theory, trade flows and factor endowments, theory-data confrontations

I. Introduction

A country, A, is said to have a comparative advantage in producing a given good, x, if the opportunity cost of producing x in terms of other goods is lower in A than in other countries. There are many reasons why A in the production of x might have a
comparative advantage; e.g., climate and the supply of natural resources. Whatever the reasons are, international trade will allow A to specialize in producing goods in which it has a comparative advantage vis-a-vis its trading partners.

Eli Heckscher (1919) and Bertil Ohlin (1933) conjectured that a country will export goods in the production of which its relatively abundant factors play a dominant role and import goods in the production of which the country’s scarce factors play a dominant role. Their reason was that a country will have a comparative advantage vis-a-vis other countries in producing goods that are intensive in the country’s abundant factors.

In 1953 Wassily Leontief published a study in which he used input-output analysis to investigate the possible empirical relevance for US trade of Heckscher and Ohlin’s (H&O’s) conjecture. Leontief summarized his principle findings in a table that I record below. The table suggests that “America’s participation in the international division of labor [in 1947 was] based on its specialization on labor intensive, rather than capital intensive, lines of production. In other words, [the U.S.A. resorted] to foreign trade in order to economize its capital and dispose of its surplus labor, rather than vice versa.” (Leontief, 1953, p. 343). Since the U.S.A. in 1947 possessed more productive capital per worker than any other country, Table 1 contradicts H&O’s conjecture.

### Domestic Capital and Labor Requirements per Million Dollars of U.S Exports and of Competitive Import Replacements (of Average 1947 Composition)

<table>
<thead>
<tr>
<th></th>
<th>Exports</th>
<th>Import Replacements</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital (dollars, in 1947 prices)</td>
<td>2,550,780</td>
<td>3,091,339</td>
</tr>
<tr>
<td>Labor (man years)</td>
<td>183.313</td>
<td>170.004</td>
</tr>
</tbody>
</table>

Table 1

Leontief’s study was criticized for theoretical as well as statistical inadequacies. S. Valavanis-Vail insisted that Leontief’s input-output approach could not be used to analyze international trade problems, and B.C.Swerling observed that the production and trade conditions prevailing in 1947 were abnormal and that Leontief’s results, therefore, were biased (Swerling, 1954 and Valavanis-Vail, 1954). Also, M. A. Diab
complained that Leontief had not paid sufficient attention to the capital-intensive natural-resource component of US imports (Diab, 1956).

In this paper I scrutinize the validity of H&O’s conjecture in a general equilibrium input-output model of international trade, search for meaningful concepts of a country’s labor and capital endowment, and confront the conjecture with data from Norwegian trade flows. In staging the trial of H&O’s conjecture, I develop ways of using general equilibrium input-output models in empirical analyses that are very different from Leontief’s analysis and from the methods that Edward Leamer developed in his book on *Sources of International Comparative Advantage* (Leamer, 1984).

My arguments are based on ideas that I presented in Chapters 26-28 of my book, *Toward a Formal Science of Economics* (Stigum, 1990). Specifically, I view the trial of H&O’s conjecture as an event in which relevant parts of a general equilibrium-theory of international trade is confronted with data to test the empirical relevance of H&O’s conjecture. Formally, the confrontation can be pictured as in Fig. 1 below. On the left side of the figure are boxes that contain information pertaining

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**Fig. 1** A Theory-Data Confrontation
to the relevant theory; i.e., the theory itself, models of the theory, and the part of the theory that is at stake in the empirical analysis. The last part comprises the ingredients by which the theory universe is constructed. On the right hand side of the figure are boxes that contain information concerning the data generating process; i.e., the sample population on whose characteristics observations are based, the observations, data that the researcher has constructed, and the data universe in which all the pertinent data variables reside. The two universes are disjoint and connected by a bridge of principles that detail how the variables in the two universes are related to one another.

In the present theory-data confrontation the theory is a theory of trade in a two-country input-output world in which one of the countries, A, is taken to be 1997 Norway and the other, B, is the community of nations with which Norway traded in 1997. I develop axioms for the two universes and the bridge that connects them, discuss the relative merits of various measures of a country’s factor endowments, and carry out the analysis that is to test the empirical relevance of H&O’s conjecture. Throughout the paper I make, without say, use of material that I presented in chapters ten, eleven and seventeen in a book on **Econometrics and the Philosophy of Economics** that Princeton University Press published in 2003.

II. **Trade in a Two-Country Input-Output Economy**

I shall scrutinize the Heckscher-Ohlin conjecture in a simple thought experiment. The world I imagine is like the world Paul Samuelson envisioned in his seminal articles on the factor-price equalization theorem (cf. Samuelson, 1948 and 1949). It features two countries, A and B, two commodities, x and y, two primary factors of production, K and L, and two natural resources, z and u. For these countries I assume that conditions I-IV below are valid.

(I) the initial endowments of primary factors in A, \( L_A \) and \( K_A \), and in B, \( L_B \) and \( K_B \), satisfy the conditions, \( 0 < L_A < K_A \) and \( L_B > K_B > 0 \).

(II) the production of x and y requires the use of both primary factors, and the production of both commodities occur under conditions of constant returns to scale.
(III) In A and B there are two natural resources, z and u, the extraction of which requires the use of both primary factors of production. Also, z and u are essential factors of production of x and y. Finally, neither z nor u is part of final demand; i.e., they are not consumables.

(IV) x and y can flow freely without transport cost both within and between countries. Also, the primary factors and the natural resources move freely within countries but not between. Finally, the markets for goods and factors are perfectly competitive.

To simplify my discussion, I denote x, y, z, and u, respectively by \(x_1, x_2, x_3,\) and \(x_4\). Similarly, I denote the prices of the four commodities by \(P_1, P_2, P_3,\) and \(P_4\). Finally, I denote the prices of L and K by \(w\) and \(q\), the supply of and demand for commodities by superscripts \(s\) and \(d\), and the commodities and primary factors that belong to A and B by subscripts \(A\) and \(B\).

In the intended interpretation of (I)-(IV) the demand functions for \(x_1\) and \(x_2\) in the two countries are rationalized by two utility functions, \(U_A(\cdot) : \mathbb{R}^2 \to \mathbb{R}_+\) for A and \(U_B(\cdot) : \mathbb{R}^2 \to \mathbb{R}_+\) for B, where

\[
U_A(x_1, x_2) = x_1^\alpha x_2^\beta, \quad U_B(x_1, x_2) = x_1^\delta x_2^\mu
\]

with \(\alpha, \beta, \delta,\) and \(\mu \in \mathbb{R}_+^+\). Also, the production functions of the four commodities are constant-returns-to-scale Leontief-type functions that satisfy the following conditions:

\[
\begin{align*}
x_1 &= \min \{L/a_{L1}, K/a_{K1}, x_3/a_{31}, x_4/a_{41}\} \\
x_2 &= \min \{L/a_{L2}, K/a_{K2}, x_3/a_{32}, x_4/a_{42}\} \\
x_3 &= \min \{L/a_{L3}, K/a_{K3}\} \\
x_4 &= \min \{L/a_{L4}, K/a_{K4}\},
\end{align*}
\]

where \((L,K) \in \mathbb{R}_+^2\) and \(a_{ij} \in \mathbb{R}_+^+, i = L, K, 3, 4,\) and \(j = 1, 2, 3,\) and \(4\). Thus, to produce one unit of \(x_1\) one needs \(a_{L1}\) units of \(L, a_{K1}\) units of \(K, a_{31}\) units of \(x_3,\) and \(a_{41}\) units of \(x_4\). Similarly for \(x_2, x_3,\) and \(x_4.\)
If the preceding conditions hold, a trade equilibrium in A and B’s world is a vector of prices and commodities, \((P, w, q, x^d_A, x^s_A, x^d_B, x^s_B)\) that satisfy equations (1)-(19) below.

