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Enterprise Input-Output Accounting: a non-deterministic approach

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Abstract

Input-Output Analysis (IOA) provides a computational structure which is interesting for many applications within supply chain analysis and business processes analysis. These applications are usually performed as static and deterministic analyses, especially as far as cost accounting is concerned.

The aim of this paper is to introduce elements of uncertainty analysis within an environmentally-extended Input-Output technological model at the enterprise level, which is common to the environmental accounting and the cost accounting. This will allow both backcasting and forecasting to aid management towards taking informed actions.

The approach is built bottom-up from the basic operations within the enterprise, describing each internal process in terms of parameters that reasonably approximate its real characteristics. Monte Carlo methods will be employed to assess how the uncertainty associated with such estimated parameters of the model – especially the price which is expected to correspond for the external purchasing of resources – may affect its expected outcomes. Besides production planning and product costing, such framework can also be employed in order to evaluate what effect different design solutions are likely to have on both the material flows which link a supply chain of production processes, and even the associated whole-life costs.

Keywords: Enterprise Input-Output Analysis, Environmental Costing, Monte Carlo Methods.

1. Introduction

Applications of Input-Output Analysis (IOA) at the enterprise level have been developing since the late 1960s, although it is not common to find them explicitly addressed in the context of widely recognized cost accounting techniques. It can be argued, however, that IOA could provide such techniques – especially process costing, Activity-Based Costing (ABC) and Life Cycle Costing (LCC) – with a formalized computational structure which can help modeling interdependences among a supply chain of operations to address simultaneously both cost and production planning. The computational structure is secured by using matrices. While this is very effective for the mathematically literate, it may also explain why IOA is rarely employed in cost accounting.

Reflecting some common features of managerial accounting systems, the applications of IOA at the enterprise lever are usually deterministic. This is of no harm if one is interested in performing an *ex-post* cost assessment. Yet, as pointed out by Emblemsvåg (2003), chasing accuracy of past figures as an apparent reduction in uncertainty can increase risk. Furthermore, given the main usage of IOA as an economic planning tool, then uncertainty is to be explicitly addressed in order to allow both backcasting and forecasting to aid management towards taking informed actions. This paper then aims at introducing elements of uncertainty analysis within an environmentally-extended Enterprise Input-Output Accounting (EIOA). We focus on its ease of application to possible real-life case studies by means of commonly used electronic spreadsheets. The "Activity-Based Life Cycle Costing" will be taken as a reference cost accounting technique throughout the paper, consisting in ten modelling steps as described in Emblemsvåg (2003, 2001). Consistently with such steps, in the following section we discuss all those preliminary issues concerned with obtaining a comprehensive physical inventory, which will serve as a basis for the cost assessment. In Section 3 the remaining steps are discussed, concerned mainly with modelling uncertainty, assigning costs to cost objects and, lastly, performing Monte Carlo simulations.

2. Preliminary material flows inventory

The first six steps of the reference model are:

1. Define the scope of the model and the corresponding cost objects.
2. Obtain and clean Bill of Materials for all cost objects.
3. Identify and quantify the resources.
4. Create an activity hierarchy and network.
5. Identify and quantify resource drivers, activity drivers and their intensities.
6. Identify the relationships between activity drivers and design changes.

This section aims at showing how these steps could be "physiologically" fulfilled by properly setting an environmentally-extended EIOA[1].

As to Step (1), the choice of the unit production processes which are linked by supplier/customer relationships, determines the system boundaries and the scope of the analysis[2]. In a matrix form, which is warranted by the computational structure of IOA, processes are listed as column headings. In order to build a forecasting tool, IOA must be employed to set up a *technological model*, in the sense of Gambling (1968); that is built bottom-upwards from the basic operations which it purports to illustrate. Environmental extensions should enter the model consistently, as suggested by Lin & Polenske (1998). Such model is common to production planning – driven by physical flows – and cost accounting, thus fully reflecting its dual nature. Since a starting point should be the process data, the approach will be adopted here which is called "Activity-level Analysis", as defined by Heijungs (2001), which implies that the relevant interdependent manufacturing unit processes are to be analytically represented, within previously defined system boundaries, in terms of those parameters that reasonably

[1] At the microeconomic level, indeed, the IOA shows peculiarities that make it different from the original macroeconomic leontevian model – as discussed in Gambling & Nour (1969).

[2] Focusing on quite aggregated processes instead of activities is mainly due to the lack of details which may characterize many small and medium enterprises, especially in those Countries - like Italy, as emerges from the work of Bhimani *et al.* (2007) - where there is a delay in the diffusion of Activity-Based Costing .

approximate their real characteristics. Such parameters are to be quantified with reference to a certain level of process operation that is considered to be meaningful for planning decision (*e.g.*, one hour, one shift, or the production time of a batch). On this basis, we can estimate the total activity levels and the amount of resources that must be used to achieve a desired amount of the whole system's net production. The perspective here is a supply-chain one[3].

As to Steps (2) and (3), assume that an hypothetical single-product manufacturing system produces two commodities (Commodity 1, measured in tons, and Commodity 2, measured in m^3), and the manufacturing system consists of two unit processes (producing Commodity 1 and Commodity 2 respectively under the assumption that the *j-th* process produces the *j-th* commodity as its main output) and one treatment process (whose output, measured in tons, is the amount of waste/by-products, produced by the other processes, which it undergoes). These processes are mutually linked by material flows. The usage phase is also included, which consists in using 20 m^2 of Commodity 2 for one year. Such a system would be represented in an Input-Output form using a *make matrix* **V** and a *use matrix* **U** (both are positive semi-definite) as follows[4]:

$$\mathbf{Z}=\left(\begin{array}{c|c}\mathbf{Z}_{I,I} & \mathbf{Z}_{I,II}\\ \hline \mathbf{Z}_{II,I} & \mathbf{Z}_{II,II}\end{array}\right)=\begin{pmatrix}\mathbf{V}_{I,I} & \mathbf{V}_{I,II}\\ \mathbf{Z}_{II,I} & \mathbf{V}_{II,II}\end{pmatrix}-\begin{pmatrix}\mathbf{U}_{I,I} & \mathbf{U}_{I,II}\\ \mathbf{U}_{II,I} & \mathbf{U}_{II,II}\end{pmatrix}=\left(\begin{array}{ccc|c}100 & 0 & 0 & 0\\ 0 & 50 & 0 & 0\\ 0 & 0 & 20 & 0\\ \hline 0 & 0 & 0 & 5\end{array}\right)-\left(\begin{array}{ccc|c}0 & 20 & 0 & 0\\ 0 & 0 & 30 & 0\\ 0 & 0 & 0 & 0\\ \hline 0 & 0 & 0 & 0\end{array}\right)=$$

$$\left(\begin{array}{cc}\text{(p1)}:\text{Commodity 1} & ton\\ \text{(p2)}:\text{Commodity 2} & m^3\\ \text{(p3)}:\text{used}-\text{commodity 2} & m^2\\ \hline \text{(p4)}:\text{treatment} & ton\end{array}\right)=\left(\begin{array}{ccc|c}100 & -20 & 0 & 0\\ 0 & 50 & -30 & 0\\ 0 & 0 & 20 & 0\\ \hline 0 & 0 & 0 & 5\end{array}\right)$$

