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Environmental Input-Output Analysis, Structural Decomposition Analysis and Uncertainty

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Abstract

Environmental Input-Output Analysis (EIOA) is a tool for environmental analysis of broad classes of sectoral activities, taking into account indirect effects in other sectors in the supply chain. The core of EIOA is an input-output table (IOT) and national accounting matrix including environmental accounts (NAMEA) for a fixed base-year. We evaluate the uncertainty in EIOA using a time series of current-price IOT and NAMEA for 13 years from 1990 to 2002. We find annual variations in the current-price IOT and NAMEA which may represent realistic changes in production or measurement error. We assume the changes are errors and apply a regression analysis to remove the trends from the underlying data and estimate the uncertainty in the raw IOT. We then calculate the emissions for various final users and sectors to estimate the uncertainties from typical EIOA investigations. Using Monte-Carlo analysis, we then investigate how well the variations in the current-price IOT and NAMEA over time may represent uncertainties. The IOT can be further converted into constant-price data for use in Structural Decomposition Analysis (SDA). We prepare three kinds of the constant price; raw constant-price data constructed from raw current price data, smooth constant-price data constructed by smoothing the raw constant-price data, and smooth constant price data constructed from smooth current price data. By comparing the variations in these three sets of constant-price data we assess how errors in the current-price data may effect constant-price data and hence SDA.

Keywords: Uncertainty, Constant prices, Structural Decomposition Analysis, Monte-Carlo.

1. Introduction

Environmental input-output analysis (EIOA) is a common methodology to evaluate the environmental repercussions of economic activity, i.e: Shu and Kagawa (2005). Its extension to Structural Decomposition Analysis, i.e: Hoekstra and van der Bergh (2002) allows the analysis of changes in emissions, economic structure, and final consumption over time. The core of an SDA is a time series of constant price input-output tables (IOT) and national accounting matrix including environmental accounts (NAMEA).

In EIOA it is rarely discussed how uncertainty in the data may lead to inaccurate analysis and policy recommendations. It is expected that the underlying data has some uncertainty, but this is rarely quantified, i.e: Lenzen (2001). Currently, we are not aware of any statistical offices that regularly publish the uncertainty in IOT or NAMEA. This is undoubtedly a key reason behind the lack of uncertainty analysis.

The uncertainty in IOA does not solely depend on the underlying data. Other factors will greatly affect the outcome of an analysis such as the selection of the base-year, the level of aggregation, and so on i.e: Lenzen (2001). Little attention is given to how results may vary depending on the base-year---was it unusually hot or cold, oil-prices lower or higher than average, reduced demand due to economic downturn, and so on. In addition, SDA studies require constant price IOT which are generally constructed using time series of current price IOT. Uncertainty will propagate from the current price IOT through to the constant price IOT and onto any SDA results. Consequently, a simple measure of the uncertainty in a given entry in an IOT for a fixed base-year may not be adequate for a full evaluation of the uncertainty in EIOA.

Uncertainty in the data may have several implications and even provide some opportunities. An initial question resulting from uncertain data is how it may affect the outcomes of analyses. For instance, will policy recommendations differ if the 2000 IOT is used instead of the 2001 data? What is the uncertainty in a particular result? Or will the randomness of errors throughout the data tend to cancel each other out, i.e. Peters (2007)?

There are three aims in this study. First, we introduce various smoothing algorithms to analysis uncertainty in the current price data. Monte-Carlo analysis estimates the uncertainty in the underlying data based on the smooth data. Second, we compute raw constant price data constructed from raw current price data, raw-smooth constant price data constructed by smoothing the raw constant price data and smooth constant price data constructed from smooth current price data. Here we discuss differences of the constant price movements, difficulty to choose a base year and results by different base years. Finally, we compare the results of SDA constructed from three constant prices. We show variations of the three results and discuss a risk of the results computed from data with variations and the necessity of smoothing in the forepart.

1.1 The Norwegian IOT

Statistics Norway (SSB) provided input-output tables, energy and environmental flow tables for 13 years from 1990 to 2002. There are 64 industry sectors and five final consumers---household, governments, capital formation, changes in stocks, and export. The IOT are constructed using the fixed product sales structure assumption (industry-technology assumption with industry-by-industry tables). In this paper we denote by $\mathbf{Z}$ monetary value matrix (64×64 matrix) and by $\mathbf{y}$ final demand matrix (64×5 matrix). The energy flow tables consist of 64 industry sectors and 16 fuel-types. Likewise, the environmental stressors cover 64 industry sectors. The current price data requires some manipulations to make the entire time series---from 1990 to 2002---consistent. Most manipulations are specific to the Norwegian IOT.

An initial analysis of the time-series data provided by SSB shows that considerable variation exists in some data points over time. Some of the variations may represent realistic changes in structure, but it is also likely that noise plays a role. It is natural that noise appears in observed data due to inaccurate reporting, poor data quality, measurement error, human error, and so on. In the case of the energy data we could not find correlations between different fuel-types; for example, substitution between different fuel-types due to price variations. Some of the variations may also relate to changes in reporting definitions. Overall, an initial analysis of the data suggests that uncertainty is a significant cause in the annual variations in data.

2. Smoothing Algorithms

To understand the uncertainty in the reported IOT using time-series we smooth the IOT using regression to determine variations about the expected values. Removing noise in this way also makes it easier to reveal trends.

To analyze a series of data, we assume that it may be represented by a trend plus noise:

$$y_i = ax_i + b + \varepsilon_i \tag{2.0}$$

where y_i is the measured variable, x_i are the independent variables, time in this study, a and b are constants, and ε_i are independent and randomly distributed "errors". Depending on the structure of the data, we sometimes use piecewise linear trends. Before smoothing, we classify the data into various types depending on the structure. In this paper, we call raw data and values calculated from the underlying data " *raw data*" and the results obtained from the smoothing algorithm "*smooth data*". This is often described as $smooth(z)$ or $smooth(y)$.

2.1 Theory

In this study, we apply a regression analysis to clear noise from the underlying data. The method of least squares is the most common linear regression analysis, which has an objective function,

$$\min. \qquad J = \sum_t (x_{(t)} - f(x_{(t)}))^2 \tag{2.1.1}$$

The objective minimizes the total square error between observing x and predicting $f(x)$ and consequently it is sensitive to outliers. We use robust regression to reduce the importance of outliers in the regression analysis, i.e: Fischler (1981) and Torr (1997).

Although the data is smoothed with robust regression, the smoothed data can still have problems. In particular, the data may have structures which are not detected by the robust regression and the regression may produce negative numbers when in IOT z_{ij} and y_{ik} are non-negative (except for “changes in stocks”'). The underlying data often has various structures that should remain in the smoothed data, fig 3-1-1. For instance, sudden changes may occur due to economic shocks, technology innovation, or even new reporting procedures or definitions. If we apply robust regression analysis in these cases the regression functions do not sufficiently represent the features of the data. Therefore, before performing the regressions we classify the data according to three distinct structures.

2.1.1 Classification 0: Standard robust regression

The least squares method minimizes the total square error and may be inefficient and biased in some cases. The least squares method is influenced to a higher degree by a few outliers. With a robust regression, less weight is placed on outliers, producing quite a different trend to the least squares method. Since many of our data set has large outliers, our standard method is a robust regression.