\[
\begin{align*}
(1) & & P \in \mathbb{R}^{++}, w \in \mathbb{R}^{++}, \text{ and } q \in \mathbb{R}^{++} \\
(2) & & x^d_A = (\alpha/\alpha+\beta)[(wL_A + qK_A)/P_1] \text{ and } x^d_B = (\beta/\alpha+\beta)[(wL_B + qK_B)/P_2] \\
(3) & & x^d_A = 0 = x^d_B \\
(4) & & x^d_B = (\delta/\mu+\delta)[(wL_B + qK_B)/P_1] \text{ and } x^d_A = (\mu/\mu+\delta)[(wL_B + qK_B)/P_2] \\
(5) & & x^d_B = 0 = x^d_A \\
(6) & & P_1 = a_{31}P_3 + a_{41}P_4 + a_{L1}w + a_{K1}q \\
(7) & & P_2 = a_{32}P_3 + a_{42}P_4 + a_{L2}w + a_{K2}q \\
(8) & & P_3 = a_{L3}w + a_{K3}q \\
(9) & & P_4 = a_{L4}w + a_{K4}q \\
(10) & & a_{L1}x^s_A + a_{L2}x^s_A + a_{L3}x^s_A + a_{L4}x^s_A \leq L_A \\
(11) & & a_{K1}x^s_A + a_{K2}x^s_A + a_{K3}x^s_A + a_{K4}x^s_A \leq K_A \\
(12) & & x^s_A - a_{31}x^s_A - a_{32}x^s_A = 0 \\
(13) & & x^s_A - a_{41}x^s_A - a_{42}x^s_A = 0 \\
(14) & & a_{L1}x^s_B + a_{L2}x^s_B + a_{L3}x^s_B + a_{L4}x^s_B \leq L_B \\
(15) & & a_{K1}x^s_B + a_{K2}x^s_B + a_{K3}x^s_B + a_{K4}x^s_B \leq K_B \\
(16) & & x^s_B - a_{31}x^s_B - a_{32}x^s_B = 0 \\
(17) & & x^s_B - a_{41}x^s_B - a_{42}x^s_B = 0 \\
(18) & & x^s_A + x^s_B = x^d_A + x^d_B \\
(19) & & x^d_A + x^d_B = x^s_A + x^s_B
\end{align*}
\]

In this system of equations, equations 2-5 record the four demand functions in A and B. Equations 6-9 insist that in trade equilibrium the prices of the four commodities equal the unit costs of producing them. Equations 10-11 and 14-15, respectively, demand that the indirect and direct use of primary factors in the production of \(x_1\) and \(x_2\) in A and B equals or is less than the supply of primary factors in the two countries. Equations 12-13 and 16-17, respectively, require that the production of \(x_3\) and \(x_4\) in A and B equals the use in each country of these commodities in the production of \(x_1\) and \(x_2\). Finally, equations 18-19 insist that in trade equilibrium the aggregate supply of \(x_1\) and \(x_2\) must equal the aggregate demand for the two commodities. There are twenty-
two equations to determine the values of twenty-two variables. However, only twenty-one of the equations are independent. The equations, therefore, allow us to determine the values of five price ratios and not the values of six prices.

One interesting aspect of Leontief’s analysis is the way he determines whether the production of a commodity is intensive in L or intensive in K. In Leontief’s analysis the factor intensity of a production function is determined by both the direct and indirect uses that it makes of the primary factors of production. In my thought experiment the direct factor requirements in the production of \( x_1 \) and \( x_2 \) are determined by the respective outputs and by the values of the components of the \( D \)-matrix below. The indirect factor requirements are determined by the outputs of \( x_1 \) and \( x_2 \) and by the \( C \) and \( F \) matrices. Finally, the factor intensities of the production of \( x_1 \) and \( x_2 \) are determined by the entries in the \( G \) matrix. The production of \( x_1 \) is relatively intensive in L if \( g_{L1}/g_{K1} > g_{L2}/g_{K2} \), and the production of \( x_2 \) is relatively intensive in K if \( g_{K2}/g_{L2} > g_{K1}/g_{L1} \).

\[
\begin{bmatrix}
    a_{L1} & a_{L2} & a_{L3} & a_{L4} & a_{31} & a_{32}  \\
    a_{K1} & a_{K2} & a_{K3} & a_{K4} & a_{41} & a_{42}  \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
    g_{L1} & g_{L2}  \\
    g_{K1} & g_{K2}  \\
\end{bmatrix}
\]

\[
D = \begin{bmatrix}
    a_{L1} & a_{L2} & a_{L3} & a_{L4} & a_{31} & a_{32}  \\
    a_{K1} & a_{K2} & a_{K3} & a_{K4} & a_{41} & a_{42}  \\
\end{bmatrix}
\]

\[
C = \begin{bmatrix}
    g_{L1} & g_{L2}  \\
    g_{K1} & g_{K2}  \\
\end{bmatrix}
\]

\[
F = \begin{bmatrix}
    a_{L1} & a_{L2} & a_{L3} & a_{L4} & a_{31} & a_{32}  \\
    a_{K1} & a_{K2} & a_{K3} & a_{K4} & a_{41} & a_{42}  \\
\end{bmatrix}
\]

\[
G = \{D + CF\} = \begin{bmatrix}
    g_{L1} & g_{L2}  \\
    g_{K1} & g_{K2}  \\
\end{bmatrix}
\]

To get a good idea of the workings of an international economy like the one I described above, I shall look at several contrasting cases. In all cases I assume that \( a_{31} = 0.2, a_{32} = 0.1, a_{41} = 0.1, a_{42} = 0.3, a_{L1} = 2.04, a_{L2} = 0.67, a_{K1} = 1.06, a_{K2} = 1.38, a_{L3} = 1.5, a_{L4} = 1.6, a_{K3} = 1.8, a_{K4} = 1.8. \) \( L_A = 100, K_A = 150, L_B = 130, \) and \( K_B = 100. \) These numbers by themselves mean nothing. The things that matter are that they ensure that \( K \) is the abundant primary factor in \( A \) and \( L \) is the abundant primary factor in \( B. \) Also, they ensure that the production of \( x_1 \) is intensive in \( L, \) the production of \( x_2 \) is intensive in \( K, \) and the production of natural resources, \( x_3 \) and \( x_4, \) is intensive in \( K, \) as evidenced in the \( D, C, F, \) and \( G \) matrices below.

\[
\begin{bmatrix}
    2.04 & 0.67 & 1.5 & 1.6 & 0.2 & 0.1 & 2.5 & 1.3  \\
    1.06 & 1.38 & 1.8 & 1.8 & 0.1 & 0.3 & 1.6 & 2.1  \\
\end{bmatrix}
\]
Case I: In this case I assume that \( \frac{\alpha}{\beta+\alpha} = \frac{\delta}{\delta+\mu} = \frac{1}{2} \). Then, with \( P_2 = 1 \), there is a unique trade equilibrium in the A and B world economy in which

\[
(P_1, P_2, w, q, x_{1A}^d, x_{2A}^d, x_{1A}^s, x_{2A}^s, x_{1B}^d, x_{2B}^d, x_{1B}^s, x_{2B}^s) = (1.63, 1, 0.57, 0.12, 23.21, 37.76, 4.74, 67.82, 26.63, 43.31, 45.11, 13.25)^1.
\]

In this equilibrium, A imports 18.47 units of \( x_1 \) and exports 30.06 units of \( x_2 \), and the H&O conjecture is valid. Also both factors are fully employed in A and B.

Case II: In this case I assume that \( \frac{\alpha}{\alpha+\beta} = 0.04 \) and \( \frac{\delta}{\mu+\delta} = 0.87 \). Then, in autarky, there is a unique equilibrium in A in which

\[
(P_1, P_2, w, q, x_{1A}^d, x_{2A}^d, x_{1A}^s, x_{2A}^s) = (1.6, 2.1, 0, 1, 3.75, 68.57, 3.75, 68.57)^2,
\]

and in which capital is fully employed and only 98.52 units of labor are used in the production of \( x_1 \) and \( x_2 \). Similarly, there is a unique autarkic equilibrium in B in which

\[
(P_1, P_2, w, q, x_{1B}^d, x_{2B}^d, x_{1B}^s, x_{2B}^s) = (2.5, 1.3, 1, 0, 45.24, 13, 45.24, 13)^3
\]

and in which labor is fully employed and only 99.68 units of capital are used in the production of \( x_1 \) and \( x_2 \). When we allow A and B to trade in \( x_1 \) and \( x_2 \), we find that there is a unique trade equilibrium in which

\[
(P_1, P_2, w, q, x_{1A}^d, x_{2A}^d, x_{1A}^s, x_{2A}^s, x_{1B}^d, x_{2B}^d, x_{1B}^s, x_{2B}^s) = (1.37, 1, 0.40, 0.23, 2.17, 71.33, 4.73, 67.82, 47.67, 9.75, 45.11, 13.25)^4.
\]

In this equilibrium both primary factors are fully employed. Also, A exports 2.56 units of \( x_1 \) and imports 3.51 units of \( x_2 \) in contradiction to H&O’s conjecture.

