Grading the I-O Coefficients Importance. A Fuzzy Approach.

A. Morillas*, L. Robles & B. Díaz
Universidad de Málaga

Facultad de CC Económicas

Plaza El Ejido s/n

29071 Málaga

Spain

*** Corresponding author: morillas@uma.es**

Abstract

In inter-industry studies, the coefficients of the production function matrices have been analyzed with different techniques in order to recognize in some way those coefficients that can be considered to be important for an economy. Many critics have been posed to the procedures, the most remarkable one being their lack of connectivity with the values of the absolute flows behind the coefficients. In our approach, we define the importance of a coefficient as a fuzzy concept, and the grade of importance takes into account those absolute flows. This grade can be considered as a membership function, which is used to define a fuzzy graph associated to an I-O matrix. We apply this new procedure to the Spanish 2000 I-O matrix and compare our results to those reached by classical methods.

1 Introduction

The relationship between the so called Leontief's or input-output economic model and graphs theory has been exploited since the seventies of the last century - see references [2],[6],[7],[8], [14], [16],[17] and [18]-. Valued and qualitative graphs have been used to explain the inter-relationship structure and the influence between the economic sectors, as they offer and demand economic goods and services one from each other.

The most common formulation for this model (demand model) is $x = (I - A)^{-1} y$, where $y \geq 0$ is the final demand vector, $x \geq 0$ the final production vector, and A is the technique coefficients matrix, $a_{ij} \in A$. These coefficients represent the proportion of the merchandise from the *i*-sector which is used by the *j*-sector to produce a unity of its own merchandise: $a_{ij} = \frac{x_{ij}}{x_j}$. So,

$0 \leq a_{ij} < 1$ and $\sum_{i=1}^{n} a_{ij} < 1$ to guarantee the economy is productive (there is added value). It also assumes that it is no-decomposable, therefore $|I - A| \neq 0$.

On the other hand, , it is possible to define a distribution coefficient as $b_{ij} = \frac{x_{ij}}{x_i}$, the proportion of merchandise from the *i*-sector that it is sold to the *j*-sector. The offer model is then: $x = (I - B)^{-1} y$, with $b_{ij} \in B$ and similar conditions for the model and coefficients. It is easy to demonstrate that, in fact, both models are related, since $a_{ij} = b_{ij} \frac{x_i}{x_j}$.

In the graph associated to this model, basically, it is assumed the sectors are the graph nodes. If $a_{ij} \neq 0$, then an edge exists reaching the *i*-node from the *j*-node, meaning that the *j*-sector demands the merchandise produced by the *i*-sector. When each edge in the graph is valued by its corresponding a_{ij}, we get a valued graph: the *absolute influenc*e graph. If the values are the b_{ij} coefficients, then we get the *relative influence* graph [17]. Also, it is possible to deal with a directed or qualitative graph by assigning a value 1 if $a_{ij} \neq 0$ ($b_{ij} \neq 0$) and 0 in other case (the edge does not exists in this case). Mostly, these kinds of graphs are applied to structural analysis.

All of these graphs are crisp. Nevertheless, when we are talking about the “importance” of elements, for example coefficients, and this concept is not univocally defined, as we will see it happens in the literature on the subject, we get obviously imprecision in its management. In our opinion, it could be a good and practical idea to introduce the fuzzy graph associated to this situation, as something eclectic that allows measuring the “grade of importance” of a coefficient in a formal, although context-dependent way. This is the principal goal of this paper. First, a literature review about the important coefficients will be made. Then, a definition for the fuzzy graph associated to an input-output matrix will be presented. Finally, a case study of the Spanish economy will be developed.