2.1.2 Classification 1: Piecewise linear fit

If the data does not satisfy classification 0, we next assess whether a piecewise linear fit is better. Since we only have a maximum of $M=13$ data points in our time-series, we only consider two segments. We partition the data into two parts to represent the features using the following procedures:

1) Divide the data into two segments from 1 to t and from $t+1$ to M ($2 \le t \le M-2$) and calculate total correlation as

$$correl(x_{(1)} : x_{(t)}) + correl(x_{(t+1)} : x_{(M)}) \qquad \textbf{(2.1.2)}$$

where $correl(x_{(1)} : x_{(t)})$ represents a correlation of a series of x from i to k. There are $M-3$ possible combinations.

2) Find a combination $correl_{(\max)}$ which maximizes eq.(2.1.2).

3) Partition the data with $correl_{(\max)}$ as the border.

4) Apply the robust regression analysis to each cluster.

2.1.3 Classification 2

Classification 1 is a method to divide data to maximize the total correlation. If we have data as dotted data in fig 3-1-1 left, then classification 1 would not divide the first 3 data from the last 10 data as solid line in the figure. Classification 1 does not catch sight of the features of the data as the correlation is small. Typically, this structure applies when there is a change in definition in the IOT. Therefore we propose the second classification as follows:

1) Find the maximum distance $d_{\max}$ in $d = \left| x_{(t)} - x_{(t-1)} \right| (2 \le t \le M)$. The number of combinations is $M-1$.

2) Partition the data with $d_{\max}$ as the border.
3) Apply the robust regression analysis to each cluster.

2.1.4 The smoothing algorithm

Figure 2-1-1

Overall, the smoothing procedure for the data follows figure 2-1-1, where ε in the figure means very small positive number and the error rate is given as

$$\frac{\sum_{t=1}^{M}\left|g(x_{(t)})-x_{(t)}\right|}{\max(x)-\min(x)} \tag{2.1.3}$$

where $g(x)$ represents the robust regression function with the modification for inappropriate values (such as zero). If the robust regression has a power correlation, then the error rate for the different classifications is determined. The algorithm with the lowest error rate is chosen.

3. Variation of underlying data

In section 3.1 and 3.2, we show the variations of underlying data and compare raw data and smooth data. In section 3.3, we discuss the variation of the errors between raw and smooth data and inspect usefulness of the errors.

3.1 Variation of final demand

Figure 3-1-1

In figure 3-1-1, the raw data are represented by circles and sold lines show the smoothed data constructed by the algorithm in figure 2-1-1. The left figure shows export on casting of metal and the right represents household expenditure on insurance and pension funding. Although the raw data have variations, the smooth data represents the features without the noise. By the classification, the features are well-represented.

Figure 3-1-2

Similar variations are found in the raw energy and emissions data. In figure 3-1-2, the circles show the total energy consumption (the sum of the different energy types) and the solid lines represent the sum of smoothed consumption of each fuel type. Thus, we are comparing the sum of smoothed components and not the individual points. The figures not only show how energy consumption can vary significantly in each sector, but also show that the features of the total energy consumption are retained with the classification and smoothing methods.

3.2 Variation in EIOA

This section compares standard EIOA calculations using the raw data and the smooth data. This process gives an indication in the likely errors caused by using the noisy raw data compared to using the smooth data.

In each of the following figures we show the results of the calculation

$$\mathbf{f} = \mathbf{F}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{y} \qquad \textbf{(3.2.1)}$$

for the raw and the smoothed data where $\mathbf{F}$ represents energy consumption intensity and $\mathbf{A}$ is technical coefficient matrix, i.e: Leontief (1986). The “raw” calculations simply substitute the raw data into each of the variables to obtain $\mathbf{f}$. For the “smooth” calculations, $smooth(z_{ij})$, $smooth(x_i)$, $smooth(y_{ik})$ and $smooth(f_{il})$ are used to calculate $smooth(\mathbf{A})$ and $smooth(\mathbf{F})$ where $smooth(x_i)$ represents a sum of individual smoothed flow from sector 1 to 64 ($\sum_{j=1}^{64} smooth(z_{ij})$). Consequently, the whole data set is smoothed, then the EIOA calculations are performed (in other words, $smooth(\mathbf{f})$ is not the results of smoothing $raw(\mathbf{f})$). Thus the following comparisons give an indication of how “average” values in the IOT may differ from a fixed-base year.

Figure 3-2-1

The calculations show how the noise in the raw data may affect the results. In figure 3-2-1 left, energy consumption for the export of renting of machinery and equipment is gradually increasing although the point in 2000 is an outlier. Thus, if 2000 was used as a base year, the energy consumption in this sector might be twice as much as if the smooth data was used. The energy consumption for the export of sea and coastal water transport, figure 3-2-1 right, shows a lot of scatter from 1990 to 2002. Depending on the base year, the emissions can vary about 40% either side of the median. These figures collectively demonstrate that the choice of base-year may effect results and hence policy recommendation.

Table 3-2-1

Table 3-2-1 shows the average errors for estimating the difference between raw and smooth energy consumption, where the error rate is defined as:

$$\frac{\sum_{t=1}^{13} \left| f_{(t)} - smooth(f_{(t)}) \right|}{\left(\max(smooth(f_{(t)})) - \min(smooth(f_{(t)})) \right) \times 13} \qquad \textbf{(3.2.2)}$$

For example, 0.164 for exports represents the sum of the absolute differences divided by the range of the values (to make the different final demands comparable). The average error for government & NPISH is larger than others, because of two quite large outliers. Except for these outliers, it is almost same as others.

3.3 Monte-Carlo Analysis

With a mean based on the smooth data and error estimates for each cell in the IO data Monte-Carlo analysis can be performed. In the Monte-Carlo analysis we use the uncertainty distributions of the monetary value of the flow z_{ij}, final demand y_{ik} and energy flow f_{il} and the normalize the data and calculate the energy consumption using Eq. (3.2.1). This process ensures that the IO data remains balanced and consistent. Because the data takes non negative numbers, we assume the data follows a log-normal distribution, with the mean from the smooth data and the standard deviation from the average of the difference between the raw and smooth data. We use $N = 1000$ in the Monte-Carlo simulation and we survey the average number of data within $\pm\sigma$ centering around $smooth(f)$. Where, σ is standard deviation from $raw(f) - smooth(f)$. The Monte-Carlo analysis gives the distribution of the variations is more peaked than in the normal distribution in table. 3-3-1.

Table 3-3-1

Figure 3-3-1

The Monte-Carlo analysis can be applied to individual sectors to show the uncertainty in a standard IO calculation. Figure 3-3-1 show a Monte-Carlo analysis applied to the export of oil extraction and the household consumption of land transport. In the figures, circles and crosses represent the raw and smooth data and the bars show $\pm\sigma$ from the average by Monte-Carlo analysis. One standard deviation gives the expected coverage of the raw data. This confirms that the use of Monte-Carlo analysis describes the variations in the data.

4. Constant price data

When we compare consumptions at two different periods, it is difficult to directly compare the consumptions and grasp the change because consumption consists of two variances, price and quantity. Indices are generally used to compare prices / quantities at different periods by fixing quantities / prices.