The preceding examples demonstrate that in an input-output model of international trade differences in demand among the two countries may lead to situations in which the H&O conjecture is not valid. Still an input-output world like
the one I have described has a built-in structure that raises an interesting question concerning the flow of goods between A and B: If $\alpha$ and $\beta$ are not too different from $\delta$ and $\mu$, can we be sure that trade flows in A and B’s world economy will accord with H&O’s conjecture? The answer to this question is of importance for a meaningful empirical analysis of the H&O conjecture. So I come back to it later.

III. A Theory Universe for a Test of H&O’s Conjecture

Next I formulate a theory universe for a test of the Hecksher-Ohlin conjecture based on Norwegian data and a version of the Input-Output model that I discussed above. In reading the details of the theory universe, note that it can serve as a basis for a theory-data confrontation with cross-section data on one or more countries. If I had had in mind analyzing n yearly observations on Norwegian industries, each of the components of $\omega_T$ would have had to be n-dimensional instead of one-dimensional.

$$\omega_T \in \Omega_T \text{ only if } \omega_T = (x^s, x^d, x_1^f, x_4^f, L, K, p, w, q, L_A, K_A, EL_A, EK_A, EL_{TC}, EK_{TC}),$$

where $x^s \in \mathbb{R}^4_+$, $x^d \in \mathbb{R}^2_+ \times \{(0,0)\}$, $x_1^f \in \mathbb{R}^2_+$, $x_4^f \in \mathbb{R}^2_+$, $(L, K) \in \mathbb{R}^8_+$, $p \in \mathbb{R}^4_+$, $(w, q) \in \mathbb{R}^2_+$, $(L_A, K_A) \in \mathbb{R}^2_+$, and $(EL_A, EK_A, EL_{TC}, EK_{TC}) \in \mathbb{R}^4_+$.

In the intended interpretation of this axiom, the country concerned, A, is a country in trade equilibrium with its trading community, TC. It produces four commodities, $x_1, \ldots, x_4$, with two primary factors of production, labor and capital. I denote by $(L_i, K_i)$ the pair of primary factors that is used in the production of $x_i$, $i = 1, \ldots, 4$, and I take $L$ to stand for labor and $K$ for capital. The first two commodities, $x_1$ and $x_2$, are consumables. The last two, $x_3$ and $x_4$, are natural resources that are not consumable. The country trades in $x_1$ and $x_2$ and uses $x_3$ and $x_4$ as factors in the production of $x_1$ and $x_2$. I denote by $x_{3i}^f$ and $x_{4i}^f$ the amount of $x_3$ and $x_4$ used as factors in the production of $x_i$, $i = 1, 2$. The symbol, $p$, denotes the price of $x$; $w$ and $q$ denote the wage of labor and the rental price of capital, and $L_A$ and $K_A$ designate country A's current stock of the respective primary factors. It is an interesting question in the theory-data confrontation of H&O’s conjecture whether the two countries' endowments of labor and capital, $EL_J$ and $EK_J$, $J = A, TC$, ought to be taken to equal
their stocks of labor and capital. I come back to that question when I present the axioms for the data universe.

Next the axiom concerning the demand for \(x_1\) and \(x_2\), HO 2, and the axioms concerning the production of the components of \(x\), HO 3 and HO 4.

**HO 2** There is a constant, \(a \in (0,1)\), such that, for all \(\bar{\omega}_T \in \Omega_T\),
\[
p_1x_1^d = a[wL_A + qK_A], \quad \text{and} \quad p_2x_2^d = (1-a)[wL_A + qK_A].
\]

**HO 3** There are positive constants, \(a_{ki}, k = 3,4, L,K\), and \(i = 1, \ldots, 4\), such that, for all \(\bar{\omega}_T \in \Omega_T\),
\[
x_i^s = \min\{(L_i/a_{Li}),(K_i/a_{Ki}),(x_{3i}/a_{3i}),(x_{4i}/a_{4i})\}, i=1,2, \quad \text{and} \quad x_i^s = \min\{(L_i/a_{Li}),(K_i/a_{Ki})\},i=3,4.
\]

**HO 4** For all \(\bar{\omega}_T \in \Omega_T\),
\[
p_1 - p_3a_{31} - p_4a_{41} = wa_{L1} + qa_{K1};
\]
\[
p_2 - p_3a_{32} - p_4a_{42} = wa_{L2} + qa_{K2};
\]
\[
p_3 = wa_{L3} + qa_{K3}; \quad \text{and}
\]
\[
p_4 = wa_{L4} + qa_{K4}.
\]

In axiom HO 2 I assume that domestic demand for \(x_1\) and \(x_2\) in \(A\) can be rationalized by a utility function like the ones I used to rationalize demand in \(A\) and \(B\) in Section II. Here \(a = (\alpha/(\alpha+\beta))\) for some values of \(\alpha\) and \(\beta\). Also, in HO 3 I assume that the production of \(x_1, \ldots, x_4\) in \(A\) satisfies the conditions that I imposed on the respective production functions in my trade model. Finally, the equations in HO 4 insist that the prices of the components of \(x\) equal the cost of producing the respective components.

Axiom HO 5 below insists that the supply of \(x_3\) and \(x_4\) equals the demand for \(x_3\) and \(x_4\). Similarly, axiom HO 6 makes sure that the production of the respective components of \(x\) does not employ more of \(L\) and \(K\) than is available. Since \(A\) trades in \(x_1\) and \(x_2\), there are no analogous conditions on the supply and demand for \(x_1\) and \(x_2\). However, HO 7 is a partial substitute. In the intended interpretation of the axioms, HO 7 insists that in trade equilibrium the value of \(A\)'s supply of \(x_1\) and \(x_2\) must equal the value of its demand for the same commodities.
HO 5 For all $\omega_T \in \Omega_T$,
\[ x_3^s - x_3^f - x_3^f = 0, \quad \text{and} \]
\[ x_4^s - x_4^f - x_4^f = 0. \]

HO 6 For all $\omega_T \in \Omega_T$,
\[ L_1 + L_2 + L_3 + L_4 \leq L_A, \quad \text{and} \]
\[ K_1 + K_2 + K_3 + K_4 \leq K_A. \]

HO 7 For all $\omega_T \in \Omega_T$,
\[ p_1 x_1^s + p_2 x_2^s = p_1 x_1^d + p_2 x_2^d. \]

In HO 8 I formulate the version of Heckscher and Ohlin’s conjecture that I intend to confront with data. In reading the axiom, note that it is ELJ and EKJ, and not the stocks of labor and capital in A and TC, on which depends the validity of H&O’s conjecture.

HO 8 For all $\omega_T \in \Omega_T$, if $\frac{EL_A}{EK_A} < (>) \frac{EL_{TC}}{EK_{TC}}$ and
\[ \left( \frac{(a_{L1} + a_{L3}a_{31} + a_{L4}a_{41})}{(a_{K1}+a_{K3}a_{31}+a_{K4}a_{41})} \right) < \]
\[ \left( \frac{(a_{L2} + a_{L3}a_{32} + a_{L4}a_{42})}{(a_{K2}+a_{K3}a_{32}+a_{K4}a_{42})} \right) \]
then
\[ x_1^s > (<) x_1^d \quad \text{and} \quad x_2^s < (>) x_2^d. \]

Also, if $\frac{EL_A}{EK_A} < (>) \frac{EL_{TC}}{EK_{TC}}$, then the preceding condition on the $a_{ki}$ coefficients with < replaced by > implies that $x_1^s < (>) x_1^d$ and $x_2^s > (<) x_2^d$.