(1)

[3] Yet, those processes must be defined first which are controlled by the so called Focal Company. The model could then be gradually extended to combine the perspectives of those partners which are actually concerned with jointly reducing, where possible, costs and impacts at the same time, even in the use and disposal phases of the commodities they produce.

[4] As usual, the "Use table" **U** gives the main inputs which feed the *j-th* unit process - included self consumption - as listed in the *j-th* column. Its generic element u_{ij} describes the requirement of intermediate economic flow *i* by process *j* to produce a certain amount of outputs as described in another table, the "Make table" **V**. The latter shows all the gross outputs per production unit, as its element v_{ij} describes the production of the economic flow *i* by the process *j*. It is assumed that the main output of process *j* is listed as the *j-th* row within the Make table. Subscripts "I" and "II" have been introduced in order to make a distinction among the relevant rows and columns that make reference, respectively, to the production processes and to the treatment ones.

Note that matrix $\mathbf{Z}$ is referred to as the technology matrix. All the flows are listed in a definite order as row headings and expressed in physical terms. Each column represents a technique which is not *pure*. The partition $\mathbf{Z}_{I,II}$ is the waste treatments demanded by production processes. It is to be calculated according to the amount of waste which is not internally recycled and undergoes the latter processes. Other elements that must be taken into account are: 1) the consumption of those resources that are absorbed by one or more processes but are not produced by any of them[5] – called externally purchased inputs (matrix **M**); 2) the cycle time, measured in machine hours (h), and other cost drivers that are related to the operation of each process and can be used to assign variable overhead costs to them (matrix **C**); 3) the net output of waste and the input of waste, recorded separately in matrices $\overline{\mathbf{N}}$ and $\underline{\mathbf{N}}$, respectively[6]; and, lastly, 4) the releases into the environment (matrix **R**). Using some hypothetical figures, the above can be put into matrix notation as follows:

$$\mathbf{B}=\left(\mathbf{B}_{\bullet,\mathrm{I}}\quad \mathbf{B}_{\bullet,\mathrm{II}}\right)=\left(\begin{array}{c|c} -\mathbf{M}_{\bullet,\mathrm{I}} & -\mathbf{M}_{\bullet,\mathrm{II}} \\ \hdashline \mathbf{C}_{\bullet,\mathrm{I}} & \mathbf{C}_{\bullet,\mathrm{II}} \\ \hline \overline{\mathbf{N}}_{\bullet,\mathrm{I}} & \overline{\mathbf{N}}_{\bullet,\mathrm{II}} \\ \hdashline -\underline{\mathbf{N}}_{\bullet,\mathrm{I}} & -\underline{\mathbf{N}}_{\bullet,\mathrm{II}} \\ \hline \mathbf{R}_{\bullet,\mathrm{I}} & \mathbf{R}_{\bullet,\mathrm{II}} \end{array}\right)=\left(\begin{array}{cc} \text{(p6) : Input1} & \text{ton} \\ \text{(p7) : Input2} & \text{m}^3 \\ \text{p(9) : Input3} & \text{L} \\ \text{(p8) : Electricity} & \text{MWh} \\ \text{(p6) : Energy} & \text{GJ} \\ \hdashline \text{Cycle time 1} & \text{h} \\ \text{Cycle time 2} & \text{h} \\ \text{Cycle time 3} & \text{h} \\ \hline \text{waste type1(out)} & \text{ton} \\ \text{waste type2(out)} & \text{ton} \\ \hdashline \text{waste type1(in)} & \text{ton} \\ \text{(p5) : waste type2(in)} & \text{ton} \\ \hline \text{(p10) : Releases} & \text{kg} \end{array}\right)=\left(\begin{array}{ccc|c} -7 & -13 & 0 & 0 \\ -2 & 0 & 0 & -8 \\ 0 & 0 & -9 & 0 \\ -23 & -15 & 0 & -20 \\ -1 & -0{,}5 & 0 & 0 \\ \hdashline 1 & 0 & 0 & 0 \\ 0 & 1{,}5 & 0 & 0 \\ 0 & 0 & 0 & 2 \\ \hline 0{,}4 & 0 & 0 & 0 \\ 0 & 0{,}8 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ -0{,}3 & 0 & 0 & 0 \\ \hline 0{,}5 & 0 & 0 & 12 \end{array}\right) \quad \textbf{(2)}$$

[5] It is important to notice that their contribution to the overall manufacturing cost, given the market price paid to suppliers, can only be influenced by resource consumption rates.

[6] The net generation of waste *k* by process *j* is recorded as the generic element $\left(\overline{N}\right)_{kj}$ of the matrix $\overline{\mathbf{N}}$, whereas the element $\left(\underline{N}\right)_{kj}$ of the matrix $\underline{\mathbf{N}}$ represents the input of the same waste into that process. Following Nakamura & Kondo (2006), each process is assumed either to produce a given waste *k* or to use it as a secondary input, *i.e.* $\forall j, \forall k : (\overline{N})_{kj} \times (\underline{N})_{kj} = 0$. Though this is not the only way to proceed, it seems most convenient for cost accounting purposes.