More widely used indices are Laspeyres and Paasche indices and chained index is more representative of economic variations, i.e Statistic New Zealand (1998) United Nations (1993). They have two kinds of indices, price index and quantity index. Price index shows the change of prices by fixing the quantities and quantity index represents the movement of quantities by fixing the prices. The consumption data converted by the price or quantity indices are called constant price data or constant quantity data. Here we mainly describe the Laspeyres constant price data and chained constant price data.

4.1 Laspeyres fixed-base indices

The perhaps most common indices in use, and most certainly the most simple ones, are the Laspeyres and Paasche indices. Suppose information on the price and quantity of n output is available for period $t = 1 \cdots M$. Denoting the output price and quantity vectors in period t as $\mathbf{p}^t = (p_1^t, \cdots, p_i^t, \cdots, p_n^t)$ and $\mathbf{q}^t = (q_1^t, \cdots q_i^t, \cdots, q_n^t)$. The Laspeyres price index ($P^t{}_L$) is computed using the following formula:

$$
\begin{aligned}
P_L^t &= p_1^t \frac{q_1^0}{\sum_{i=1}^n p_i^0 q_i^0} + p_2^t \frac{q_2^0}{\sum_{i=1}^n p_i^0 q_i^0} + \cdots + p_n^t \frac{q_n^0}{\sum_{i=1}^n p_i^0 q_i^0} \qquad \textbf{(4-1-1)} \\
&= \sum_{i=1}^n w_i^0 \frac{p_i^t}{p_i^0}
\end{aligned}
$$

where $w_i^t = p_i^t q_i^t \Big/ \sum_{j=1}^{n} p_j^t q_j^t$ is output i's nominal output share. It shows that the Laspeyres output price is the period 0 share-weighted sum of price ratios. The Paasche output price index uses period t prices as the weight, in contrast to the Laspeyres output price index that use period 0 prices as weight.

When we wish to grasp time series price changes with a common year, constant price is constructed from P_L^t or P_P^t. Assuming that we already have computed an index, the value of consumption with quantity in period t and price in base period zero, $\sum_{i=1}^{n} p_i^0 q_i^t$, is computed as the value of current price consumption in period 0 times P_L^t index with period 0 as the base year and period t as the end year:

$$\sum_{i=1}^{n} p_i^t q_i^0 = P_L^t \times \sum_{i=1}^{n} p_i^0 q_i^0 \qquad \textbf{(4-1-2)}$$

When computing values of consumption in each period with the same base period and quantity, we get a time series of consumption measured with quantity in constant period 0.

4.2 Chained index

Although the above fixed-weight indices are common and simple, when studying long time series, the structure of prices and quantities in the base period become progressively less relevant as one is moving further away from the base period, and the bias of fixed-weight indices is likely to increase correspondingly. The reason is the information on price movements and weighting changes in the intervening periods are ignored.

This problem may be overcome by changing base period from time to time. A chained index compares prices between two periods taking into account information on weighting changes in the intervening periods, i.e Statistic New Zealand (1998) United Nations (1993). Rebased P_L price indices are shown below:

$$\begin{aligned} P_L^{1/0} &= \frac{\sum_{i=1}^{n} p_i^1 q_i^0}{\sum_{i=1}^{n} p_i^0 q_i^0} \\ &\vdots \\ P_L^{t/t-1} &= \frac{\sum_{i=1}^{n} p_i^t q_i^{t-1}}{\sum_{i=1}^{n} p_i^{t-1} q_i^{t-1}} \end{aligned} \qquad \textbf{(4-2-1)}$$

The next step is to link the indices for each period, which have individually different base periods, together to a continuous time series of indices with a common base period.

This procedure is called chaining, and is done by multiplying the indices for adjacent periods:

$$P_{LCH}^{t/0} = P_L^{1/0} \times P_L^{2/1} \times \cdots \times P_L^{t/t-1} \qquad \textbf{(4-2-2)}$$

Here the subscript *LCH* indicates the Laspeyres chained index. In general the result is different from the fixed base index. The expenditure in a period t with a base period 0 by a chained index is

$$\sum_{i=1}^{t} p_i^t q_i^0 = P_L^{1/0} \times P_L^{2/1} \times \cdots \times P_L^{t/t-1} \times \sum_{i=1}^{n} p_i^0 q_i^0 \quad \textbf{(4-2-3)}$$

Economical changes since a base year is included in the calculation of indices in each period by updating the weights. Therefore it is advantage that chained index reflects the change of economical structure since the base year as compared with the Laspeyres index which has a fixed weight. In this study we adopt the Laspeyres chained index for constant price.

4.3 Raw, raw-smooth and smooth constant price data

Figure 4-3-1

Although we perform chained constant price, it is difficult to how to specify a base year. The comparison of export on manufacture of basic precious and non-ferrous metals with base year 1999 and 2000 is shown in figure 4-3-1. Because of sudden increase of the price in 2000, the raw constant price with base year 2000 is much higher than that with 1999. While the smooth constant prices with base period 2000 is not subject to it and similar to the constant price with base year 1999. It might make easy to choose a base year.

Here we compute constant price with base year 2000 and compare the difference between raw constant price data constructed from raw current price data, raw-smooth constant price data constructed by smoothing the raw constant price data and smooth constant price data constructed from smooth current price data.

Figure 4-3-2

Figure 4-3-2 shows household expenditure on extraction of crude petroleum and natural gas and manufacture of machinery and equipment n.e.c. respectively. The figures compare three constant prices. In figure 4-3-2 left, raw constant price (black circles) is nearly zero before 1994 and increases constantly after sudden boost in 1995 although it has some variations. The feature is well-represented with two straight lines by the raw-smooth constant price (solid line). On the other hand, the smoothed constant price data dotted line represents gradual rise with a curve. The similar tendency is shown in figure 4-3-2 right. The raw-smooth data linearly represents increase until 1997 and decrease after 1998. The slope of the line from 1990 to 1997 may be a little steep because of an outstanding data in 1997. The smooth data shows it with a gradual curve and is not concerned by the point in 1997.

Figure 4-3-3

Capital formation on manufacture of other non-metallic mineral products is shown in figure 4-3-3. It clearly shows the difference of two ways to represent the features. The raw-smooth data shows the decline of the data from 1991 to 2002 and then the points from 1995 to 1997 are regarded errors. Meanwhile the smooth constant price shows the rise tendency from 1995 to 2002 with a curve.

To compare the raw-smooth and the smooth constant prices to the raw constant price we define the difference as follows:

$$\frac{\sum_{t=1}^{M}\left|smooth(y_{(t)}) - raw(y_{(t)})\right|}{\left(\max(raw(y_{(t)})) - \min(raw(y_{(t)}))\right) \times M} \quad \textbf{(4-3-1)}$$

It is the difference between raw and smooth constant prices. For example the differences in figures 4-3-2 left and right and 4-3-3 are 0.117, 0.122, and 0.149. The difference between raw and raw-smooth constant prices is computed with the same definition and the results in the figures are 0.072, 0.064 and 0.199 respectively.