There are several interesting aspects of the theory universe that I have delineated above. First of all, HO 1-HO 8 does not constitute a complete set of axioms from which we can deduce a theory of a country’s trade flows. I have picked out pertinent elements of the family of models of the world economy that I discussed in section II. The models of the given world economy ensure that the axioms of my theory universe have meaningful models. Secondly, I have not insisted on possible values of $a, L_A$ and $K_A$ that, with the appropriate choice of values for $EL$ and $EK$, will ensure the validity of H&O’s conjecture in a trade model like the one I described in section 1. Even so, in my test of the empirical relevance of H&O’s conjecture the
estimated values of \( a, L_A \) and \( K_A \) will play a significant role. Thirdly, the incompleteness of HO 1-HO 8 raises a serious question. How am I to determine which factor, L or K, is the relatively abundant factor in A? We shall see.

IV. A Data Universe for a Test of Heckscher and Ohlin’s Conjecture

In this section I describe a data universe for a test of the H&O conjecture. How to look at the data without doing harm to the import of the theory-data confrontation is a serious problem in many situations. My search for an appropriate data universe in which to try the empirical relevance of Heckscher and Ohlin’s conjecture provides a good illustration of how uncomfortable the problem can be. The data I possess comprise a 1997 input-output table for Norway with twenty-three industries and endogeneous imports, the 1997 costs of production due to wages and salaries for eighty-two Norwegian industries, and the 1997 kroner value of the stock of capital in thirty-eight Norwegian industries.\(^5\) I need to create data to measure the inputs and outputs of four industries that I can relate to the four \( x_8 \) in my theory universe. Also, I need to find a way to measure final demand for the products of the two non-natural-resource industries, and to measure the 1997 stocks of labor and capital in Norway. Finally, I need to construct estimates of the production coefficients on whose existence I insist in axiom HO 3 of the theory universe, and I have to find a good way of measuring the endowments of labor and capital in 1997 Norway and its trading community. That is a tall order when considering the fact that the resolution of my problem is not to affect the outcome of the test of H&O’s conjecture.

Leaving out the details, I resolve my problem in the following way. Let \( Z_i, i = 1, \ldots, 23 \), designate the twenty-three industries in the 1997 Norwegian Input-Output Table. The industries, \( Z_{20} \) to \( Z_{22} \) concern various operations of public administration, and I lump them together in a ‘public sector,’ \( Z^5 \). Industry \( Z_3 \) comprises mining and oil production. With the help of the Central Bureau of Statistics (CBS), I split the industry in two and obtain an industry for mining, \( Z^3 \), and an industry for oil production, \( Z^4 \). Also with the help of the CBS, I combine all industries in \( \{ Z_1, Z_2, Z_4, \ldots, Z_{19} \} \) with more exports than imports and all industries with more imports than exports in two industries, \( Z^1 \) and \( Z^2 \), and construct an input-output table with endogeneous imports for \( Z^1, Z^2, Z^3, Z^4 \) and \( Z^5 \).

Finally, I ask the CBS to provide me with data on the total costs of salaries and wages and on the values of the stocks of capital in the five industries, \( Z^1, \ldots, Z^5 \). With the constructed
input-output table and the latter data on hand, I can divine a suitable data universe for a
test of the H&O conjecture. Here it is.

\[
\text{HO 13 } \omega_p \in \Omega_p \text{ only if } \omega_p = (X^s, X^d, A, M, W, Q, AI, MI, AE, ME, AC, MC, AG,
MG, A\Sigma, M\Sigma, W\Sigma, Q\Sigma, SL, SL\Sigma, SK, SK\Sigma, w^*, q^*, B, c, SL\Sigma_{TC}, SK\Sigma_{TC}, L^c, K^c, U, V, \epsilon),
\]

where \(X^s \in \mathbb{R}^5_+, X^d \in \mathbb{R}^5_+, A = (A_{ij})\) and \(M = (M_{ij})\) are, respectively, \(5 \times 5\) and \(4 \times 5\) real-
valued matrices with nonnegative components, \(W \in \mathbb{R}^{++}_5, Q \in \mathbb{R}^{++}_5, AI \in \mathbb{R}^+_5, MI \in \mathbb{R}^+_4,
AE \in \mathbb{R}^+_5, ME \in \mathbb{R}^+_4, AC \in \mathbb{R}^+_5, MC \in \mathbb{R}^+_4, AG \in \mathbb{R}^+_5, MG \in \mathbb{R}^+_4, A\Sigma \in \mathbb{R}^+_5,
M\Sigma \in \mathbb{R}^+_4, W\Sigma \in \mathbb{R}^{++}_5, Q\Sigma \in \mathbb{R}^{++}_5, (SL, SK) \in \mathbb{R}^{++}_5, (SL\Sigma, SK\Sigma) \in \mathbb{R}^{++}_2,
w^* \in \mathbb{R}^{++}_5, q^* \in \mathbb{R}^{++}_5, B\) is a \(2 \times 9\) real-valued matrix with nonnegative entries, \(c \in \mathbb{R}^{++}_4, (SL\Sigma_{TC}, SK\Sigma_{TC}, L^c, K^c) \in \mathbb{R}^{++}_4,\)
and \((U, V, \epsilon) \in \mathbb{R}^{12}\).

In the intended interpretation of the axiom, the components of \(X^s\) and \(X^d\) denote the
supply of and the final demand for the products of the respective industries in \(Z\), where \(Z = (Z_1, Z_2, Z_3, Z_4, Z_5)\). The components of \(Q\) and \(SL\) and \(SK\) designate, respectively, the
gross surplus and the stocks of labor and capital in the respective industries in \(Z\), and
\(SL\Sigma_{TC}\) and \(SK\Sigma_{TC}\), and \(L^c\) and \(K^c\), respectively, denote the stocks of labor and capital in
\(TC\), and the amount of labor and capital used in the production of \(X_1^d\) and \(X_2^d\). Finally,
\(w^*\) and \(q^*\) and \(U, V, \) and \(\epsilon\) record, respectively, the average wage rate, the average rental
price of capital, and factors that are not accounted for, \(B\) is a matrix of coefficients that are
to play the HO 8-aijs' role in my test of the empirical relevance of H&O’s conjecture, and
c is a vector of appropriately chosen constants. The remaining components of \(\omega_p\) are
entries in an input-output matrix the relevant parts of which I display below. In this
matrix, \(Z = (Z_1^i, Z_2^i, Z_3^i, Z_4^i, Z_5^i)\) as above, and \(MZ = (MZ_1^i, MZ_2^i, MZ_3^i, MZ_4^i)\). Also, \(MZ^i\) is
short for ‘imports of products pertaining to industry \(Z^i’; j=1,\ldots,4\), \(I\) and \(E\) are short for
‘gross investment’ and ‘export,’ \(C\) and \(G\) are short for ‘household consumption’ and
‘government,’’ and \(\Sigma\) is short for ‘sum.’

<table>
<thead>
<tr>
<th>(Z)</th>
<th>(I)</th>
<th>(E)</th>
<th>(C)</th>
<th>(G)</th>
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<tbody>
<tr>
<td>(Z)</td>
<td>A</td>
<td>AI</td>
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<td>(MZ)</td>
<td>M</td>
<td>MI</td>
<td>ME</td>
<td>MC</td>
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<td>Wages</td>
<td>W</td>
<td>(W\Sigma)</td>
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<td>Gross Surplus</td>
<td>Q</td>
<td>(Q\Sigma)</td>
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</table>
Axioms HO 14 and HO 15 interpret $\Sigma$ as a sum and provide recipes for computing $w^*$ and $q^*$, the Us, Vs, and $\varepsilon$s, and the components of the B matrix.