Through matrix $\mathbf{A} = \left(\frac{\mathbf{Z}}{\mathbf{B}} \right)$ those volume-related variables are assumed to have been identified and quantified, consistently with Step (3) of the model. According to Step (5), in particular, the elements of matrix **A** quantify the resource and activity drivers[7]. It is important to notice that matrix **C** must not be used to assign non-volume-related overheads. Such operation could be performed, instead, by recording those period-related (even not physical) cost drivers within a separate matrix **F** *that can be taken into account only after the material flows and the volume-related cost drivers will have been balanced against* the production plan for the period considered[8]. Note that matrix **F** consists of four columns and of a number of rows which depends upon the overhead issues that are to be assigned. For the sake of simplicity, only variable overheads will be herein considered. As to Step (6), a distinction could be made between design dependent and design independent cost drivers, like in Bras & Emblemsvåg (1996).

The balancing procedure is based upon the above mentioned deterministic data. It consists in calculating the activity levels at which the production processes must operate in order to generate a net output that meet an exogenous final demand. The latter could be, for example, the amount of final and/or intermediate products that have been planned to be produced in a month. This production plan sets the reference flow vector. If we consider, for example, only the production and usage of Commodity 2 in our example, such vector reads $\mathbf{y}_{\mathbf{I}} = (0 \quad 0 \quad 20.000)^T$, where superscript T stands for the vector transposed, and the scaling vector can be determined accordingly as $\mathbf{Z}_{\mathbf{I},\mathbf{I}}^{-1}\mathbf{y}_{\mathbf{I}} = \mathbf{s}_{\mathbf{I}}$, provided that the inverse of **Z** exists. In our example $\mathbf{s}_{\mathbf{I}} = (120 \quad 600 \quad 1.000)^T$. The recycling ratio for each waste type k is determined as $r_k = \sum_{j=1}^{n} (\underline{N})_{kj} \times s_j \Big/ \sum_{j=1}^{n} (\overline{N})_{kj} \times s_j$ where n is the number of production processes (in our example, n=3 due to the presence of the use phase). Such ratios can be collected in a

[7] The former are mainly recorded in matrices **M** and $\underline{N}$, whereas the latter are mainly recorded in matrices **V**, **U** and **C**. The drivers recorded in Matrix **M** actually have a mixed nature.

[8] This could be useful in order to take separately into account the fixed quote of an overhead such as the Electric energy consumption, whereas the variable part is assigned according to a measure (MWh) obtained form the installed power and the operating times of the machineries.

vector, which reads $\mathbf{r} = (0 \quad 0{,}75)^T$. The percentage of the output of waste which is not recycled within the system considered, is $(\mathbf{I} - \hat{\mathbf{r}})$, where $\mathbf{I}$ is an identity matrix of appropriate dimensions. Such quantity will undergo the treatment process. The latter produces an output that is in fact a treatment service, measured by the physical amount, expressed as weight, of the waste it processes. This is an example of "activity driver" consumed by the other processes. If there is not a one-to-one relationship among the waste types and the treatment processes, then a matrix $\mathbf{Q}$ is to be exogenously defined whose element q_{lk}, where $0 < q_{lk} < 1$, indicates the amount of k-th waste type which undergoes the l-th treatment. Assume $\mathbf{Q} = (1 \quad 1)$. The demand for waste treatment is $\tilde{\mathbf{Z}}_{\mathbf{I,II}} = -\mathbf{Q}(\mathbf{I} - \hat{\mathbf{r}}) \times (\overline{\mathbf{N}} \times \hat{\mathbf{s}}) = (-48 \quad -444)$. If $\mathbf{Z}_{\mathbf{I,I}} \times \hat{\mathbf{s}} = \tilde{\mathbf{Z}}_{\mathbf{I,I}}$, then another scaling vector $\mathbf{s}_{\mathbf{II}} = \begin{pmatrix} \tilde{\mathbf{Z}}_{\mathbf{I,I}} & \mathbf{Z}_{\mathbf{I,II}} \\ \tilde{\mathbf{Z}}_{\mathbf{I,II}} & \mathbf{Z}_{\mathbf{II,II}} \end{pmatrix}^{-1} \times \begin{pmatrix} \mathbf{y}_{\mathbf{I}} \\ \mathbf{0} \end{pmatrix}$ that takes into account also the treatment process must be calculated. The overall balanced system is obtained by stacking

$$\tilde{\mathbf{Z}} = \begin{pmatrix} \tilde{\mathbf{Z}}_{\mathbf{I,I}} & \mathbf{Z}_{\mathbf{I,II}} \\ \tilde{\mathbf{Z}}_{\mathbf{I,II}} & \mathbf{Z}_{\mathbf{II,II}} \end{pmatrix} \times \hat{\mathbf{s}}_{\mathbf{II}} = \left(\begin{array}{ccc|c} 12000 & -12000 & 0 & 0 \\ 0 & 30000 & -30000 & 0 \\ 0 & 0 & 20000 & 0 \\ \hline 48 & -444 & 0 & 492 \end{array}\right)$$

and

$$\tilde{\mathbf{B}} = \left(\begin{array}{c|c} -\mathbf{M}_{\bullet,\mathbf{I}} \times \hat{\mathbf{s}}_{\mathbf{I}} & -\mathbf{M}_{\bullet,\mathbf{II}} \\ \hdashline \mathbf{C}_{\bullet,\mathbf{I}} \times \hat{\mathbf{s}}_{\mathbf{I}} & \mathbf{C}_{\bullet,\mathbf{I}} \\ \hline \hat{\mathbf{r}}\overline{\mathbf{N}}_{\bullet,\mathbf{I}} \times \hat{\mathbf{s}}_{\mathbf{I}} & \overline{\mathbf{N}}_{\bullet,\mathbf{II}} \\ \hdashline -\underline{\mathbf{N}}_{\bullet,\mathbf{I}} \times \hat{\mathbf{s}}_{\mathbf{I}} & -\underline{\mathbf{N}}_{\bullet,\mathbf{II}} \\ \hline \mathbf{R}_{\bullet,\mathbf{I}} \times \hat{\mathbf{s}} & \mathbf{R}_{\bullet,\mathbf{II}} \end{array}\right) \times \hat{\mathbf{s}}_{\mathbf{II}} = \left(\begin{array}{ccc|c} -840 & -7800 & 0 & 0 \\ -240 & 0 & 0 & -787{,}2 \\ 0 & 0 & -9000 & 0 \\ -2760 & -9000 & 0 & -1968 \\ -120 & -300 & 0 & 0 \\ \hline 120 & 0 & 0 & 0 \\ 0 & 900 & 0 & 0 \\ 0 & 0 & 0 & 196{,}8 \\ \hline 0 & 0 & 0 & 0 \\ 0 & 36 & 0 & 0 \\ \hdashline 0 & 0 & 0 & 0 \\ -36 & 0 & 0 & 0 \\ \hline 60 & 0 & 0 & 1180{,}8 \end{array}\right) \qquad \mathbf{v} = \left(\begin{array}{c} -16 \\ -30 \\ -17{,}2 \\ -0{,}6 \\ -80 \\ \hline 0{,}12 \\ 0{,}57 \\ 1{,}2 \\ \hline -0{,}5 \\ -0{,}2 \\ \hdashline 0{,}5 \\ 0{,}2 \\ \hline 0{,}017 \end{array}\right)$$

where the vector v includes the unit standard resource costs, the predetermined overhead coefficients and the price at which the wastes are sold internally. All numbers are in terms of € per each unit. This quantifies the variable consumption intensities[9]. It must be noted that, in the example, the unit cost of direct materials in the usage phase has been calculated as the present value of an annually recurring uniform amount of, say 1€/L, for 40 years at 5%.