Table 4-3-1

Table 4-3-1 is averages and medians of the differences of 64 sectors by each final demand. As shown in Eq. (4-3-1), it shows the distance from the raw data. It is natural that the raw-smooth data fits the raw constant price more than the smooth data because the raw-smooth data is smoothed based on the raw constant price.

The averages of the raw-smooth constant price are similar to the medians, while the averages of the smooth constant price are much larger than the medians. It means differences between the raw and raw-smooth constant prices distribute as normal distribution, whereas a few of the smooth constant price have large differences from the raw constant price. In the other words, the smooth constant prices fit to the raw constant data as well as the raw-smooth constant data except for a few data.

Besides, the smooth constant price represents the feature with a curve and it may be more realistic price movement than to be represented by one or two liner functions. In addition, to compare smooth constant price to raw-smooth constant price, we define the dissimilarity as follows:

$$\frac{\sum_{t=1}^{M}\left|smooth(y_{(t)}) - raw\_smooth(y_{(t)})\right|}{\left(\max(raw\_smooth(y_{(t)})) - \min(raw\_smooth(y_{(t)}))\right) \times M} \quad \textbf{(4-3-2)}$$

Table 4-3-2

The average and median of the dissimilarity are shown in table 4-3-2. The median is much less than the average. It means smooth constant prices are very similar to raw-smooth constant price except for a few constant prices. For instance, the dissimilarities of figures 4-3-2 left, right and 4-3-3 are 0.074, 0.127 and 0.379 respectively. It is said the smooth constant price is relatively similar to the raw-smooth constant price and approximate it with realistic curves. Besides the raw-smooth constant price may have variation by the base period same as the raw constant price in figure 4-3-1, because it is constructed from the raw constant price. Consequently the smooth constant price is not influenced by the variations that is brought by difference of the base year, and shows the movement of prices with realistic curve.

4.4 EIOA with constant price data

Figure 4-4-1

We compute energy consumption with the same manner as current price energy consumption. Figures 4-4-1 left and right show energy consumptions of capital formation on machinery and equipment n.e.c and export on other business activity. The dotted lines (energy consumption from smooth constant price data) are smoother than the solid line (energy consumption from raw-smooth constant price data) though it is not smooth curve.

Table 4-4-1

Table 4-4-2

We compute the difference and dissimilarity with eq.(4-3-1) and eq.(4-3-2) and the averages and medians are shown in tables 4-4-1 and 4-4-2. The averages of differences between energy consumptions from the raw and raw-smooth constant prices are almost same as the median of those, while the averages of difference between the raw and smooth constant price are quite larger than the medians. That is the differences between the raw and smooth constant prices are not so much different from those between the raw and raw-smooth constant prices except for some data. The same can be said in Table 4-4-2. The medians of dissimilarity are about half of the average and 0.1 or thereabout. It means many of the energy consumption from the smooth constant price are similar to those from the raw-smooth constant price. The dissimilarities in figures 4-4-1 left and right are 0.103 and 0.129 respectively.

Here it is important that the energy consumption constructed from the raw and the raw-smooth constant prices may be vulnerable to difference of base years.

5. SDA

There are some techniques that are used for decomposing the development in emission indicators at the sector levels. Structural decomposition analysis (SDA) is one of the techniques, i.e: Rørmose (2005) and Hoekstra (2002). SDA has been applied, for example, to analyze the demand and technological driving forces of energy use, CO2 emission and various other pollutant and resources.

5.1 Mathematical derivation

Environmental stressor / energy indicators are introduced by three variances, environmental stressor / energy intensity, changes in absolute quantities and output elasticity.

$$\mathbf{f} = \mathbf{F} \cdot \mathbf{L} \cdot \mathbf{y} \qquad \textbf{(5-1-1)}$$

Where $\mathbf{L} = (\mathbf{I} - \mathbf{A})^{-1}$ is the Leontief inverse matrix. The additive decomposition form is given as

$$\Delta\mathbf{f} = \Delta\mathbf{F} \cdot \mathbf{L} \cdot \mathbf{y} + \mathbf{F} \cdot \Delta\mathbf{L} \cdot \mathbf{y} + \mathbf{F} \cdot \mathbf{L} \cdot \Delta\mathbf{y} \qquad \textbf{(5-1-2)}$$

The first term, the intensity effect, measures the influence of changing physical flow per unit of monetary output. The second and third effects are the input-output coefficient and final demand effects.

The discrete approximation of a continuous integral function of $\Delta\mathbf{f}$ is represented by the parametric equation

$$\Delta\mathbf{f} \approx (\mathbf{w}_{1(t-1)} + \alpha_1 \cdot \Delta\mathbf{w}_1)\Delta\mathbf{F} + (\mathbf{w}_{2(t-1)} + \alpha_2 \cdot \Delta\mathbf{w}_2)\Delta\mathbf{L} + (\mathbf{w}_{3(t-1)} + \alpha_3 \cdot \Delta\mathbf{w}_3)\Delta\mathbf{y} \quad \textbf{(5-1-3)}$$

where the w represents weights and the α are parameters. The sizes of weights are determined by their value in period $t-1$ and t and the parameter. The parameter is specified by a choice in index. For example, when $\alpha = 1$, $w_{1(t)}\Delta\mathbf{F} + w_{2(t)}\Delta\mathbf{L} + w_{3(t)}\Delta\mathbf{y}$. It leads to the Paasche index because it use the previous year as weights. If $\alpha = 0$, it means the Laspeyres index because base year is used for weights. The decomposition in Eq.(5-1-3) can be decomposed into more than three determinant effects.

5.2 Model for Norwegian SDA

With three constant price, raw constant price, raw-smooth constant price and smooth constant price in section 4-3, decomposition is carried out of changes in emission between 1990 and every single year from 1991 to 2002 subsequently. The base year is kept constant at 1990, the target year gradually run through the entire time span.
The model for this SDA study is defined as:

$$\mathbf{f} = P \cdot \mathbf{F} \cdot \mathbf{L} \cdot \mathbf{y}_s \cdot \mathbf{y}_{v/cpt} \qquad \textbf{(5-2-1)}$$

The notations in eq.(5-2-1) are as follows:

P : Population
$\mathbf{F}$: Emission or energy consumption intensity
$\mathbf{L}$: Leontief inverse, Inversed matrix of intermediate deliveries
$\mathbf{y}_s$: Final demand coefficient ($y_{sij} = y_{ij} / \sum_i y_{ij}$)
$\mathbf{y}_{v/cpt}$: Per capita final demand.

With eq.(5-2-1), it is possible to use the decomposition method laid out in section 5.1 to get the changes of isolated elements.