**HO 14** For all $\omega_p \in \Omega_p$, \(A_{j1} + A_{j2} + A_{j3} + A_{j4} + A_{j5} + U_j + A_{i1} + AE_j + AC_j + AG_j = A_\Sigma_j, \quad j = 1, \ldots, 5; \quad M_{j1} + M_{j2} + M_{j3} + M_{j4} + M_{j5} + MI_j + ME_j + MC_j + MG_j = M_\Sigma_j, \quad j = 1, \ldots, 4; \quad W_1 + W_2 + W_3 + W_4 + W_5 = W_\Sigma; \quad Q_1 + Q_2 + Q_3 + Q_4 + Q_5 = Q_\Sigma; \quad SL_1 + SL_2 + SL_3 + SL_4 + SL_5 = SL_\Sigma; \quad SK_1 + SK_2 + SK_3 + SK_4 + SK_5 = SK_\Sigma ; \quad w^* = W_\Sigma/SL_\Sigma, \) and \(q^* = Q_\Sigma/SK_\Sigma. \) Also, \(V_i = A_\Sigma_i - ((W_i + Q_i)+(A_{j5}/A_\Sigma_j)(W_{j5} + Q_{j5})+(A_{j3_i} + \varepsilon_{j3i})+(A_{j4_i} + \varepsilon_{j4i})), \) \(i = 1, 2; \) and \(V_i = A_\Sigma_i - ((W_i + Q_i)+(A_{j5}/A_\Sigma_j)(W_{j5} + Q_{j5})), \) \(i = 3, 4. \)

The entries in the B matrix are to play the role of the HO 8’s $a_{ij}$s in my test of H&O’s conjecture. In HO 15 I propose a possible way of measuring the respective components of B. The bridge principles that I formulate in HO 9 – HO 12 in Section VI will ensure that the respective $b_{ij}$ can play the role that I have assigned to them.

**HO 15** For all $\omega_p \in \Omega_p$, the components of $\varepsilon$, $c$, and $B$ satisfy the relations,

\[
\begin{align*}
   c_{i}b_{Li} &= W_{i}/A_{\Sigma_i} \quad \text{and} \quad c_{i}b_{Ki} = Q_{i}/A_{\Sigma_i}, \quad i = 1, \ldots, 4, \quad c_{i}b_{3i} = (A_{3i} + \varepsilon_{3i})/A_{\Sigma_i} \quad \text{and} \quad c_{i}b_{4i} = (A_{4i} + \varepsilon_{4i})/A_{\Sigma_i}, \\
   b_{L5} &= W_{5}/A_{\Sigma_5}, \quad b_{K5} = Q_{5}/A_{\Sigma_5}, \quad \text{and} \quad c_{i}b_{5i} = A_{5i}/A_{\Sigma_i}, \quad i = 1, \ldots, 4.
\end{align*}
\]

\[
\begin{align*}
   \varepsilon_{3i} &= (A_{3i}/(A_{31} + A_{32}))c_{3}A_{3i} - A_{3i}, \quad i = 1, 2; \quad \text{and} \quad \varepsilon_{4i} = (A_{4i}/(A_{41} + A_{42}))c_{4}A_{4i} - A_{4i}, \quad i = 1, 2.
\end{align*}
\]

Also, the $c_i$ are chosen such that

\[
\begin{align*}
   (b_{Li} + b_{L5}b_{5i}) + (b_{Ki} + b_{K5}b_{5i}) + b_{3i} + b_{4i} = 1, \quad i = 1, 2, \quad \text{and} \\
   (b_{Li} + b_{L5}b_{5i}) + (b_{Ki} + b_{K5}b_{5i}) = 1, \quad i = 3, 4
\end{align*}
\]

Finally,

\[
\begin{align*}
   B &= \begin{pmatrix}
   b_{L1} & b_{L2} & b_{L3} & b_{L4} & b_{L5} & b_{31} & b_{41} & b_{51} & b_{53} \\
   b_{K1} & b_{K2} & b_{K3} & b_{K4} & b_{K5} & b_{32} & b_{42} & b_{52} & b_{54}
\end{pmatrix}
\]
The variables that in the data universe are to play the roles that the components of \( x^s \) and \( x^d \) play in the theory universe are \( X_i^s \) and \( X_i^d \), \( i = 1, \ldots, 4 \). I give precise definitions of the latter variables in HO 16.

**HO 16** For all \( \omega_p \in \Omega_p \),

\[
\begin{align*}
X_i^s &= (W_i + Q_i) + (A_{5i}/A_{5}) (W_5 + Q_5) + (A_{3i} + \varepsilon_{3i}) + (A_{4i} + \varepsilon_{4i}), \quad i = 1, 2; \\
X_i^s &= (W_i + Q_i) + (A_{5i}/A_{5}) (W_5 + Q_5), \quad i = 3, 4; \\
X_1^d &= (U_1 + A_{11} + A_{C_1} + A_{G_1}) + A_{2} + X_4^s; \\
X_2^d &= (U_2 + A_{12} + A_{C_2} + A_{G_2}) + A_{2} + X_3^s + (1 - (\Sigma_{1\leq j \leq 4}(A_{5j}/A_{5}))) (W_5 + Q_5) - A_{55}; \\
X_3^d &= 0; \\
X_4^d &= 0.
\end{align*}
\]

I proposed the axioms for the data universe with the idea in mind that I was to try the empirical relevance of H&O’s conjecture for Norwegian trade flows. Hence the axioms delineate the ways in which I, with the data that I received from the Central Bureau of Statistics, have constructed the variables that I needed for the test.

**V. The Measurement of Factor Endowments**

In HO 12 below I suggest two ways to measure a country’s endowments of labor and capital, \( EL \) and \( EK \). There are other ways to measure these variables, and they are all equally controversial. Therefore, we need to discuss what characteristics measures of factor endowments must have in order to be appropriate for a test of H&O’s conjecture.

Constructing appropriate measures for a test of H&O’s conjecture is exceedingly difficult. So a thought comes to mind. I could admit that I do not know how to measure factor endowments in a meaningful way and infer from the results of my empirical analysis which factor is the abundant factor in 1997 Norway. It might be labor. If it is, the conclusion would not be quite as startling as Leontief’s inference that labor was the abundant factor in 1947 USA. Still, it would require sophisticated economic reasoning to explain. Chances are good that my reasoning would fare no better than Leontief’s insistence that labor was at least three times as productive in 1947 USA as it was in the rest of the world. Besides, no matter how ingenious my arguments, proceeding Leontief’s
way would be a bad way to try the empirical relevance of H&O’s conjecture. Therefore, I shall reject the thought and search for good measures of the endowments of labor and capital in 1997 Norway.

I can measure a country’s endowments of labor and capital in many different ways. In the trade model in Section II, the natural candidates for measures of A’s factor endowments are \( L_A, K_A \), and the equilibrium values of \( wL_A \) and \( qK_A \). From them we obtain four possible measures of the relative abundance in A of the two factors, \( \frac{L_A}{K_A} \), \( \frac{wL_A}{K_A} \), \( \frac{wL_A}{qK_A} \), and \( \frac{L_A}{qK_A} \). In case I (case II) the respective values were, \( \frac{L_A}{K_A} = 0.67 \) (0.67), \( \frac{wL_A}{K_A} = 0.38 \) (0.267), \( \frac{wL_A}{qK_A} = 3.17 \) (1.159), and \( \frac{L_A}{qK_A} = 5.56 \) (2.899). The first two measures suggest that capital is the relatively abundant factor in A. The last two indicate that labor is the relatively abundant factor in A.

In international trade it is far from obvious that it is possible to determine the relative abundance of a country’s factors of production simply by measuring the country’s own stocks of such factors. Hence the divergent views on the relative abundance of labor and capital in A that I obtained above were to be expected. To determine the relative abundance of a factor in one country, I must compare the country’s factor endowments with the factor endowments of its major trading partners. Thus, in the trade model of Section II we must compare the measures of relative factor abundance in A with the corresponding measures for B. In case I (case II): \( \frac{L_B}{K_B} = 1.3 \) (1.3), \( \frac{wL_B}{K_B} = 0.74 \) (0.52), \( \frac{wL_B}{qK_B} = 6.17 \) (2.261), and \( \frac{L_B}{qK_B} = 10.83 \) (5.652). The first and the last two measures suggest that labor is the abundant factor in B while the second measure insists that capital is the abundant factor in B. Note, therefore, that in all cases the value of the measure in B is larger than the equivalent measure in A. So, no matter how I decide to measure factor endowments in the given trade model, I will conclude that labor is the abundant factor in B and capital the abundant factor in A.