2 Antecedents. Measuring the importance of the coefficients in an I-O table.

It can be observed that in any country’s intermediate matrix[1] *Z*, the number of large flows is relatively low. In the Spanish IO tables for the year 1995, for example, the 18 highest

---

[1]This matrix shows the connections between the different branches of an economy. The rows of this matrix consist of the outputs, which concern resources supplied by a given sector to each of the sectors of activity. The columns consist of the inputs from the different sectors, i.e. the consumption per sector required for production.

intermediate flows comprise 25% of the total, and the 82 highest ones entail half, while the remaining ones (4818) account for the other 50% [12]. We have found similar figures for the 2000 tables, with the 20 highest intermediate flows accounting for 25% and the 4808 lowest ones for 50% of the total. On the other hand, analyzing the coefficients matrices, it could be assumed that the biggest or more “important elements” will be those backed by large intermediate flows, but as we shall see, this is not always the case. There is a simple reason underlying this statement: the fact that coefficients are ratios in which the denominator is the production. As a result, they only measure the relationship between numerator and denominator, with independence of their values. Thus, a coefficient a_{ij} may be large and at the same time belong to a branch of little importance and in consequence its influence will be minor despite its large size.

Sensitivity studies have been applied to classify the coefficients according to their importance or influence, highlighting those than can cause the highest change in production. In consequence, a difference has been established between “important coefficients” and simply big coefficients. These important coefficients, often just a small number of them, have been called *Most Important Coefficients* (MICs). The importance or influence of a sector within the productive system is assessed according to the number of MICs that it contains. .

The MICs are those coefficients whose relative variations cause a bigger error or deviation in terms of total production of the branches of activity – [4],[9],[10] and[15]-. In these studies, a coefficient a_{ij} is important if a variation of the coefficient under 100% provokes a change that is greater than a pre-established level p% – 0.5% or 1% is generally used – in the total production of some of the branches. In the literature, different authors have classified the MICs according to their size/importance, establishing thus different groups, amongst which they highlighted, for example, those with the smallest r_{ij}, (less than 20%) because of their special relevance, calling them “the most important of the important ones”.

The studies on the sensitivity of the coefficients has usually been carried out by computing w_{ij}, the degree of importance of coefficient a_{ij}, in the following way: $w_{ij}(p) = a_{ij}\left[l_{ji}\,p + 100\left(\frac{l_{ii}}{x_i}\right)x_j \right]$, where p is the maximum percentage of variation that it will provoke in the production of any sector x_j (in other words, “*acceptable”* limit of error), l_{ij} an element of the inverse matrix $(I-A)^{-1}$ and x_j the production of sector j.

Nevertheless, if we analyze the definition of w_{ij}, if self-consumption (l_{ii}) is eliminated from the matrix Z, as is usually done for different reasons, the l_{ii}, element of the diagonal of the inverse matrix, will be equal to 1. In addition, the result of the product $a_{ij}\ \frac{x_j}{x_i}$ will be the allocation coefficients $b_{ij.}$ As a consequence, the definition of w_{ij} stated above basically consists on the addition of very small $l_{ij}\frac{p}{100}$ numbers to coefficients b_{ij} . Thus, MICs could in fact be considered as elements that are very close to the biggest or most significant b_{ij}, and so, for this purpose, size and importance would be directly related. *If* b_{ij} *is big* – or r_{ij} small – *the corresponding* a_{ij} *is important*.

Studies of coefficient sensitivity in the 1995 and 2000 SIOT (Simetric I-O tables) produce 480 MICs for 1995 and 504 for 2000[2], approximately 10% of the table in both cases, which is a small but usual number in this kind of works.

Elasticity interpretation (b_{ij} can be considered elasticities) is a delicate issue and can reflect situations that differ greatly if it is not analyzed in the context of the absolute numbers with which it is calculated. The increase in production will depend on the absolute values of the flows that are behind the coefficients. This is the reason why it might be advisable to differentiate MICs depending on whether or not they are backed by large intermediate flows.

3 Fuzzy Graph Associated to an I-O Matrix (FUGA)

As we have just discussed, there is not yet a consensus on how to define the "important coefficients", and many of the solutions offered have serious limitations. In our opinion, importance is not a precise, but a fuzzy concept, which admits a graduation in its conceptualization or definition. In that sense, we consider that the use of fuzzy logic and a membership function is the appropriate way to tackle this issue.