Although weights can be calculated by economic method in eq.(5-1-3), here we introduce a way to derive them with the structural decomposition method. The additive identity splitting method is used to get an idea of what the w's should be:

$$
\begin{aligned}
\Delta f &= f_1 - f_0 \\
&= P_1F_1L_1y_{s1}y_{v/cpt1} - P_0F_0L_0y_{s0}y_{v/cpt0} \\
&= \Delta PF_1L_1y_{s1}y_{v/cpt1} + P_0F_1L_1y_{s1}y_{v/cpt1} - P_0F_0L_0y_{s0}y_{v/cpt0} \\
&= \Delta PF_1L_1y_{s1}y_{v/cpt1} + P_0\Delta FL_1y_{s1}y_{v/cpt1} + P_0F_0L_1y_{s1}y_{v/cpt1} - P_0F_0L_0y_{s0}y_{v/cpt0} \\
&\quad\vdots \\
&= \Delta PF_1L_1y_{s1}y_{v/cpt1} + P_0\Delta FL_1y_{s1}y_{v/cpt1} + P_0F_0\Delta Ly_{s1}y_{v/cpt1} + P_0F_0L_0\Delta y_sy_{v/cpt1} + P_0F_0L_0y_{s0}\Delta y_{v/cpt}
\end{aligned}
\quad \textbf{(5-2-2)}
$$

Each of components represents the contribution of the Δ-component to the total change in f. We notice the pattern of the coefficients. But this pattern is not unique. If the number of variance increases, the pattern would be a huge number. Dietzenbacher and Los (1998) propose that a way to reduce the variance is to look at the mean of so-called "mirror images". Let n be number of variable and k represent the number of subscript 0 values in a coefficient. According to Dietzenbacher and Los (1998), the number of coefficient for each k and the weights are given by eq.(5-2-3) and. eq. (5-2-4) respectively.

$$\frac{(n-1)!}{[(n-1-k)!\cdot k!]} \quad \textbf{(5-2-3)}$$

$$(n-1-k)!\cdot k! \quad \textbf{(5-2-4)}$$

In Norwegian SDA, the results of eq.(5-2-3) and eq.(5-2-3) with $n=5$ are represented in table 5-2-1.

Table 5-2-1

Making reference to table5-2-1, Δf for Norwegian SDA is computed as the average of all 120 components represented by 16 different decomposition of f as follows:

$$
\begin{aligned}
\Delta f = {}& \tfrac{1}{120}\cdot[\\
&(24\Delta P F_0 L_0 y_{s0} y_{v/cpt0} + 24 P_0 \Delta F L_0 y_{s0} y_{v/cpt0} + 24 P_0 F_0 \Delta L y_{s0} y_{v/cpt0} + 24 P_0 F_0 L_0 \Delta y_s y_{v/cpt0} + 24 P_0 F_0 L_0 y_{s0} \Delta y_{v/cpt}) +\\
&(6\Delta P F_0 L_0 y_{s0} y_{v/cpt1} + 6 P_0 \Delta F L_0 y_{s0} y_{v/cpt1} + 6 P_0 F_0 \Delta L y_{s0} y_{v/cpt1} + 6 P_0 F_0 L_0 \Delta y_s y_{v/cpt1} + 6 P_0 F_0 L_0 y_{s1} \Delta y_{v/cpt}) +\\
&(6\Delta P F_0 L_0 y_{s1} y_{v/cpt0} + 6 P_0 \Delta F L_0 y_{s1} y_{v/cpt0} + 6 P_0 F_0 \Delta L y_{s1} y_{v/cpt0} + 6 P_0 F_0 L_1 \Delta y_s y_{v/cpt0} + 6 P_0 F_0 L_1 y_{s0} \Delta y_{v/cpt}) +\\
&(6\Delta P F_0 L_1 y_{s0} y_{v/cpt0} + 6 P_0 \Delta F L_1 y_{s0} y_{v/cpt0} + 6 P_0 F_1 \Delta L y_{s0} y_{v/cpt0} + 6 P_0 F_1 L_0 \Delta y_s y_{v/cpt0} + 6 P_0 F_1 L_0 y_{s0} \Delta y_{v/cpt}) +\\
&(6\Delta P F_1 L_0 y_{s0} y_{v/cpt0} + 6 P_1 \Delta F L_0 y_{s0} y_{v/cpt0} + 6 P_1 F_0 \Delta L y_{s0} y_{v/cpt0} + 6 P_1 F_0 L_0 \Delta y_s y_{v/cpt0} + 6 P_1 F_0 L_0 y_{s0} \Delta y_{v/cpt}) +\\
&(4\Delta P F_0 L_0 y_{s1} y_{v/cpt1} + 4 P_0 \Delta F L_0 y_{s1} y_{v/cpt1} + 4 P_0 F_0 \Delta L y_{s1} y_{v/cpt1} + 4 P_0 F_0 L_1 \Delta y_s y_{v/cpt1} + 4 P_0 F_0 L_1 y_{s1} \Delta y_{v/cpt}) +\\
&\qquad\qquad\qquad\qquad\qquad\qquad \cdots \qquad\qquad\qquad\qquad\qquad\qquad +\\
&(24\Delta P F_1 L_1 y_{s1} y_{v/cpt1} + 24 P_1 \Delta F L_1 y_{s1} y_{v/cpt1} + 24 P_1 F_1 \Delta L y_{s1} y_{v/cpt1} + 24 P_1 F_1 L_1 \Delta y_s y_{v/cpt1} + 24 P_1 F_1 L_1 y_{s1} \Delta y_{v/cpt})]
\end{aligned}
\tag{5-3-4}
$$

5.3 Comparison of SDA

In this section, we compare decompositions from the three constant prices in section 4-3. In figures 5-3-1 and 5-3-2, the three figures are decompositions computed from the raw constant price, the raw-smooth constant price and the smooth constant price respectively.

Figure 5-3-1

The bold lines in figure 5-3-1 indicate the total change in CO2 emissions in Norway from export as compared to the level in 1990. In the figure from the raw constant price, this line declines from 1990 to 1993 and repeats increase and decrease until 2002. CO2 in 2002 is 10% more than in 1990. In figures from the raw-smooth and the smooth constant prices, these lines are very cloth to zero until 1995 and increase gradually after sudden increase in 1996. The reason of the sudden increase is the rise of emission intensity. In 1996 consumption of heavy fuel oil for sea and coastal water transport suddenly increased about 30%. The consumption of heavy fuel oil gradually decreases after 1996, on one hand middle distillates increase gradually. Therefore after increase of emission intensity in 1996, it declines not rapidly but slowly. In the figure from the raw-smooth constant price CO2 increases to 13% for 13 years from 1990 to 2002, while the figure from the smooth constant price shows CO2 increase of 18%. The most significant in terms of increasing the CO2 emissions is per-capita final demand. The three figures show the isolated effect from per-capita final demand is increases of more than 40% in CO2 emissions. In the figure from the raw constant price, it is shown by rough line and they are shown by gradual curve and straight line in figures from the raw-smooth and smooth constant prices.

Figure 5-3-2

Figure 5-3-2 shows decomposition of energy consumption. The three figures indicate the total energy consumptions of more than 5% increase for 13 years. It is shown with rough line in the figure from the raw constant price and linearly in the figures from the raw-smooth constant price. In the figure from the smooth constant price, the line is close to zero from 1990 to 1992 and increase with relatively steep sloop after that. The only one decline element is emission coefficient. The emission coefficient from the raw constant price has variations and that from the raw-smooth constant price shows constant decline. In the figure from the smooth constant price, it has a sharp decline from 1990 to 1992 and gradual decrease after that. As comparing these three results, the trends are similar as a whole. But when we wish to predict the change in a short term, it is difficult to do it with the result from the raw constant price because the changes in each year are variable. Besides, we should remember that results from the raw and raw-smooth constant price may have variations by the base year.