When a country, A, has more than one trading partner, comparing A’s factor endowments with the endowments of each and every trading partner, need not lead to a meaningful assessment of the relative abundance of A’s factors. There is, however, a better way to proceed. With reasonable measures of factor endowments on hand, I can estimate the total endowment of each factor in A’s trading community and judge the factor in A that has the largest percentage of this total to be the abundant factor. Then, with TC denoting the trading community and with just two factors of production, L and K, I would judge capital to be the relatively abundant factor in A if the measure of \( \frac{E_L_A}{E_L_{TC}} \) is smaller than the measure of \( \frac{E_K_A}{E_K_{TC}} \). In HO 12 below I propose two different ways
to measure factor endowments. Whether the chosen way of measuring EL and EK makes a difference as far the empirical assessment of the relative abundance of A’s two primary factors is a question that I attend to in my empirical analysis of H&O’s conjecture.

VI. Bridge Principles for a Test of H&O’s Conjecture

The bridge principles for the theory-data confrontation of the H&O conjecture are recorded below. In reading them, recall that the bridge principles concern vectors in the sample space, $\Omega$, and that $\Omega \subset \Omega_T \times \Omega_P$. Remember also that the variables in the data universe are variables that I have constructed for the purpose of testing the empirical relevance of H&O’s conjecture with Norwegian input-output data. Therefore, the formal appearance of the bridge principles not withstanding, HO 9 - HO 12 describe the way I perceive the relationship between my theory variables and my Norwegian data variables. In reading them, note that I conceive of two possible measures of the endowments of labor and capital.

**HO 9** For all $(\omega_T, \omega_P) \in \Omega$, $p_jx_j^s = X_j^s$, $j = 1,\ldots,4$: $p_jx_j^d = X_j^d$, $j = 1,\ldots,4$; and $p_jx_{ji}^f = A_{ji} + \epsilon_{ji}$, $j = 3,4$, and $i = 1,2$.

**HO 10** For all $(\omega_T, \omega_P) \in \Omega$, $L_A = \sum_{1 \leq i \leq 5} S L_i$, $wL_A = \sum_{1 \leq i \leq 4} (W_i + (A_5/A)W_5)$, $K_A = \sum_{1 \leq i \leq 5} S K_i$, and $qK_A = \sum_{1 \leq i \leq 4} (Q_i + (A_5/A)Q_5)$.

**HO 11** For all $(\omega_T, \omega_P) \in \Omega$, $w_{aL}p_j = b_{Lj} + b_{L5}b_{5j}$, $q_{aK}p_j = b_{Kj} + b_{K5}b_{5j}$, $j = 1,\ldots,4$, $p_3a_3/p_j = b_{3j}$ and $p_4a_4/p_j = b_{4j}$, $j = 1,2$.

**HO 12** Either, for all $(\omega_T, \omega_P) \in \Omega$,

$$(EL_A, EK_A, EL_{TC}, EK_{TC}) = (SL\Sigma, SK\Sigma, SL\Sigma_{TC}, SK\Sigma_{TC}),$$

or, for all $(\omega_T, \omega_P) \in \Omega$,

$$(EL_A, EK_A, EL_{TC}, EK_{TC}) = (SL\Sigma, SK\Sigma_A, L^c, K^c).$$

Note that HO 11 provides an example of bridge principles that vary with models of HO 1 – HO 8. Note, also, that it follows from HO 10 that, for all $(\omega_T, \omega_P) \in \Omega$, w differs from $w^*$ and q differs from $q^*$. In different words, I assume that my observations on w
and q are marred by errors. How $L^c$ and $K^c$ can be used to measure $EL_{TC}$ and $EK_{TC}$ is a question that I will return to below.

The bridge principles allow me to establish several interesting properties of the variables in the data universe. I shall begin with a theorem, T 1, that shows how to use entries in the B matrix to determine the relative factor intensities of the production processes in $Z^1$ and $Z^2$. With T 1 and estimates of EL and EK we can see if the 1997-Norwegian Trade Flows are in accord with the HO 8-version of H&O’s conjecture.

**T 1** If HO 1–HO 7 and HO 9–HO 16 are valid, then for all $(\omega_T, \omega_P) \in \Omega$ and $j = 1, 2$,

$$\frac{(w/q)(a_{Lj} + a_{L3a3j} + a_{L4a4j})/(a_{Kj} + a_{K3a3j} + a_{K4a4j})}{[(b_{Lj} + b_{L3b3j})(b_{L4} + b_{L3b34})b_{4j}]/[(b_{Kj} + b_{K3b3j})(b_{K4} + b_{K3b34})b_{4j}]}.$$

Judging from our discussion of factor endowments above, it might seem impossible to use input-output tables and data on trade flows and factor endowments in one country alone to test the validity of H&O’s conjecture. It is, therefore, interesting to note that it is possible when the demand functions of the country in question, A, and its trading partners can be rationalized by one and the same homothetic utility function. In that case capital is the abundant factor in A if and only if $K_A/L_A > K_c/L_c$, where $K_c$ and $L_c$, respectively, denote the stocks of capital and labor that in A are needed to produce the domestically consumed commodities. I state and prove this proposition in section VIII.2.

The idea of the proposition I owe to Edward Leamer (cf. Leamer, 1980, pp. 497-498).

In the universe of HO 1 – HO 8 both primary factors are fully employed, and $K^c$ and $L^c$ are given by the following two equations:

$$L^c = (a_{L1} + a_{L3a31} + a_{L4a41})x_1^d + (a_{L2} + a_{L3a32} + a_{L4a42})x_2^d,$$

$$K^c = (a_{K1} + a_{K3a31} + a_{K4a41})x_1^d + (a_{K2} + a_{K3a32} + a_{K4a42})x_2^d.$$

The next theorem shows how to measure $qK^c/wL^c$ in the data universe.
T 2 If HO 1–HO 7 and HO 9 – HO 16 are valid, then for all \((w_T, w_P) \in \Omega\),

\[
(qK^c/wL^c) = \left[ a \left( (b_{K1} + b_{K5b51}) + (b_{K3} + b_{K5b53})b_{31} + (b_{K4} + b_{K5b54})b_{41} \right) + \\
(1-a) \left( (b_{K2} + b_{K5b52}) + (b_{K3} + b_{K5b53})b_{32} + (b_{K4} + b_{K5b54})b_{42} \right) \right] / \\
\left[ a \left( (b_{L1} + b_{L5b51}) + (b_{L3} + b_{L5b53})b_{31} + (b_{L4} + b_{L5b54})b_{41} \right) + \\
(1-a) \left( (b_{L2} + b_{L5b52}) + (b_{L3} + b_{L5b53})b_{32} + (b_{L4} + b_{L5b54})b_{42} \right) \right] 
\]

The validity of the theorem follows from HO 11 and the easily established equality,

\[
qK^c/wL^c = \left[ a \left( (qa_{K1}/p_1) + (qa_{K3}a_{31}/p_1) + (qa_{K4}a_{41}/p_1) \right)p_1x_1^d + \\
(1-a) \left( (qa_{K2}/p_2) + (qa_{K3}a_{32}/p_2) + (qa_{K4}a_{42}/p_2) \right)p_2x_2^d \right] / \\
\left[ a \left( (wa_{L1}/p_1) + (wa_{L3}a_{31}/p_1) + (wa_{L4}a_{41}/p_1) \right)p_1x_1^d + \\
(1-a) \left( (wa_{L2}/p_2) + (wa_{L3}a_{32}/p_2) + (wa_{L4}a_{42}/p_2) \right)p_2x_2^d \right] 
\]

In this context, T2 is particularly interesting since \(K_A/L_A > K^c/L^c\) if and only if

\(qK_A/wL_A > qK^c/wL^c\). My observations and the bridge principles will enable me to
estimate the value of \(qK_A/wL_A\), and T2 tells me how to estimate \(qK^c/wL^c\).