MICs and coefficients b_{ij} are so similar, that we are going to use coefficients b_{ij} matrix, B (their empirical correlations are almost exactly one). These coefficients have a clearer interpretation in terms of elasticities. On the other hand, the fact that in [12] the MICs obtained for the TIOE95 (9.7% of the coefficients, 480 coefficients), and the 504 MICs obtained for the TIOE2000 account for almost the 77% of the intermediate consumption, highlights the first conclusion when looking at them, that all the coefficients that are not trivial (very close to 0) are considered important, and that is in our opinion, the worst critic that can be made to a technique that is trying to find the important coefficients in and I-O table. In fact, as they mention, for the ones with a low intermediate consumption, the interest that they can have based on their high

---

[2] We have replicated the study carried out in [12] for the TIOE2000, resulting 504 MICs.

relationship is lowered by the low values they have. They carry out a sensibility analysis to re-classify those 480 coefficients into three groups to somehow solve the aforementioned problem.

In our approach, we take these two ideas into account, working on the construction of a membership function in which not only the value of the coefficient, but also the percentage o intermediate consumption that each coefficient represents, is taken into account.

We inspired our approach in the Lorenz curve and Gini index. Once we have ordered the coefficients in an ascending order, we compute the percentage of intermediate consumption accumulated until that coefficient, and that is the membership value to the "important coefficients cluster". We consider then the larger coefficients that account for the 50% of the internal consumption the "very important coefficients", VICs. This is now an objective definition of the important sectors in an economy, although, of course, a higher percentage can be considered.

In our application to the Spanish Input-Output tables (TIOE2000) with 70 sectors, (that is, 4900 coefficients) a 3% of the larger coefficients account for the 50% of the intermediate consumption (148 coefficients). The Lorenz curve can be seen in ANNEX1. All the higher MICS according to the sensibility analyses are included among them. Ours is a smaller set of coefficients and adds a grade of importance for each coefficient, so that no sensibility studies are needed to "label" or re-classify the coefficients.

The matrix with the memberships of each coefficient to the "important" coefficients group is then used to define a fuzzy graph. B is a finite set with elements $\{b_1,\ldots,b_n\}$. The pair[3] $G(B,\mu)$ is a crisp nodes fuzzy edges graph on B, where $B=\{b_i\}$ is a set of nodes and $\mu_{ij} : B\times B\rightarrow[0,1]$ is a fuzzy relation that defines the value of the edge going from node b_i to node b_j and represents the grade of importance of that connection. Since a fuzzy graph is an expression of a fuzzy relation, it is frequently expressed as a fuzzy matrix.

Different α-cuts in the fuzzy relation matrix can be considered as credibility thresholds of the importance of the coefficients. This way, only the remaining connections, with an importance degree higher than the value specified by the threshold, will be considered.

A brief summary of the algorithm follows.

The FUGA has the following features:

- It operates on the allocation coefficients matrix, B.

---

[3] Fuzzy graphs have been studied since its introduction in 1973 by Kaufman in [1], based on Zadeh's definition of fuzzy relations. Since then, many other contributions have been made to the field, see, for example: [3], [11], [13] or [19].

- It takes into account the percentage of the total intermediate inputs that each coefficient represents, by providing the “grade of importance” of the coefficients selected.
- It provides a visual display of the graph with the most important coefficients, for different $\alpha - cuts$ or levels of importance.
- It gives an intuitive and objective definition to select the very important coefficients, which are those representing the 50% of the total intermediate inputs.

The algorithm FUGA computes the membership of each coefficient b_{ij} to the cluster of “very important coefficients”, VICs. To reach that goal, these are the steps needed:

1.- Both the distribution coefficients matrix B and the intermediate consumption matrix Z are transformed into column vectors, with elements B_i and Z_i , $i=1...k*k$, being k the number of sectors in the I-O matrix considered.

2.- Column vectors B and Z are ordered according to the column vector B ascending order.

3.- For each coefficient, the membership to the “important” coefficients group is computed:

$$\mu_q = \frac{\sum_{i=1}^{q} Z_i}{\sum_{i=1}^{k \cdot k} Z_i}, \quad 0 \le \mu_q \le 1$$

4.- Initially, the coefficients with $\mu_q \ge 0.5$ are considered the VICs . They are the greater distribution coefficients that account for the 50% of the intermediate consumption. A higher threshold can be fixed, and a tighter cluster of important coefficients will be reached.