The whole system, $\tilde{\mathbf{A}} = \begin{pmatrix} \tilde{\mathbf{Z}} \\ \tilde{\mathbf{B}} \end{pmatrix}$, can be graphically represented as in Figure (1), thus fulfilling the Step (4) of the reference model[10].

Figure 1

3. Modeling Uncertainty

Step (7) of the reference model consists in modelling uncertainty. All the process-related operating parameters, which have been depicted so far in a deterministic manner, are in real life subject to uncertainty. As pointed out by Heijungs and Suh (2002), Chapter 6, and Hendrickson *et al.* (2006), Appendix IV; within Input-Output computational structures, a change affecting one element of the technology matrix affects all the elements within its inverse and propagates through the scaling factors. If, perturbation is systematically introduced within the technology matrix, each of its elements must then be defined in stochastic terms, as a specific probability distribution[11]. Such mathematical analysis might be, however, cumbersome and unnecessary.

Uncertainty distributions should then be assigned to most uncertainties associated with both cost drivers and consumption intensities. This is performed in the so called “assumption cells” in the Excel spreadsheet. Thus, some assumption cells

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[9] If fixed ones also existed, another similar vector of consumption intensities should be defined consistently with the number of rows of matrix **F**.

[10] The network of processes is described in terms of conditions (the circles – representing local states of the systems, like the availability of the inputs, the production of intermediate or final outputs etc.) and events (the rectangles – representing the operation of a process) . The notation is that of Eqs. (1) and (2).

[11] Wu & Chang (2003), that explicitly addressed the non-deterministic application of IOA for usage in cost accounting, characterized, instead, each element within their overall system matrix **A** in terms of a lower and a higher bound, and then applied the grey numbers theory.

within the spreadsheet equivalent matrix $\tilde{\mathbf{A}}$ will be chosen according to their relevance for real-life business planning processes. In particular, the uncertainty distribution will apply only with reference to some cost drivers, like:

- The gross main output as recorded within the make matrix **V**, thus reflecting the different efficiency of the process.
- The amount of waste/scrap.
- Some consumption intensities, which are specified as the entries of the vector ν of cost coefficients.

It should be noted that the second and the third issue can affect the balancing procedure described in Section 2, involving the technology matrix inverse, which yields the scaling factors.

3.1 Estimating the cost of processes, products and waste.

According to Step (8) of the model, to estimate the direct (variable) cost of each production process, the relevant resource drivers are to be multiplied by their consumption intensities. In this way, direct materials costs are traced and conversion costs (*i.e.* labour and production overheads) are traced to each process according to drivers which are available within the above described input-output scheme. The following

$$\boldsymbol{\omega} = \mathbf{v} \times \tilde{\mathbf{B}} = \begin{pmatrix} 31.903 & 154.706 & 154.432 & 25.033 \end{pmatrix} \qquad \textbf{(3)}$$

yields, in a deterministic way, the analogous of leontevian IOA's value added vector. Although it cannot be intended as a "value added" properly.

As to Step (9), the EIOA is structured so that the cost of manufacturing the main output of a unit process is *transferred* to the downstream processes, as a separate category of direct material costs. This concurs with how prices are computed within leontevian macroeconomic IOA, where the primary factors' costs are incurred by each producing sector in addition to payments for input purchased from other producing sectors. The cost of producing the main outputs is assumed to offset the *transfer price* for all the inputs supplied by the other processes and the costs related to a) externally purchased inputs, b) waste/by-products treatment and c) inter-process recycling and d) labour and overheads.

$$\mathbf{p} = \boldsymbol{\omega} \times \tilde{\mathbf{A}}^{-1} = \left(2{,}86 \quad 7{,}05 \quad 18{,}30 \mid 50{,}92\right)^T \tag{4}$$

This shows how manufacturing cost has been accumulated in each of the unit processes within the supply chain, including those costs incurred to run the treatment processes internally.

Emblemsvåg & Bras (2002) point out that capturing waste generation and the costs associated to waste is one of the core environmental dimensions in the model herein assumed as a reference. In order to estimate the value of resources employed by a process just to produce a given waste, the latter should be seen as a joint product. Consequently, an *assignment* procedure should be carried out to split the former process into two independent unit processes, one producing the same output as before and one producing the former by-product as a main output[12]. Assume this applies to Process 1. The entries of the new vectors are obtained once the allocation factors λ and (1–λ), where $0 < \lambda < 1$, has been determined according to proper criteria and multiplied by the entries[13] of the first column of matrix **A**. Furthermore, due to the aim of cost assignment and cost tracing, the following is required: (a) within the process vector producing the former by-product, the same flow is entered twice, as both a main output and as a by-product, exactly as before; (b) the downstream processes using the former main output shall also use the economic flow obtained after the allocation has been carried out. Assuming an assignment criterion based on mass has been adopted and Process 1 is assigned $\lambda = 0{,}996$, the Equations (3) and (4) reads

$$\boldsymbol{\omega} = \mathbf{v} \times \tilde{\mathbf{B}} = \left(31.776 \quad 127{,}10 \quad 154.705 \quad 154.431 \mid 25.033\right) \tag{5}$$

$$\mathbf{p} = \boldsymbol{\omega} \times \tilde{\mathbf{A}}^{-1} = \left(2{,}65 \quad 53{,}57 \quad 7{,}06 \quad 18{,}31 \mid 50{,}92\right)^T \tag{6}$$

[12] This is similar to the so called *allocation* procedure within the input-output-based computational structure of Life Cycle Assessment (LCA) as described by Heijungs & Suh (2002).
[13] Except those ones corresponding to the Main Product 1 and the Waste type 1.

where the fictitious process producing the waste is now assigned the whole cost of its treatment, though only for illustrative purposes.