VII. The Data and the 1997 Norwegian Trade Flows

To try the empirical relevance of the H&O conjecture for the 1997 Norwegian trade
flows, I shall begin by listing the 1997- values of the entries in the input-ouput matrix that

\[
\begin{array}{cccccccc}
Z^1 & Z^2 & Z^3 & Z^4 & Z^5 & Z^d & Ex \\
Z^1 & 78.812.197 & 70.110.820 & 876.490 & 4.721.016 & 13.698.156 & 67.112.483 & 149.405.661 \\
Z^2 & 82.322.282 & 196.287.679 & 1.279.485 & 15.171.473 & 38.915.025 & 303.373.923 & 86.131.933 \\
Z^3 & 433.644 & 2.364.815 & 179.422 & 158.211 & 116.491 & 260.420 & 2.012.144 \\
Z^4 & 2.708.902 & 82.322.282 & 1.279.485 & 15.171.473 & 38.915.025 & 303.373.923 & 86.131.933 \\
Z^5 & 98.389.000 & 236.533.000 & 1.262.000 & 13.688.000 & 159.733.000 & 21.084.000 & 0 \\
M^2 & 58.304.804 & 75.819.366 & 493.131 & 1.365.088 & 8.489.909 & 2.729.295 & 0 \\
M^4 & 0 & 1.433.297 & 0 & 0 & 0 & 1.559.703 & 0 \\
W & 98.389.000 & 236.533.000 & 1.262.000 & 13.688.000 & 159.733.000 & 21.084.000 & 0 \\
Q & 84.746.000 & 206.356.000 & 1.095.000 & 159.868.000 & 21.084.000 & 0 & 0
\end{array}
\]
I displayed on p. 12. The entries are in units of kr. 1.000 and the $Z^d$-column records the sum of the pertinent values of I, C, and G:

\[
\begin{array}{c|c|c|c|c}
A\Sigma_1 & A\Sigma_2 & A\Sigma_3 & A\Sigma_4 & A\Sigma_5 \\
A\Sigma & 447.802.000 & 890.311.000 & 5.543.000 & 216.025.000 & 268.837.000 \\
M\Sigma_1 & M\Sigma_2 & M\Sigma_3 & M\Sigma_4 \\
M\Sigma & 55.107.000 & 304.652.000 & 3.397.000 & 2.993.000 \\
Z^1 & Z^2 & Z^3 & Z^4 \\
U & 63.065.177 & 164.829.200 & 17.853 & 11.110.770 \\
V & 247.580.348 & 279.004.221 & 3.137.500 & 41.238.384 \\
\varepsilon_3 & -60.792 & -332.167 \\
\varepsilon_4 & 13.312.946 & 142.948.143 \\
\end{array}
\]

We can use these entries to calculate the values of $X^{s}_j$ and $X^{d}_j$ for $j=1,\ldots,4$.

\[
\begin{array}{c|c|c|c|c}
Z^1 & Z^2 & Z^3 & Z^4 \\
X^s & 200.221.652 & 611.306.779 & 2.405.500 & 174.786.616 \\
X^d & 171.416.044 & 640.102.989 & 0 & 0 \\
\end{array}
\]

They show that

\textit{(I) Net exports are positive in $Z^1$ and negative in $Z^2$.}

Then the values of the $c_i$, $i=1,\ldots,4$, and the components of the B matrix.

\[
\begin{array}{c|c|c|c|c}
Z^1 & Z^2 & Z^3 & Z^4 \\
c & 0,4470 & 0,6867 & 0,4339 & 0,8091 \\
\end{array}
\]
\[
\begin{array}{cccccc}
\mathbf{b_L} & 0.4915 & 0.3869 & 0.5248 & 0.0784 & 0.5942 \\
\mathbf{b_K} & 0.4233 & 0.3376 & 0.4552 & 0.9146 & 0.0784 \\
\mathbf{b_3} & 0.0018 & 0.0033 \\
\mathbf{b_4} & 0.0745 & 0.2615 \\
\mathbf{b_5} & 0.0134 & 0.0157 & 0.0300 & 0.0105 
\end{array}
\]

With the data displayed above, the 1997 estimate of the values of the fraction,

\[
\frac{[(b_{Lj}+b_{Ls}b_{Sj})+(b_{L3}+b_{Ls}b_{S3})b_{3j}+(b_{L4}+b_{Ls}b_{S4})b_{4j}]}{[(b_{Kj}+b_{Ks}b_{Sj})+(b_{K3}+b_{Ks}b_{S3})b_{3j}+(b_{K4}+b_{Ks}b_{S4})b_{4j}]},
\]

in T1 equals 1.0272 for \( j = 1 \) and 0.7247 for \( j = 2 \). From this and T1 I infer that

(II) The production of \( X_1 \) is relatively labor-intensive, and the production of \( X_2 \) is relatively capital-intensive.

VIII. The H&O Conjecture and Norwegian Trade Flows

In our discussion of relative factor endowments above I considered several measures; e.g., \((K/L), (K^c/L^c)\), \((qK/wL)\), and \((qK^c/wL^c)\). I shall, next, use HO 9 – HO 12, the two ways of measuring \( EL \) and \( EK \) in HO 12, and the data I have to determine the empirical relevance of H&O’s conjecture for Norwegian trade flows. I begin with \( EL = SL \sum \) and \( EK = SK \sum \).

VIII.1 Case I: \( EL = SL \sum \) and \( EK = SK \sum \).

In 1997 Norway \( SL \sum \_A = 2.212.700 \) and (in millions of Norwegian kroners) \( SK \sum \_A = 3.109.342 \) So, if it makes sense to measure factor endowments of labor and capital, respectively, by the data-universe’s version of \( L_A \) and \( K_A \),

(IIIa) 1997-Norway seems to have had a relative abundance of capital.
However, it makes little sense to try Heckscher and Ohlin's conjecture on the basis of the ratio, \( L_A/K_A \), alone. We must compare this ratio with the corresponding ratio for Norway's trading community. Unfortunately, I do not have the required 1997 data for Norway's trading community. So I shall be content to recount pertinent details of the factor endowments in Norway and its trading partners in 1975 that Edward Leamer published in 1984.

In his 1984 study of *Sources of International Comparative Advantage* Edward Leamer carried out an analysis of the 1975 trade flows and endowments of various factors in forty-seven countries based on data that Harry Bowan had constructed (cf. Leamer, 1984 and Bowan, 1981). Table 2 below presents the pertinent results. In reading the table, note that Leamer’s measure of a country’s endowment of labor is a measure of the country’s stock of labor; i.e., of \( L_A \). Also, Leamer’s measure of a country’s endowment of capital is a measure of the value of the country’s net stock of capital; i.e., of \( K_A \).

I have extracted the data in Table 1 from Table B.1 on pp. 221-227 in Leamer’s book. According to Table B.2 on pp. 228-229 in the same book, Capital is measured in millions of U.S. dollars and constitutes an estimate of the net stock of capital for a country. The estimate was obtained by summing gross domestic investment flows over the period 1948-1975 and applying depreciation factors that were based on assumed average asset lives. Also, Labor is measured in thousands of economically active members of the pertinent populations with Labor 1 comprising the “professional technical and related workers,” with Labor 3 comprising the illiterate workers, and with Labor 2 comprising all the rest. The capital data were derived from the World Bank’s 1976-*World Tables*, and the labor data were from ILO’s *Labor Force Projections* 1965-1985.