5.- A fuzzy importance graph associated to the I-O matrix can be plotted, with the memberships μ_q being the values for the edges connecting any two nodes. Different $\alpha - cuts$ can be considered.

4 Results for the 2000 Spanish Input-Output table

It is a common topic in the MICs literature to define the most important sectors in relation with the number of important coefficients they have [5]. So, in order to simplify the exposition of our results, we only present here the partial results regarding these more important sectors and their relationships. Using as a criterion to define important sectors those having a number of VICs above the average, we have identified them for the Spanish Economy and obtained its associated importance fuzzy graph. In this process, we have respected the hierarchies of influence among sectors resulting in the complete VICs reduced fuzzy graph. This is the reason why there are twelve levels in the causal structure of the graph (ANNEX 2). The importance degrees for the edges in this fuzzy graph have been plotted in three categories, standing for

Low, Medium and High importance, respectively. The exact value of the importance degree for each one of the edges can be seen in ANNEX 3.

In any reduced graph, the strong connected components or blocks in its initial graph are considered as a new node, in order to ensure a strict causal structure. There are five of them in our case of study. The sectors for each component are in the legend in ANNEX 2, and the names of the 70 branches considered in the 2000 I-O table are presented in ANNEX 4. The first one is related wit agriculture activities (c1); the second one with terrestrial transport (c2); the third one with business and communications services (c3); the forth one with motor vehicles (c4), and, finally, the fifth with metal products (c5). Actually, any statistic description of the Spanish economy should describe them as the most important clusters of sectors. So, we can say the obtained graph fits well the economic information on the subject.

Tree levels of importance degree have been plotted in the graph. The following considerations about the Spanish economy have to be done:

- Hotel and restaurant services (44), Real estate services (54), Construction (40) and the agricultural block (c1) are leading the hierarchy in the more important influence relationships. They define a dominating set in the graph.
- The most important subgraph, both in number of edges and in their importance degrees, is headed by Real estate services (54) and Construction sectors (40).
- The best influence transmitting node in the graph seems to be the Manufacture of chemicals and chemical products (23). It involves 11 VICs.
- Crude petroleum and natural gas (5) and Gas, steam and hot water supply (10) sectors are both very important as their products are much demanded. Also, Rubber and plastic products (26) and Glass (24) have a good number of VICs as demanded basic products.

All these characteristics are congruent with the literature describing the Spanish sectoral structure.

5 Conclusions

The research about the most important coefficients in an Input-Output table by using the well known MICs definition is not satisfactory enough. First of all, there is an evident relationship between that definition and the one given by the b_{ij} coefficients in the relative influence graph. In both cases, MICs and b_{ij}, are likely an elasticity. Second, the MICs definition does not take in account the absolute value of the transaction between two sectors. Third, the amounts of intermediate inputs they usually imply are almost equivalent to the total value of these inputs.

So, it cannot be justified to call them the “most important coefficients” when actually the only thing done is to throw away the insignificant ones.

Finally, because there is not any precise way to define the “importance degree”, we consider it is convenient to deal with a fuzzy approach. In this approach we have used the b_{ij} coefficients as a very adequate proxy variable of the MICs (their correlations always are very close to 1). By introducing the absolute value for each one of them, in our fuzzy definition of the importance degrees, a same-size coefficient can have an “importance” value quite different if it is supported by a higher or lower quantity of intermediate input. Furthermore, in our definition of the important coefficients underlies an intuitive idea: they will be those with higher elasticity and accounting the half of the total intermediate inputs. We have called them the Very Important Coefficients (VICs). Moreover, the α-cut concept can be used to study the graph in different importance degrees, always defined between 0 y 1, which is much more convenient.

References

[1] A. Kaufmann (1973). *Introduction a la th'eorie des sous-ensembles flous*. Paris. Masson.

[2] A. Morillas (1983): *La teoría de grafos en el análisis input-output*. Secretariado de Publicaciones. Universidad de Málaga.