Table 5-3-1

The total changes of CO2 in 2002 as compared to the level in 1990 and the ratios by each isolated effect are shown in table 5-3-1. The results from the raw constant price shows total increase in CO2 emission of 7.95 Mt in 2002 compared to 1990. The results from the raw-smooth and smooth constant prices represent increase of 8.71 Mt and 10.00 Mt respectively. The result from the raw constant price is smaller than others. As shown in figure 5-3-1 left, total change of CO2 emission moves up and down and it declines in 2002. When we discuss a result from the raw constant price at one period, we should mind that it is noisy and may include temporary increase or decrease. For example, the emission from the raw constant price in 2001 compared to 1990 would be much different from that in 2002 as shown in figure 5-3-1 left. On this point, both of the results from the raw-smooth and smooth constant prices are reliable. But the result from the raw-constant price may have problem by the base year.

In the results from the raw constant price, export accounts for 46.3% of the total CO2 increase (3.68Mt). Other results also show export is a serious factor in causing a rise of CO2 emission. Export account for about 70% of whole CO2 emission in the results from the smooth constant price. The decomposition results show per capita final demands in export bring much increase of CO2. For instance, the table from the raw constant price represents per capita final demands in export generate 216.4 % of total CO2 emission (216.4% of 7.95Mt). Overall, improvement of emission intensity decreases CO2 emission but emission by per capita final demands increase much more than the reduction.

Eventually it is said when we compare the figures and table from three different constant prices, the result from smooth constant price is the most reliable.

6. Conclusion

We have proposed an algorithm for time-series data and discussed variations of IOT. Monte-Carlo analysis was used to replicate the uncertainty. In section 4 we computed constant price data with a Laspeyres chained index and showed the importance of the base year from data with variations. By comparing three kinds of constant price data, it was revealed that smooth constant price data shows the most realistic movement of expenditure and energy consumption. Comparison of decompositions of CO2 and energy consumption from the three constant price data was discussed in section 5. We showed that the results of the decomposition with raw constant price have variation and it may bring large errors. The results from the raw constant price and raw-smooth constant price data have a risk in choosing the base year. Overall, it was shown the analysis of data with variations and the necessity of removing the variation at the first stage. Analysis with uncertainty is an important area of future work.

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Tables:

Table 3-2-1

Export	Households	Government	Capital formation
0.164	0.175	1.685	0.174

Average error between raw and smooth energy consumption

Table 3-3-1

	Export	Households	Government	Capital formation
$\pm\sigma$	68.1	70.7	76.4	77.8
$\pm 2\sigma$	92.7	94.1	95.6	94.8
$\pm 3\sigma$	98.3	98.7	99.1	98.4

Distribution of data with noise

Table 4-3-1

		Export	Households	Government	Capital formation
average	raw-smooth	0.087	0.090	0.067	0.085
	smooth	0.226	0.153	0.130	0.126
median	raw-smooth	0.082	0.092	0.071	0.087
	smooth	0.144	0.127	0.092	0.101

Average of difference between raw and raw-smooth / smooth constant prices

Table 4-3-2

	Export	Households	Government	Capital formation
average	0.215	0.133	0.120	0.112
median	0.132	0.100	0.064	0.038

Average of Dissimilarity

Table 4-4-1

		Export	Households	Governme	Capital formation
average	raw-smooth	0.110	0.119	0.100	0.102
	smooth	0.246	0.187	0.197	0.166
median	raw-smooth	0.101	0.115	0.095	0.099
	smooth	0.148	0.156	0.118	0.117

Comparison of energy consumption from raw constant price
and that from raw-smooth / smooth constant price

Table 4-4-2

	Export	Households	Governme	Capital formation
average	0.249	0.199	0.175	0.154
median	0.128	0.134	0.09	0.089

Comparison of energy consumption from raw-smooth constant price
and that from smooth constant price

Table 5-2-1

k	subscript for the components in the coefficient				# of different coefficient	weitht
	1st	2nd	3rd	4th		
0	0	0	0	0	1	24
1	0	0	0	1	4	6
	0	0	1	0		
	0	1	0	0		
	1	0	0	0		
2	0	0	1	1	6	4
	0	1	0	1		
	1	0	0	1		
	0	1	1	0		
	1	0	1	0		
	1	1	0	0		
3	0	1	1	1	4	6
	1	0	1	1		
	1	1	0	1		
	1	1	1	0		
4	1	1	1	1	1	24