3.2 Perform Monte Carlo analysis

Once the above described deterministic model has been fully represented within the spreadsheet, each process is assigned uncertainty information by entering this information into the assumption cells in the spreadsheet. The forecasted dependent variable is also defined. Assumption cells can be thought of as source variables whereas the forecasted dependent variables can be thought of as response variables.

As illustrated by Bras & Emblemsvåg (1996), those parameters, among the above mentioned categories, that will serve as source variables must be further specified. The need to select a relatively small number of variables which can be defined as important has been mathematically discussed in Bullard & Sebald (1977). However, when using Monte Carlo methods this restriction does not apply. Models including thousands of variables are mathematically as straightforward as a model containing just seven variables, see for example Emblemsvåg (2003). The aspects concerning the computational performance of running such a resource-intensive numerical method as Monte Carlo Analysis has also been discussed by Peters (2007). However, since the ease of implementation is in the focus here, a simulation has been carried out by using the common Excel spreadsheet and Crystal Ball 7.3.1 (Demo version).

For illustrative purposes, a few examples of uncertainty specification attached to the model parameters have been showed in Table 1 and Table 2, regarding, respectively, some cost drivers and some consumption intensities.

Table 1 and Table 2

It should be noticed that a negative correlation should be specified between the Main Product 2 and the amount of Waste type 2, which is nearly -1.

The unit production cost for each of the process-stage[14] serve as a response variable. Monte Carlo Analysis being a sampling technique, a number of iteration is performed, using the appropriate software tools, in order to numerically simulate the changes caused to the results, given the variability of the assumption cells. With the aid of the software, each outcome will be described in terms of a calculated probability distribution as shown in Figure 2.

Figure 2

From such results one can see, for example, that there is a 10,74% probability that the unit Life-Cycle Cost of the Main Product 2 will not exceed 18€/ m^2.

Further, by performing a sensitivity analysis, like the one shown in Figure 3, one can see that the unit Life-Cycle Cost is highly influenced by the standard cost of Input 3 and, in a negative manner, by the discount rate which is used to calculate the present value of an annually recurring uniform amount.

Figure 3

From Figure 3 one can also see that the unit cost of Main Product 1 can be influenced to a greater extent (in the sense of a reduction) by an increase in Process 1 efficiency, *i.e.* an increase of the quantity of "Main Product 1" which can be obtained from the same amount of externally purchased input, and to a less extent by a reduction in the resource consumption and by the consumption intensities of such externally purchased inputs, as well.

Finally, the cost of the scrap can be very slightly influenced, in the sense of a reduction, by increasing the efficiency of Process 2 (assuming a negative correlation between the net output of the later and the scrap).

So far, the purpose of the analysis has been mainly to find out which factors have the greatest impact on the total costs and what information should be paid extra attention to. This has been accomplished by tracing the different contributors using

[14] This is intended as the equivalent of the leontevian price model, though applied to the enterprise level. It can be seen a determined consistently with the process costing principles.

sensitivity analysis, and by choosing mainly bounded and symmetric distributions. This, however, can be seen as only one of the different ways in which Monte Carlo methods can be employed.

It should be noted, indeed, that sensitivity charts, like the ones shown in this papers, not only can be used to identify the most critical drivers of performance. It can also be a highly effective tool in identifying what information should be pursued to increase the quality of the analysis. Emblemsvåg (2003), Appendix A, distinguishes among *Tracing models* and *Uncertainty models*. The former have been mainly employed here, to show that for cost management, and the corresponding continuous improvement efforts, adding uncertainty means providing information about the possible distortion problems in the models. Quoting the author, in such case ”we do not need accurate data. We need satisfactory process description that reflect the cause-and-effect relationships and data that are roughly correct”. On the other hand, *Uncertainty models* aim at finding out what information generates the most uncertainty and how this affects the forecasts. This implies that uncertainty is modelled as accurately as possible, in order to see how one issue which may be equal, in magnitude, to another one, actually differ in importance, being associated to a larger uncertainty. Consider for example, the Input 3. Being used in the usage stage, it has been attached a larger uncertainty. This is shown in Figure 4

Figure 4

From the sensitivity charts related to the Unit Life-Cycle Cost, we have seen, indeed, that such material cost is found more important than other factors.

Before we close the paper off, we would like to illustrate briefly how an application of this kind of numerical analysis could concern issues like the accounting implications of the EU Emissions Trading Directive. Assume that the releases into the environment as accounted for in matrix **R** are estimates of the CO_2 emissions for each process. There is a consumption intensity measured by the last entry of vector $\mathbf{v}$, which we call p^*, that corresponds to such driver. The latter is likely to have been calculated as a predetermined overhead cost rate by dividing the estimated cost of emissions in the period – that is not matched by the government grant consisting in the initial allocation of allowances free of charge, expressed as q^* – and the estimated plant emission of the period[15]. Now assume that p^*, is a function of the following kind:

$$\begin{cases} p^* = 17€/\text{ton} & \text{if } q > q^* \\ p^* = 0€/\text{ton} & \text{if } q \le q^* \end{cases}$$

where $\mathbf{q} = (\mathbf{R}_\mathbf{I} \times \hat{\mathbf{s}}_\mathbf{I} \mid \mathbf{R}_\mathbf{II}) \times \mathbf{s}_\mathbf{II} = 1{,}24\text{ton}$ is the overall amount of CO_2 emissions in the period. Now assume that the condition $q > q^*$ is satisfied and that the expected average price of allowances, 17€/tCO_2eq, is characterized as a triangular distribution ranging from 5€/tCO_2eq to 35€/tCO_2eq. This can be seen, for example, from a survey carried out by Point Carbon (2007) as to the price of allowances in 2010. By performing a sensitivity analysis, like the one shown in Figure 3, one can see that the expected average price of allowances is not among the larger contributors to the forecast cells variance.