[3] A. Rosenfeld (1975) "Fuzzy graphs". In: Zadeh, L.A., Fu, K.S., Shimura, M. (eds) *Fuzzy Sets and Their Applications*, Academic Press, New York, 77-95.

[4] A. V. Sebal (1974) *An analysis of the sensitivity of large scale input-output models to parametric uncertainties*, Center for Advanced Computation, nº 122, University of Illinois.

[5] A.S. García Muñiz, F. Aroche Reyes. & C. Ramos Carvajal (2007): Determinación de coeficientes importantes por niveles tecnológicos: una aproximación desde el modelo de Miyazawa. *Investigaciones Económicas*, Vol. XXXI(1), 161-190.

[6] J. Campbell (1972): “Growth pole theory, digraph analysis and the interindustry relation-ships”. *Tijdschrift Voor Eonomie en Social Geografie*. 62, pp. 69-87.

[7] J. Campbell (1975): “Application of graph theoretic analysis to interindustry relation-ships”. *Regional Science and Urban Economics*. 5, pp. 91-106.

[8] J. Hauknes (1999): “Norwegian Input-Output Clusters and Innovation Patterns”, en Roelandt, T. den Hertog, P. (eds). *Boosting Innovation: The Cluster Approach.* Organisation for Economic Cooperation and Development, Paris.

[9] J. Jilek (1971) The selection of the most important coefficients, *Economic Bulletin for Europe*, 23, pp. 86-105.

[10] J. Schintke & R. Stäglin (1985) Stability of import input coefficients, *Lecture notes in economics and mathematical systems*, vol. 251, pp. 129-139.

[11] J.N. Mordeson & P.S. Nair, (2000) *Fuzzy Graphs and Fuzzy Hypergraphs*. Studies in Fuzziness and Soft Computing, vol. 46. Springer-Verlag.

[12] L. Robles & J. Sanjuán (2005) Sectores claves. Coeficientes grandes y coeficientes importantes. *Proceedings of I Jornadas Españolas de I-O*. Oviedo, Spain, September 2005.

[13] M. Delgado, J.L. Verdegay & M.A. Vila, On valuation and optimisation problems in fuzzy graphs: a general approach and some particular cases. *ORSA J. Comput.* **2** 1 (1990), pp. 74–83.

[14] M. Gilli (1979): *Etude et Analyse des Structures Causales dans les Modéles Economiques.* Ed. Peter Lang, Berna.

[15] M. Sekulic (1968) Applications for input output models to the structural analysis of the Yugoslav economy, in: Fourth International Conference on Input Output Techniques, Geneva.

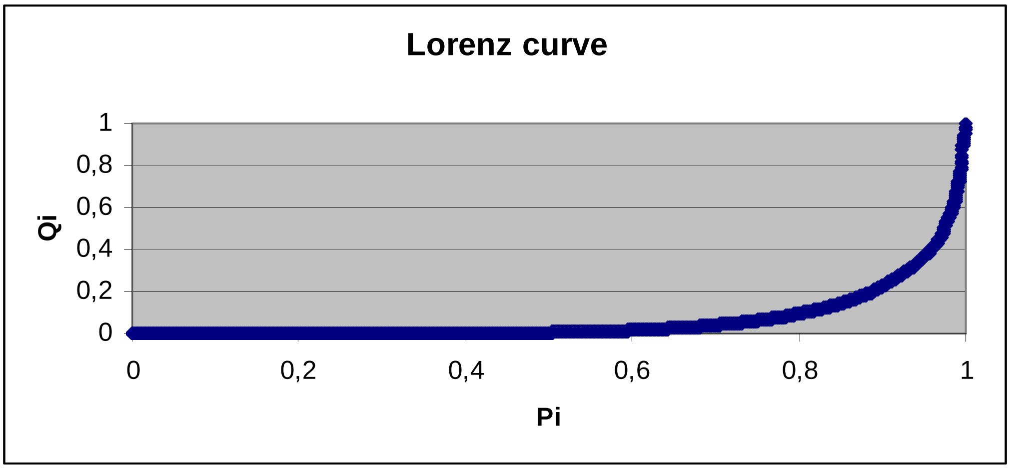
[16] P.B. Slater (1977): "The Determination of Functionaly Integrated Industries in the United State Using a 1967 Interindustry Flow Table". *Empirical Economics*, 2, pp. 1-19.

