A new kind of production multipliers to assess scale and structure effects of demand shocks in input-output frameworks

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Paper to be presented at the:

International IO Meeting on Managing the Environment
Seville (Spain), July 9 – 11, 2008

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Abstract:

The main purpose of this paper is to develop a new kind of input-output multiplier particularly well suited to quantify the impacts of final demand changes (in consumption, investment or exports) on the sectoral output growth potential of an economy. Instead of using the traditional output multipliers, given by the elements of the Leontief inverse, solving an appropriate optimization problem provides what can be called input-output Euclidean distance multipliers. This method does not impose unitary final demand shocks with a fixed (predetermined) structure, allowing the "IO economy" to change along the spectrum of all possible structures. It can be very helpful in measuring interindustry linkages, choosing (a certain kind of) key sectors in a national or regional economy and managing the environment.

JEL classification: C67; D57

Keywords: multiplier analysis; structural change

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Financial support by FCT (*Fundação para a Ciência e a Tecnologia*), Portugal is gratefully acknowledged. This paper is part of the Multi-annual Funding Project (POCTI/0436/2003)

1. Introduction

When studying the structure of a national or regional economy according to the Leontief model hypothesis, a central role is attributed to final demand multipliers, i.e. the elements of the Leontief inverse used to measure the impacts of change(s) in one (or several) component(s) of final demand on output, value added or employment.

However, the use of this kind of multiplier, dating back to Rasmussen (1956), suffers from one important drawback, namely that it is limited to particular changes in final demand, such as a unitary shock in each sector and zero elsewhere in the case of backward multipliers, and a unitary shock in all sectors at once in the case of forward multipliers. This limitation, pointed out by Skolka (1986), reduces the usefulness of the Rasmussen multipliers.

It can even be argued that the use of traditional multipliers leads to an inadequate invasion of macroeconomic concepts into the territory of a genuine multisectoral analysis. Let us consider, for instance, a unit increase in total final demand. From a macroeconomic point of view, it is by definition irrelevant to know in advance how this monetary unit is distributed among sectors, because these sectors are not individually considered. But from a multisectoral point of view, it is crucial to know if this unit is, for example, directed entirely to one particular sector or otherwise distributed evenly among all the sectors.

In the first case, the new situation (after the increase in final demand) is far more different from the initial one than in the second case. This difference does not exist in an aggregate macroeconomic analysis. In a disaggregated intersectoral analysis, however, it should not be ignored.

For this kind of comparison between different situations, the traditional Leontief/Rasmussen multipliers are inappropriate, because they are unable to compare the impacts of changes in final demand on output (value added, employment, energy consumption), giving rise to new vectors equidistant from the initial vector.

One interesting approach to this problem is the work of Ciaschini (1989, 1993) and Ciaschini and Socci (2007), based on the so-called singular value decomposition method.

In this paper, a different and easier approach is adopted. By solving an appropriately designed optimization problem, two important advantages are obtained. Firstly, the final demand structure subsequent to a final demand shock is not fixed in advance, thereby overcoming an important limitation of traditional linkage measures. Secondly, the maximum output impact can be decomposed into two significant effects: a homothetic scale effect, depending on the magnitude of the positive shock applied to a pre-existing final demand structure, and a structure effect, resulting from output maximizing changes in sectoral final demand.

This method, explained and formalized in section 2, gives rise to a new kind of multipliers, that can be termed *Euclidean distance multipliers* and may prove to be helpful in measuring interindustry linkages and choosing key sectors in a national or regional economy.

An empirical application of the method is made here using input-output data for Portugal and Spain (national level) and the respective islands of Azores and Baleares (section 3). The paper concludes with a summary of the main results (section 4).

2. Intersectoral euclidean distance multipliers

Context of analysis

Consider the solution of the standard Leontief model **x = Ly,** where **x** and **y** are vectors of output and final demand and **L** is the Leontief inverse (for a detailed presentation of this model, see Miller and Blair, 1985).

When this solution is used for studying the potentialities for growth of an economy in response to final demand shocks, at least three problems can be considered.

The first one is to find, for a new situation, the largest increase in production resulting from a unitary increase in final demand, supposing that, in this new situation, no sector will decrease its final demand in relation to the initial level. This problem is easily solved using the Rasmussen multipliers. The unitary increase in final demand should be allocated to sector i in such a way that the Rasmussen multiplier $\sum_j l_{ji}$ is maximum (l_{ji} is the generic element of the matrix **L**).