4. Closure

Applications of IOA at the enterprise level for cost accounting are interesting for integrating production and cost planning. In order to allow both backcasting and forecasting to aid management towards taking informed actions, the uncertainty is to be explicitly addressed in order to reduce risk and also being intelligently able to request more information. This paper has discussed how to develop and easy-to-implement non-deterministic computational structure for Enterprise Input-Output Accounting, which can be implemented also within small and medium enterprises with the aid of commonly used spreadsheet. Such EIOA has been applied within the framework of a cost accounting model which is called “Activity Based Life Cycle Costing”. The environmental extensions have also been considered. Focus has been on the assessment of the costs which are likely to be incurred to produce and treat a waste, and on identifying the most critical drivers of performance.

References

Bhimani, A., Gosselin, M., Ncube, M., Okano, H. (2007) **Activity Based Costing: how far have we come internationally?**, Cost Management, 21, pp.12-17

[15] For the sake of simplicity, and because of nature of the matrix operations, it is assumed that the government grant only reduces the average cost of allowances per ton of CO_2 equivalent. That is, the allowances initially allocated are not excluded from the determination of the overhead cost rate.

Bras, B. & Emblemsvåg, J (1996) **Activity-Based Costing and uncertainty in designing for the life-cycle**, in Huang, Q., Design for X: Concurrent Engineering Imperatives (London, Chapman & Hall).

Bullard, C.W, & Sebald, A.V. (1977) **Effects of Parametric Uncertainty and Technological Change on Input-Output Models**, The Review of Economics and Statistics, 59, 75-81.

Emblemsvåg, J (2003) **Life Cycle Costing. Using Activity-Based Costing and Monte Carlo Methods to manage future costs and risks** (Hoboken, Wiley).

Emblemsvåg, J. & Bras, B. (2002) **Activity-Based Cost and Environmental Management. A different Approach to the ISO 14000 Compliance** (Kluwer Academic Publishers).

Emblemsvåg, J (2001) **Activity-based Life Cycle Costing**. Managerial Auditing Journal, 16, pp.17-27

Gambling, T.E. (1968) **A technological model for use in Input-Output Analysis and cost accounting**, Management Accounting (USA), 50, pp. 33-38.

Gambling T.E., &_Nour A. (1969) **A note on Input-Output Analysis: Its Uses in Macro-Economics and Micro Economics**, The Accounting Review, 45, pp.98-102.

Heijungs, R. & Suh, S. (2001) **The computational structure of Life Cycle Assessment.** (Dordrecht, Kluwer).

Heijungs, R. (2001) **A Theory Of The Environment And Economic Systems. A Unified Framework for Ecological Economic Analysis and Decision Support**. (Cheltenham, Edward Elgar).

Hendrickson, C.T., Lave, L.B. Matthews H.S. (2006) **Environmental Life Cycle Assessment of Goods and Services. An Input-Output Approach** (Washington, Resource For the Future).

Lin, X. & Polenske, K.R. (1998) **Input-output modeling of production processes for business management**, Structural Change and Economic Dynamics, 9, pp. 205-226.

Nakamura, S. & Kondo, Y. (2006) **A waste input-output life-cycle cost analysis of the recycling of end-of-life electrical home appliances**, Ecological Economics,57, pp. 494-506.

Peters, G.P. (2007) **Efficient Algorithms for Life Cycle Assessment, Input-Output Analysis and Monte Carlo Methods**, The International Journal of Life Cycle Assessment, 12, 373-380.

Point Carbon (2007) **Carbon 2007. A new climate for carbon trading.** Carbon Market Insights, 4th annual conference, Copenhagen 13 - 15 March 2007.

Wu, C.C. & Chang, N.B. (2003) **Grey Input-Output analysis and its application for environmental cost allocation,** European Journal of Operational Research, 145, pp. 175-201.

Tables:

Table 1

Process	Cost driver	Distribution	Abs(Mean)	Min	Max
Proc 2	Main product 2	Triangular	50	45	55
Proc 1	Electricity	Normal	23	20,7	25,3
Proc 3	Cycle time	Triangular	2	1,8	2,2
Proc 1	Waste type 2	Triangular	0,8	0,72	0,88

Example of uncertainty modelling for some Cost drivers

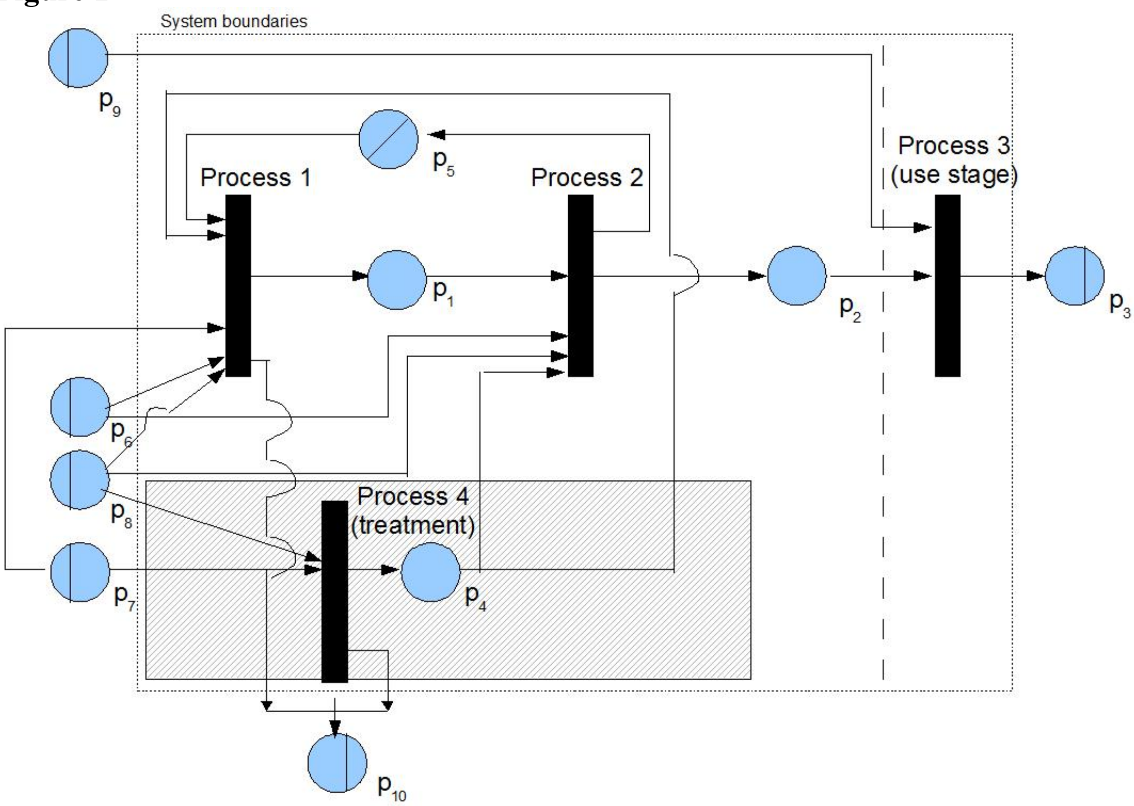
Table 2

Process	Consumption Intensity	Distribution	Mean	Min	Max
Proc 1,2	Thermal energy €/GJ	Triangular	80	72	88
Proc 2	Overhead rate €/h	Normal	0,57	0,513	0,627