[17] R. Lantner (1972): "L'Analyse de la Dominance Economique", *Revue d'Economie Politique*, nº 2, pp. 216-283

[18] R. Lantner (2001): "Influence Graph Theory Applied to Structural Analysis", en Lahr, M.L. y Dietzenbacher, E. (eds.): *Input-Output Analysis: Frontiers and Extensions*, Palgrave, New York, cap. 15.

[19] R.T. Yeh & S.Y. Bang,(1975) Fuzzy relations, fuzzy graphs, and their applications to clustering analysis, in L.A. Zadeh, K.S. Fu, K. Tanaka, M. Shimura, eds: *Fuzzy Sets and their Applications to Cognitive and Decision Processes*, Academic Press, New York, 1975, pp. 125-149.

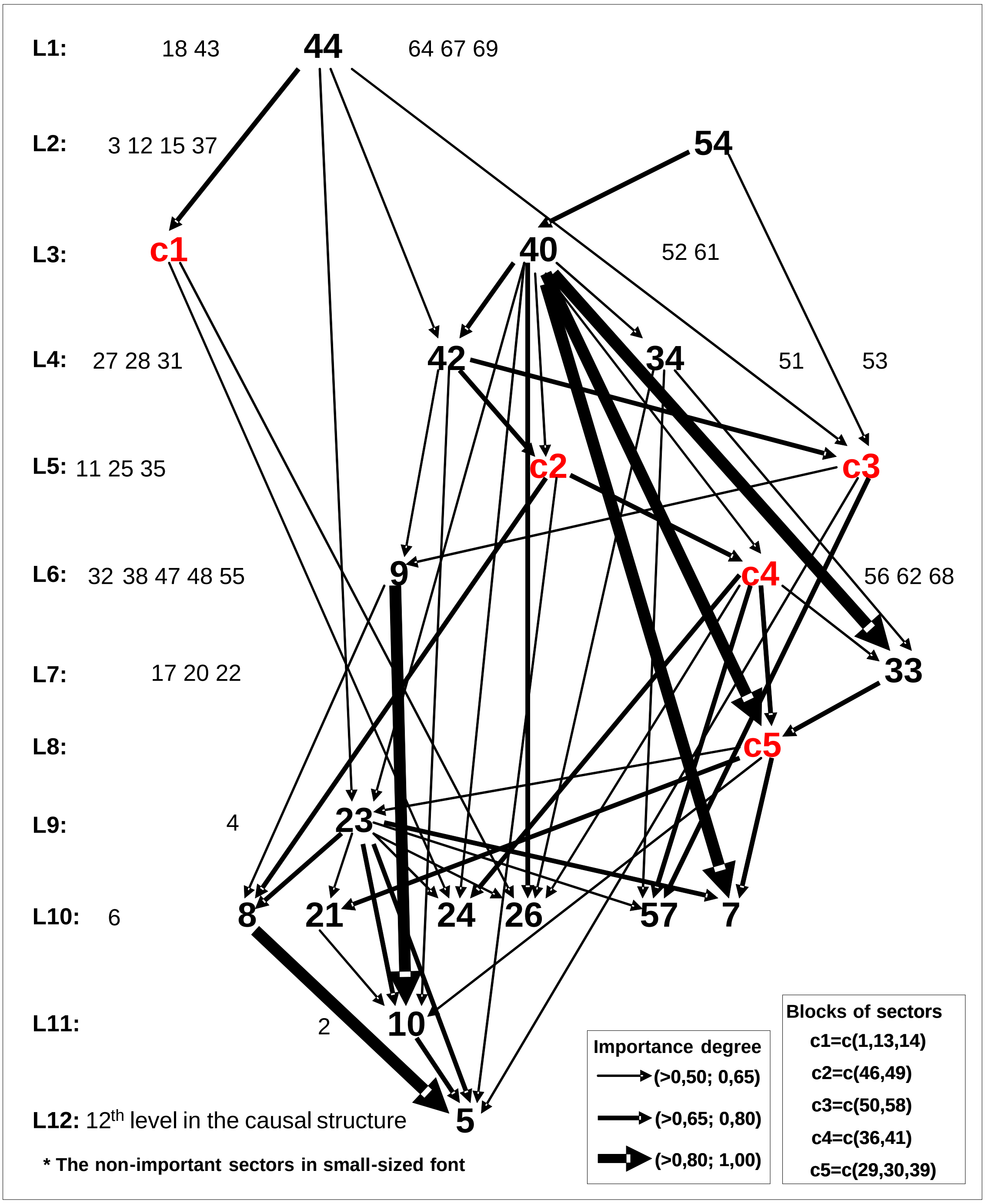
ANNEX 1



ANNEX 2

IMPORTANCE REDUCED FUZZYGRAPH*

The most important sectors and their causal relationships



ANNEX 3

REDUCED GRAPH IMPORTANCE DEGREES MATRIX

Most important sectors

	44	54	c1	40	34	42	c2	c3	9	c4	33	c5	23	7	8	21	24	26	57	10	5
44																					
54																					
c1	0.75																				
40		0.71																			
34				0.63																	
42	0.62			0.67																	
c2				0.59		0.74															
c3	0.52	0.62				0.65															
9						0.53		0.50													
c4				0.55			0.71														
33				0.85	0.52					0.58											
c5				0.88						0.79	0.68										
23	0.54			0.53								0.54									
7				0.89								0.72	0.73								
8							0.67		0.58				0.71								
21			0.60									0.68	0.57								
24				0.61						0.74			0.58								
26			0.63	0.76	0.52					0.60			0.51								
57					0.63			0.67		0.68			0.63								
10						0.50			0.80			0.57	0.67			0.53					
5							0.59	0.60					0.71		0.97					0.74	

ANNEX 4

1	Agriculture, livestock and hunting	25	Manufacture of cement, lime and plaster	49	Other transport related services
2	Forestry, logging and related service activities	26	Manufacture of glass and glass products	50	Post and telecommunications
3	Fishing	27	Manufacture of ceramic products	51	Financial intermediation
4	Mining of coal and lignite; extraction of peat	28	Manufacture of other non-metallic mineral products	52	Insurance