Number of coefficients and their weights

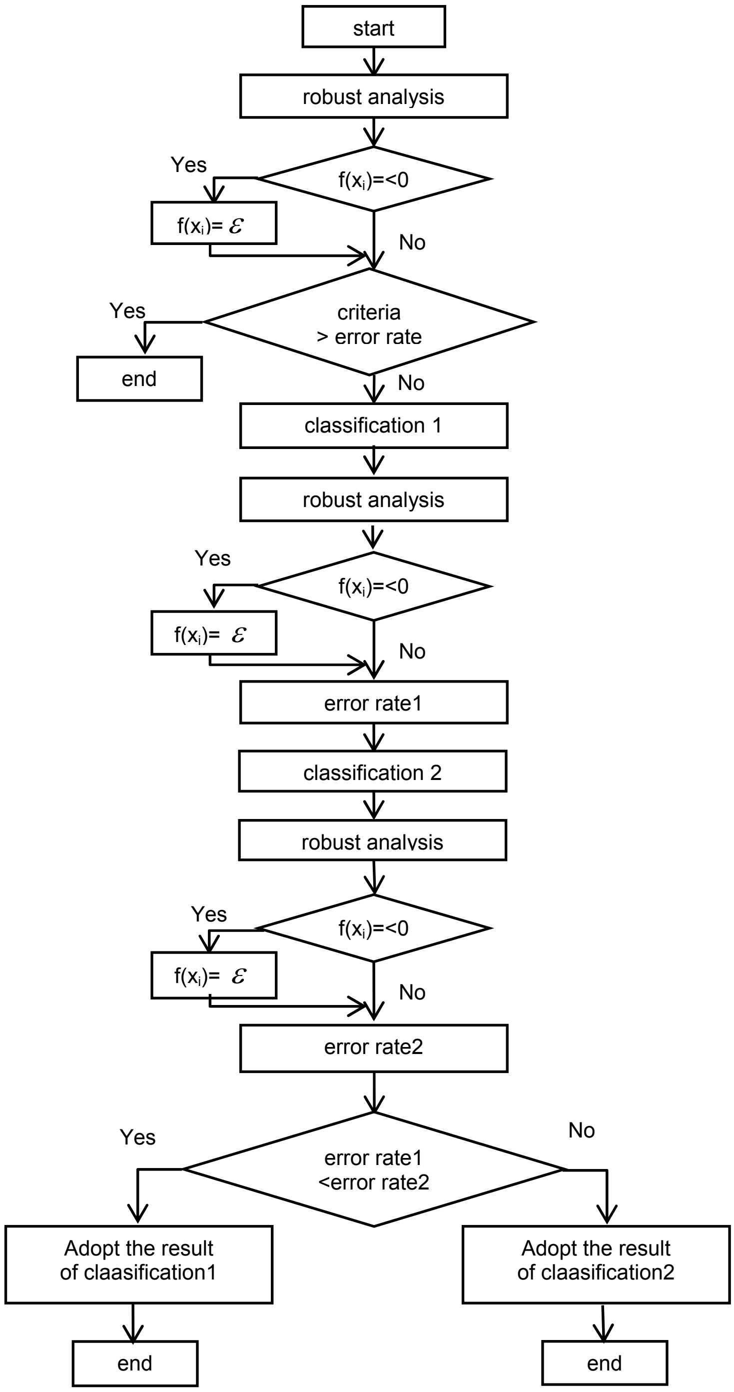
Table 5-3-1

		total (%)	total (Mt)	ΔP (%)	ΔF (%)	ΔL (%)	Δy_s (%)	$\Delta y_{v/cap}$ (%)
raw constant price	Export	46.3	(3.68)	28.1	-94.9	-4.7	-98.5	216.4
	Households	19.3	(1.53)	5.8	-16.4	7.6	-0.4	22.7
	government NPISH	9.3	(0.74)	2.1	-4.7	2.3	1.4	8.2
	GFCF	-0.5	(-0.04)	2.8	-6.6	-0.4	-6.0	9.8
	CIS	25.6	(2.03)	-1.0	2.3	-0.6	63.3	-38.3
	total (%)	100.0		-37.8	-120.4	4.1	-40.2	218.7
	total (Mt)		(7.95)	(3.00)	(-9.57)	(0.33)	(-3.19)	(17.38)
raw-smooth constant price	Export	61.9	(5.39)	25.6	-66.8	-9.3	-85.3	197.6
	Households	20.3	(1.77)	5.2	-13.4	7.2	0.3	21.1
	government NPISH	7.8	(0.68)	1.9	-3.4	2.6	1.0	5.8
	GFCF	3.4	(0.29)	2.5	-5.3	-0.2	-7.0	13.3
	CIS	6.7	(0.59)	-0.1	0.3	0.9	52.6	-47.0
	total (%)	100.0		35.1	-88.6	1.1	-38.4	190.8
	total (Mt)		(8.71)	(-3.05)	(-7.72)	(0.10)	(-3.34)	(16.62)
smooth constant price	Export	69.2	(6.94)	21.8	-68.0	-13.5	-55.6	184.5
	Households	19.2	(1.93)	4.5	-9.3	4.7	2.2	17.1
	government NPISH	5.4	(0.54)	1.5	-3.6	2.1	0.4	5.0
	GFCF	-1.5	(-0.16)	2.4	-7.2	0.0	-8.0	11.4
	CIS	7.7	(0.78)	-0.2	5.7	-0.3	41.3	-38.7
	total (%)	100.0		29.9	-82.4	-7.0	-19.8	179.3
	total (Mt)		(10.03)	(3.00)	(-8.27)	(-0.70)	(-1.98)	(17.98)

Change of CO_2 emission, 2002 compared to 1990

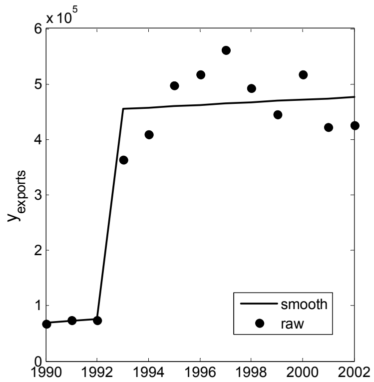
Figures:

Figure 2-1-1

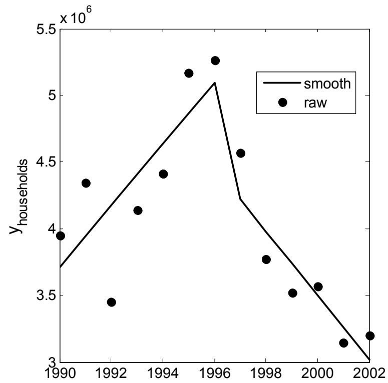


Smoothing algorithm

Figure 3-1-1

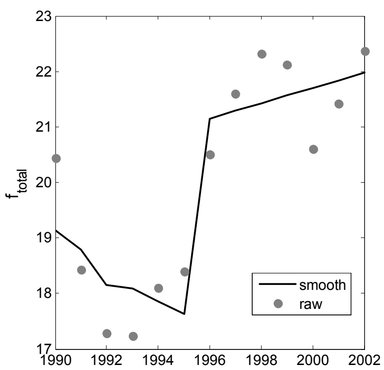


Export on casting of metals

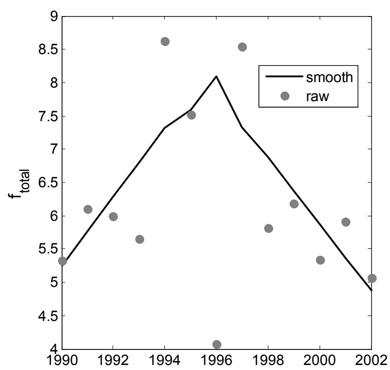


Household expenditure on insurance and pension funding, except compulsory social security

Figure 3-1-2

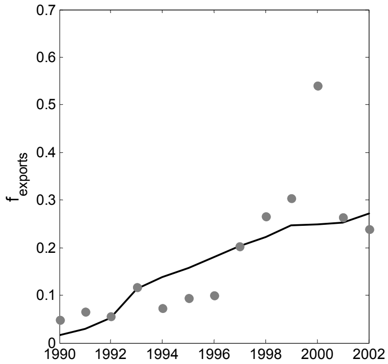


Total energy consumption by fishing, operation of fish hatcheries and fish farms; service activities incidental to fishing

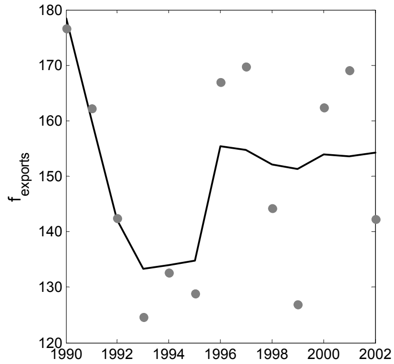


Total energy consumption by production and distribution of electricity

Figure3-2-1

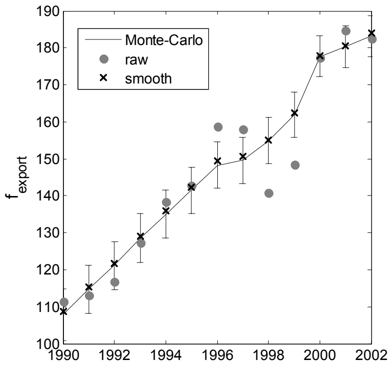


Export energy consumption on renting of renting of machinery and equipment without operator and of personal and household goods