Examples of uncertainty modelling for some Consumption intensities

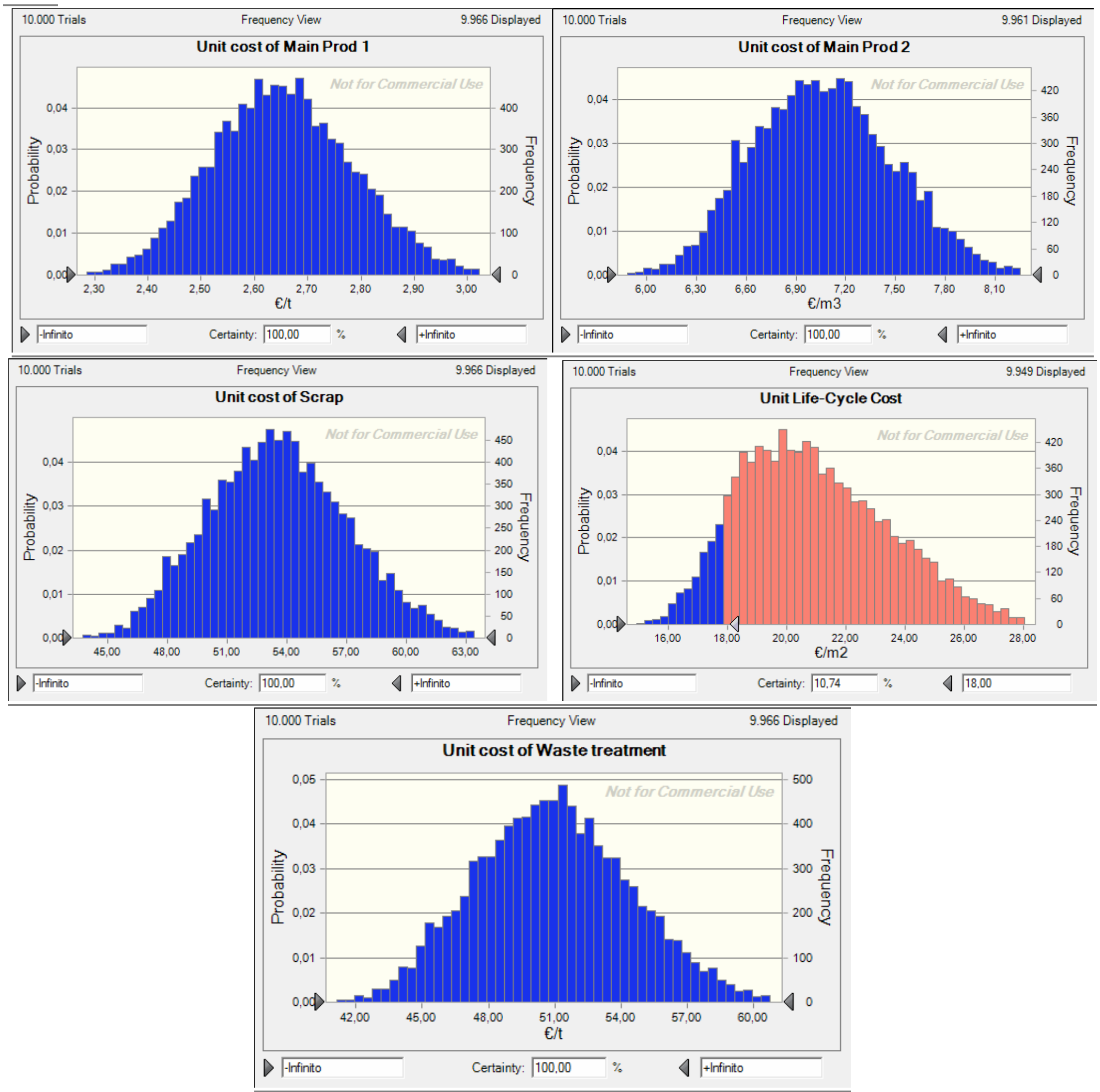
Figures:

Figure 1



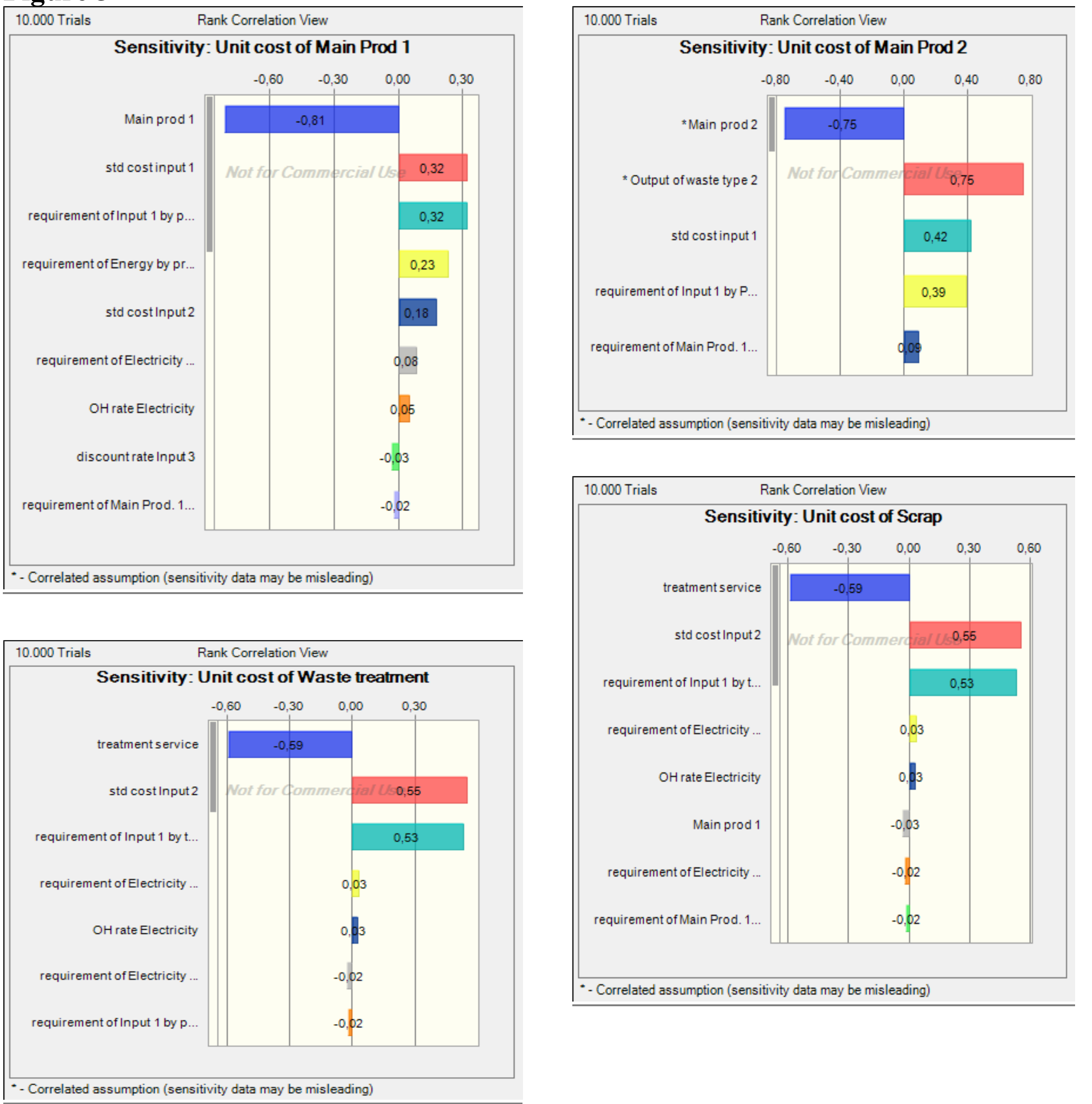
A network of interdependent processes

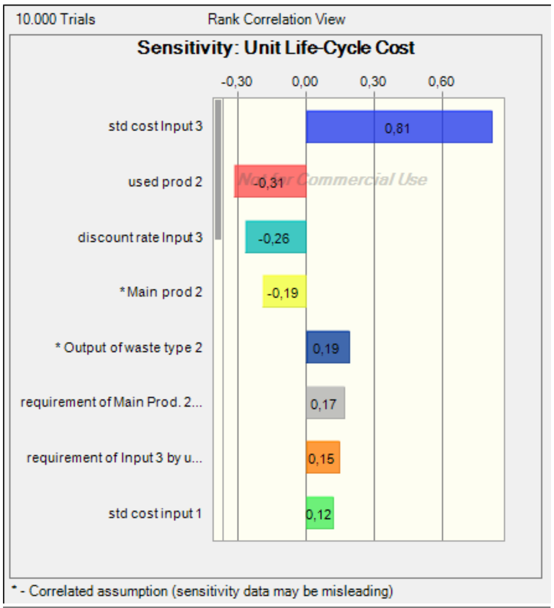
Figure 2



Uncertainty distribution of the outcomes of the model, using Crystal Ball 7.3 (demo) to perform Monte Carlo Analysis

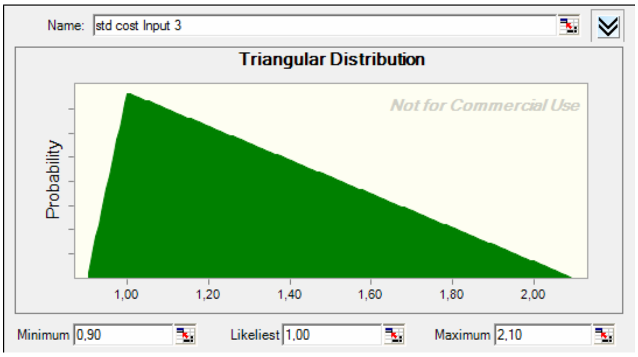
Figure 3


10.000 Trials
Rank Correlation View
Sensitivity: Unit cost of Main Prod 1
-0,60 -0,30 0,00 0,30
Main prod 1
std cost input 1
requirement of Input 1 by p...
requirement of Energy by pr...
std cost Input 2
requirement of Electricity ...
OH rate Electricity
discount rate Input 3
requirement of Main Prod. 1...
-0,81
0,32
0,32
0,23
0,18
0,08
0,05
-0,03
-0,02
Not for Commercial Use
* - Correlated assumption (sensitivity data may be misleading)
10.000 Trials
Rank Correlation View
Sensitivity: Unit cost of Main Prod 2
-0,80 -0,40 0,00 0,40 0,80
* Main prod 2
* Output of waste type 2
std cost input 1
requirement of Input 1 by P...
requirement of Main Prod. 1...
-0,75
0,75
0,42
0,39
0,09
Not for Commercial Use
* - Correlated assumption (sensitivity data may be misleading)
10.000 Trials
Rank Correlation View
Sensitivity: Unit cost of Waste treatment
-0,60 -0,30 0,00 0,30
treatment service
std cost Input 2
requirement of Input 1 by t...
requirement of Electricity ...
OH rate Electricity
requirement of Electricity ...
requirement of Input 1 by p...
-0,59
0,55
0,53
0,03
0,03
-0,02
-0,02
Not for Commercial Use
* - Correlated assumption (sensitivity data may be misleading)
10.000 Trials
Rank Correlation View
Sensitivity: Unit cost of Scrap
-0,60 -0,30 0,00 0,30 0,60
treatment service
std cost Input 2
requirement of Input 1 by t...
requirement of Electricity ...
OH rate Electricity
Main prod 1
requirement of Electricity ...
requirement of Main Prod. 1...
-0,59
0,55
0,53
0,03
0,03
-0,03
-0,02
-0,02
Not for Commercial Use
* - Correlated assumption (sensitivity data may be misleading)



Sensitivity analysis of the outcomes of the model, using Crystal Ball 7.3 (demo) to perform Monte Carlo Analysis

Figure 4



Modelling assumption cells for uncertainty and information management purposes, using Crystal Ball 7.3 (demo)