5	Extraction of crude petroleum and natural gas	29	Manufacture of basics metals	53	Activities auxilliary to financial intermediation
6	Mining of metal ores	30	Manufacture of fabricated metal products	54	Real estate activities
7	Other mining and quarrying	31	Manufacture of machinery and equipment n.e.c.	55	Renting of machinery, personal and household goods
8	Refined petroleum products	32	Manufacture of office machinery and computers	56	Computer and related activities
9	Production and distribution of electricity	33	Manufacture of electrical machinery	57	Research and development
10	Manufacture of gas	34	Manufacture of electronic equipment	58	Other business activities
11	Collection, purification and distribution of water	35	Manufacture of precision and optical instruments	59	Market education
12	Manufacture of meat products	36	Manufacture of motor vehicles	60	Market health and social work
13	Manufacture of dairy products	37	Manufacture of other transport equipment	61	Market sewage
14	Manufacture of other food products	38	Manufacture of furniture	62	Market recreational, cultural and sporting activities
15	Manufacture of beverages	39	Recycling	63	Other service activities
16	Manufacture of tobacco products	40	Construction	64	Public Administration
17	Manufacture of textiles	41	Sale and retail of motor vehicles	65	Non-market education
18	Manufacture of of fur	42	Wholesale trade and commission trade	66	Non-market health and social work
19	Manufacture of leather and leather products	43	Retail trade	67	Non-Market sewage
20	Manufacture of wood and wood products	44	Hotel and restaurant services	68	Non-market activities of membership organization
21	Manufacture of pulp, paper and paper products	45	Railway transport	69	Non-market recreational, cultural and sporting activities
22	Publishing and printing	46	Other land transport; transport via pipelines	70	Private households with employed persons
23	Manufacture of chemicals and chemical products	47	Water transport		
24	Manufacture of rubber and plastic products	48	Air transport		