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>YEAR</th>
<th>CAPITAL</th>
<th>LABOR 1</th>
<th>LABOR 2</th>
<th>LABOR 3</th>
<th>L1+L2+L3</th>
</tr>
</thead>
<tbody>
<tr>
<td>AUSL</td>
<td>75</td>
<td>140302</td>
<td>648.91</td>
<td>5079.5</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>AUST</td>
<td>64792</td>
<td>299.42</td>
<td>2932.1</td>
<td></td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>BLUX</td>
<td>93700</td>
<td>464.43</td>
<td>3385.7</td>
<td></td>
<td>43</td>
<td></td>
</tr>
<tr>
<td>CAN</td>
<td>227522</td>
<td>1311.49</td>
<td>8183.2</td>
<td></td>
<td>57</td>
<td></td>
</tr>
<tr>
<td>DEN</td>
<td>46585</td>
<td>342.34</td>
<td>2037.3</td>
<td></td>
<td>14</td>
<td></td>
</tr>
</tbody>
</table>
The data in Table 2 suggest that in 1975 capital was the abundant factor in Norway. To wit. With N for Norway and TC for the totality of Norway’s trading community,

\[
\frac{K_N}{K_{TC}} = 0.009306 \quad \text{and} \quad \frac{L_N}{L_{TC}} = 0.004727; \\
\frac{K_N}{L_N} = 35,162.41 \quad \text{and} \quad \frac{K_{TC}}{L_{TC}} = 17,860.04.
\]

If the relationship between \( \frac{K_N}{L_N} \) and \( \frac{K_{TC}}{L_{TC}} \) here is not too different from the relationship between the corresponding 1997 factor endowments, we must conclude that the H&O conjecture has little relevance for Norwegian trade flows.

VIII.2 Case II: \( EL = L^c \) and \( EK = K^c \).

To overcome the problem that lack of data on the factor endowments of Norway's trading partners causes, I shall adopt Leamer’s heroic assumption that the demand functions of
the pertinent countries can be rationalized by one and the same homothetic utility function (Leamer, 1984, Assumption 6 on p.2). In the case of A and its trading partners, in the intended interpretation of the theory universe the assumption implies that there is a constant $a \in (0,1)$ and a positive constant, $s$, such that, with the obvious notation,

\[
\begin{align*}
    p_1x_{1A}^d &= a[wL_A + qK_A] \\
    p_2x_{2A}^d &= (1-a)[wL_A + qK_A], \\

    p_1x_{1TC}^d &= a[wL_{TC} + qK_{TC}] \\
    p_2x_{2TC}^d &= (1-a)[wL_{TC} + qK_{TC}] \text{ and} \\

    s &= \frac{[p_1x_{1A}^d + p_2x_{2A}^d][p_1x_{1TC}^d + p_2x_{2TC}^d]}{[wL_A + qK_A]/[wL_{TC} + qK_{TC}]} = \\
    &\frac{[(wL_{TC}^c)(L_A/L_{TC}^c) + (qK_{TC}^c)(K_A/K_{TC}^c)]/[wL_{TC}^c + qK_{TC}^c]}.
\end{align*}
\]

The last equality follows from the fact that in trade equilibrium the Leontief world of the universe of HO 1-HO 8 must be such that $wL_{TC} + qK_{TC} = wL_{TC}^c + qK_{TC}^c$. From it we can deduce that,

\[
s \in \left[\min((L_A/L_{TC}^c), (K_A/K_{TC}^c)), \max((L_A/L_{TC}^c), (K_A/K_{TC}^c))\right].
\]

With similar arguments and the equality $wL_A + qK_A = wL_A^c + qK_A^c$ it follows that

\[
s^{-1} \in \left[\min((L_{TC}/L_A^c), (K_{TC}/K_A^c)), \max((L_{TC}/L_A^c), (K_{TC}/K_A^c))\right].
\]

Now, in trade equilibrium the Leontief world of the universe of HO 1-HO 8 with Leamer's assumption must be such that

\[
L_A^c = sL_{TC}^c \text{ and } K_A^c = sK_{TC}^c.
\]

From these equalities, the preceding observations, and simple algebra follows the validity of theorem T 3

**T 3** If HO 1- HO 7 are valid, and if demand in A and its trading community, TC, can be rationalized by the same utility function, then, in the intended interpretation of the axioms, it must be the case that either

\[
\begin{align*}
    (L_A/ K_A) &= (L_A^c/ K_A^c) = (L_{TC}/K_{TC}) \text{ or} \\
    (L_A/ K_A) &< (>) (L_{TC}/ K_{TC}) \text{ if and only if } (L_A/ K_A) < (>) (L_A^c/ K_A^c)
\end{align*}
\]
For the country in question, A, in the theory universe of HO 1 – HO 8,

\[(q/w)[(K_A/L_A) - (K^C/L^C)] = [(qK_A/wL_A) - (qK^C/wL^C)].\]

In the data universe, the value of the right-hand side equals

\[
\left[ \sum_{1 \leq i \leq 4} (Q_i + (A_5i/A\sum_5)Q_5) / \sum_{1 \leq i \leq 4} (W_i + (A_5i/A\sum_5)W_5) \right] -
\]

\[
\left[ \frac{\left( a\left( b_{K1} + b_{K5}b_{51} \right) + \left( b_{K3} + b_{K5}b_{53} \right)b_{31} + \left( b_{K4} + b_{K5}b_{54} \right)b_{41}\right)\right)}{\left( a\left( b_{L1} + \left( b_{L5}b_{51} \right) + \left( b_{L3} + b_{L5}b_{53} \right)b_{31} + \left( b_{L4} + b_{L5}b_{54} \right)b_{41}\right)\right)}\right] /
\]

Now, \(a/(1-a) = X_1^d/X_2^d\) and \(X_1^d/X_2^d = 0.2678\). Thus, \(a = 0.2112\). This value of \(a\) and the estimated values of the components of \(B\) imply that the value of the right-hand side of the last difference equals 1,2639. Since the value of this ratio is smaller than the value of the left-hand side of the same difference, 1,2647, I can conclude that

\[(IIIb) \text{ In 1997-Norway capital was the relatively abundant factor of production}\]

If Leamer’s assumption is not too way off the mark, (IIIb) and (I) and (II) above throw doubt on the empirical relevance of H&O’s conjecture.

**IX. Concluding Remarks**

It can be demonstrated that, except for rounding errors that multiply in products and fractions, the logical consequences of HO 1-HO 7 and HO 9-HO 11 are data admissible. Hence I have managed to isolate the H&O conjecture in staging its trial. It, therefore, looks like the conclusion to be drawn from the two case studies is that the H&O conjecture has little empirical relevance for the 1997 Norwegian Trade Flows. However, looks may deceive. Testing the empirical relevance of a poorly specified conjecture is difficult. My empirical analysis of H&O’s conjecture confronts a family of models of HO 1-HO 8 with data. When formulating these axioms I did not describe the contours of a two-country world economy that would have allowed me to delimit the family of models of the present theory universe. Consequently, I cannot be sure that the estimates of the relevant parameters that my data yield belong to a model of HO 1-HO 8 that pertain to a country in trade equilibrium with the rest of the world.
Footnotes

1. If I had been a numerical analyst, I would have been ashamed of presenting the equilibria in Cases I and II without providing accuracy measures for the values of the respective components. However, for the purposes of the chapter the accuracy measures are not important. Hence, I shall leave it to the reader to get an indication of the inaccuracies by computing, as I have done, the values of the missing variables. The values of the missing variables in the Case I trade equilibrium are as follows:

\[(P_3, P_4, x_3^A, x_4^A, x_3^B, x_4^B) = (1.07, 1.13, 7.73, 20.82, 10.35, 8.49)\]

2. In the Case II autarky equilibrium in A the missing values are as follows:

\[(P_3, P_4, x_3^A, x_4^A) = (1.8, 1.8, 7.61, 20.95)\]

3. In the Case II autarky equilibrium in B the missing values are as follows:

\[(P_3, P_4, x_3^B, x_4^B) = (1.5, 1.6, 10.35, 8.42)\]

4. In the Case II trade equilibrium the missing values are as follows:

\[(P_3, P_4, x_3^A, x_4^A, x_3^B, x_4^B) = (1.01, 1.05, 7.73, 20.82, 10.35, 8.49)\]


6. Axioms HO 9-HO 12 concern the bridge principles that I have adopted for a test of Heckscher and Ohlin's conjecture. They are stated and discussed in Section VI.
7. This judgment is in accord with Edward Leamer's definition of a relatively abundant factor (Leamer, 1980, p. 497).

8. In different words, my data satisfy the relations that the logical consequences of HO 1-HO 7 and HO 9-HO 11 predicate.

References