The second problem is to find the largest increase in production resulting from a unitary increase in final demand, assuming that the final demand for each sector can vary and supposing that, in the new situation, this variation will not lead to a negative final demand for that sector (a negative final demand for a given sector has no meaning, with the possible exception of the existence of large stocks for that sector in the initial situation – a case that we rule out). Again, it is easy to deal with this problem. All of the final demand (the total value of final demand in the initial situation plus one additional monetary unit) should be allocated to sector i of the largest $\Sigma_j\ l_{ji,}$ while for the other sectors final demand should be zero.

These two problems are easily solved, but both are of limited interest because of their lack of realism, which is, of course, more pronounced in the case of the second problem. For the first problem, the macroeconomic bias is clear. It is assumed that it is possible to increase the final demand of any sector by one monetary unit and at the same time keep final demand constant for the other sectors, an assumption that a genuine multisectoral analysis cannot accept.

This is why it is worth considering a third (alternative) problem, namely to find the variations of the vector of final demand within the neighbourhood of a given initial vector that will maximize (or minimize) the distance of the resulting vector of production in the new situation in relation to the initial production vector.

One important characteristic of this third problem is the use of the Euclidean distance between vectors to measure the variations in relation to the initial situation. A vector resulting from concentrating all of the increase in final demand in one sector is at a greater distance from the original final demand vector than a vector that results from evenly distributing an increase in final demand of the same magnitude, which means that the Euclidean distance effectively distinguishes between two situations that must be treated as different. So, a genuinely multisectoral analysis should focus on the comparison between final demand variations that give rise to new vectors located at the *same distance* from the original vector. In the same way, the output impact of these final demand variations should be measured by the Euclidean distances between the new and the original output vectors.

Note that this kind of multipliers is different from the usual ones. The standard use of multipliers calculates the effect on production of an increase in one monetary unity (m.u.) in final demand. This increase in one m.u. may be distributed by sectors according to the structure of final demand or, as mentioned before, can be allocated to just one sector supposing that the other sectors keep constant their respective contributions to final demand.

Our problem is different and should not be seen with the eyes of the preceding analysis. What we intend to do is to study how the production of a given economy deviates from an initial vector of production when final demand suffers a shock that leads to a new final demand vector that is at a distance of one m.u. from the previous one. This is not a planning problem as it often is the preceding one (at least the second of the two cases mentioned). Our methodology may be a useful tool to study the behaviour of the production system of an economy. It is indeed important for a number of reasons to evaluate the sensitivity of an economy to demand shocks. There are economies where the scope of variation of output in response to a unitary variation of final demand is larger than in other economies. Economies of the first type are in this very specific sense more sensitive than the others.

Methodology

In studying the structure of a national (or regional) economy, let us suppose that we have to find the vector that maximizes the total output attainable in the next period. Formally, let us call the initial final demand vector $\mathbf{y^s}$ and the corresponding output vector $\mathbf{x^s}$, given by the input-output relation $\mathbf{x^s} = \mathbf{Ly^s}$. Given a neighborhood β of $\mathbf{y^s}$, $V(\mathbf{y^s},\beta)$, the objective is to find the vector $\mathbf{y^*} \in V$ such that the distance between $\mathbf{x^*}(\mathbf{y^*})$ and $\mathbf{x^s}$ is maximum.

Note that this is not a case of calculating the output growth resulting from a unitary increase in final demand. This problem is easily dealt with by using traditional multipliers. In this case, what we want is to find, from among all the vectors at a certain distance of $\mathbf{y^s}$, the vector that maximizes the variation of the resulting output vector in relation to the initial vector, $\mathbf{x^s}$.

Let us consider, for the sake of simplicity, that $\beta = 1$. In this case, a vector at a unitary distance of $\mathbf{y^s}$ is not necessarily a final demand vector in which the sum total of all its elements exceeds the sum total of all the elements of the initial vector by exactly one monetary unit. This is only true when all of the (unitary) increase in final demand is concentrated in one sector. In general, and excluding this particular case, it is a vector that represents a monetary expenditure that is more than one unit higher than the total expenditure of vector $\mathbf{y^s}$.

Particularly in studies of economic growth it is much more interesting to consider the output impacts of final demand vectors at a given distance from an initial vector than merely considering the output growth of unitary increases in final demand.