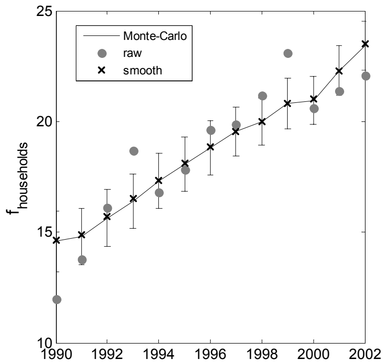


Export energy consumption on sea and coastal water transport

Figure 3-3-1

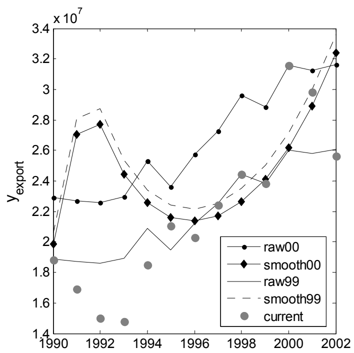


Export energy consumption on extraction of crude petroleum and natural gas; service activities incidental to oil and gas extraction excluding surveying



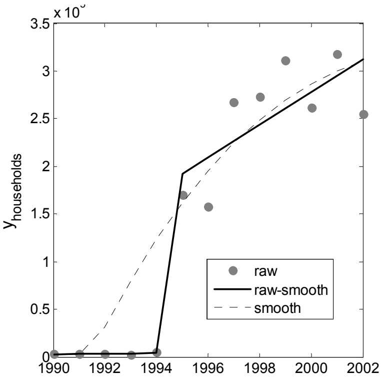
Household energy consumption on other land transport

Figure 4-3-1

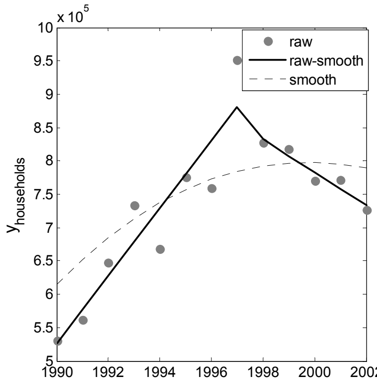


Difference of constant price by base year

Figure 4-3-2

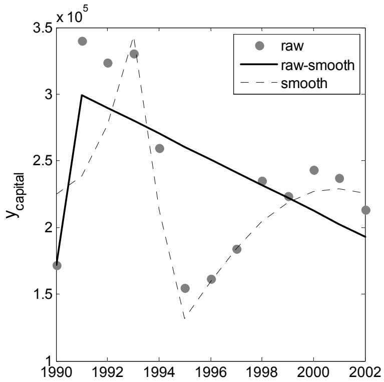


Household expenditure on extraction of crude petroleum and natural gas; service activities incidental to oil and gas



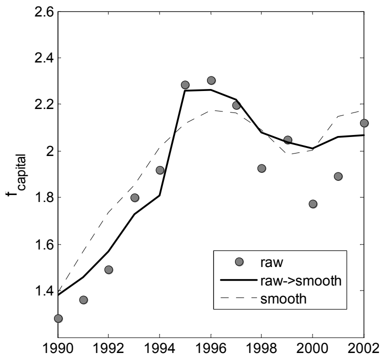
Household expenditure on manufacture of machinery and equipment n.e.c.

Figure 4-3-3

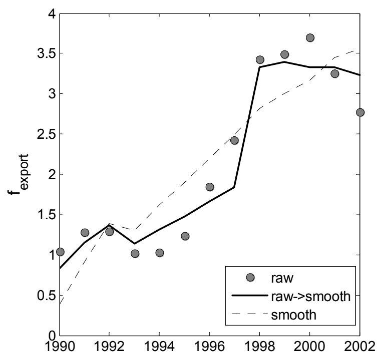


Capital formation on manufacture of manufacture of other non-metallic mineral products

Figure 4-4-1

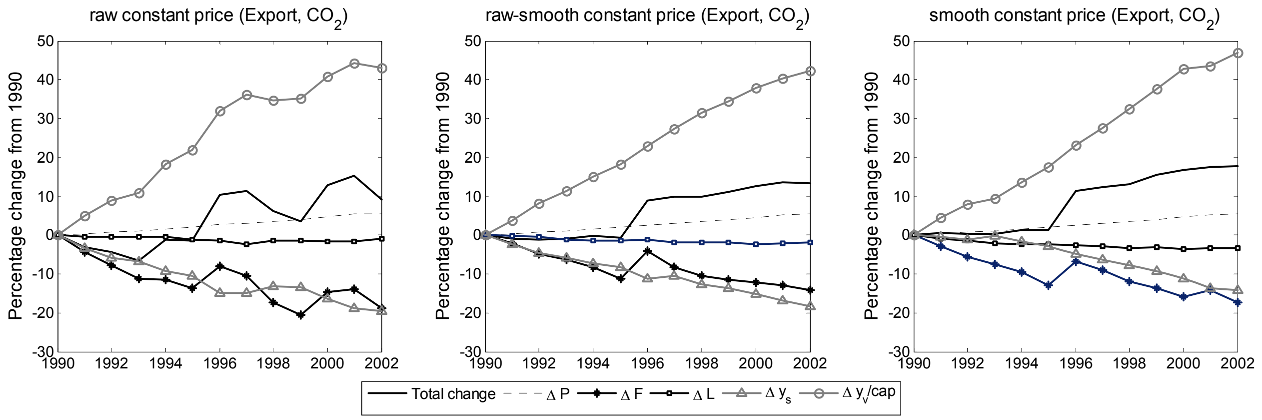


Energy consumption of capital formation on manufacture of machinery and equipment n.e.c.



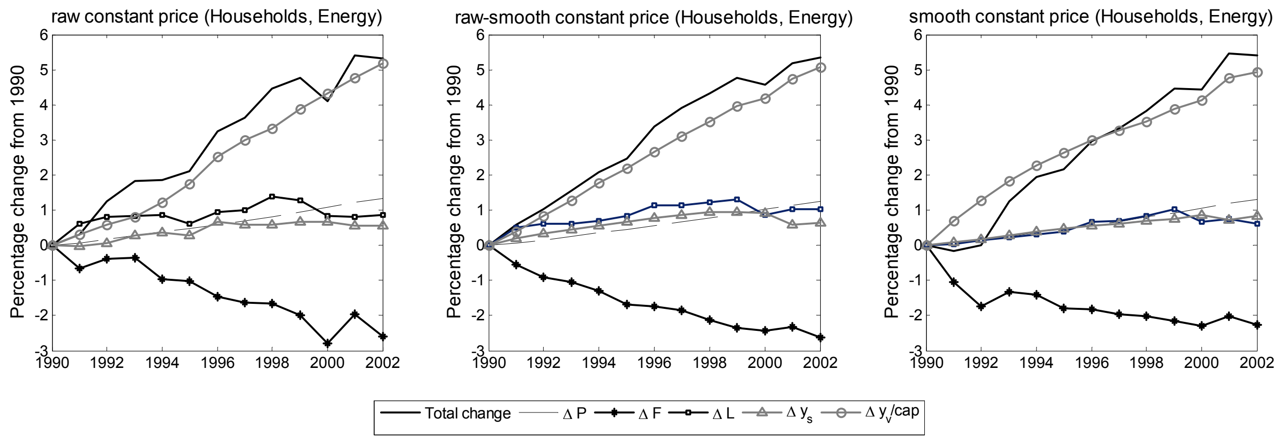
Energy consumption of export on other business activity

Figure 5-3-1



Decomposition of CO2 emission from export

Figure 5-3-2



Decomposition of energy consumption from household