Suppose that we want to study the impact upon the distance from the initial output vector $\mathbf{x^s}$ to the vector $\mathbf{x^*}$ of a change in final demand from $\mathbf{y^s}$ to $\mathbf{y^*}$, in which:

$$\sum \left(y_j^* - y_j^s\right)^2 = \beta^2$$

It is a case of maximizing (with β equal to *1*, according to our hypothesis):

$$(\mathbf{x}^* - \mathbf{x}^s)' (\mathbf{x}^* - \mathbf{x}^s), \text{ (the prime means transpose)}$$

subject to:

$$(\mathbf{y}^* - \mathbf{y}^s)' (\mathbf{y}^* - \mathbf{y}^s) = 1$$

As $\mathbf{x^s} = \mathbf{L}\, \mathbf{y^s}$, the corresponding *Lagrangean* is:

$$(\mathbf{y}^* - \mathbf{y}^s)' \, \mathbf{L'L} \, (\mathbf{y}^* - \mathbf{y}^s) - \lambda[(\mathbf{y}^* - \mathbf{y}^s)' \, (\mathbf{y}^* - \mathbf{y}^s)]$$

After differentiating and equalizing to zero:

(1) $$\mathbf{L'L} \, (\mathbf{y}^* - \mathbf{y}^s) = \lambda(\mathbf{y}^* - \mathbf{y}^s)$$

Since **L'L** is symmetric, all its eigenvalues are real. Since it a case of maximizing a definite positive quadratic form, all the eigenvalues are positive.

Furthermore, multiplying both members of (1) by $(\mathbf{y}^* - \mathbf{y}^s)'$ and considering only vectors **y** such as $(\mathbf{y}^* - \mathbf{y}^s)' \, (\mathbf{y}^* - \mathbf{y}^s) = 1$, we have:

$$(\mathbf{y}^* - \mathbf{y}^s)' \, \mathbf{L'L} \, (\mathbf{y}^* - \mathbf{y}^s) = \lambda$$

and so the maximum distance between $\mathbf{x}^*$ and $\mathbf{x}^s$ is obtained for the greatest value of λ, i.e. for the greatest eigenvalue, and the minimum distance for the smallest one.

An economy is more variable in terms of its final demand structures, the greater the amplitude of variation of the distance between $\mathbf{x}^*$ and $\mathbf{x}^s$ in response to a unitary final demand shock.

The amplitude of variation attainable for the distance between $\mathbf{x}^*$ and $\mathbf{x}^s$ can be measured by the difference $s(\mathbf{L'L}) = (\lambda_{max} - \lambda_{min})$, i.e. the *spread* of **L'L**, and it is certainly an important property of each technological structure **A** (the input coefficients matrix) and its corresponding Leontief inverse, $\mathbf{L} = (\mathbf{I}-\mathbf{A})^{-1}$.

Some linear algebra results can be used to further advance research into this property of technological structures.

It is known (Marcus & Minc*,* 1992, p.167) that:

$$2 \max c_{ij} \leq \mathrm{s}(\mathbf{L'L}) < [2\|\mathbf{L'L}\|^2 - 2/\mathrm{n}\,(tr\,\mathbf{L'L})^2]^{1/2}$$

in which by c_{ij} ($i \neq j$) we mean the off-main diagonal elements of **L'L**, and in which the norm is Euclidean, i.e. with any **N**, $\|\mathbf{N}\| = (\sum n_{ij}^2)^{1/2}$.

It is easy to see that $tr\,\mathbf{L'L} = \|\mathbf{L}\|^2$.

Furthermore, because of the properties of the general norm and Euclidean norm:

$$\|\mathbf{L'L}\| \leq \|\mathbf{L}\|.\|\mathbf{L'}\| = \|\mathbf{L}\|^2$$

so that,

$$2 \max c_{ij} \leq s(\mathbf{L'L}) < (2-2/n)^{1/2}\,\|\mathbf{L}\|^2 \approx \sqrt{2}\,\|\mathbf{L}\|^2$$

This demonstrates the importance, for this analysis, of the maximum value of the off-main diagonal values of **L'L** and of the summation of the square elements of **L**.

An increase in the value of **L** elements (i.e. the elements of **A**) necessarily leads to an increase in the elements of **L'L,** since **L** is a matrix of positive elements. If the increase is sufficiently intense, this implies that there will be an increase in the amplitude of the possible output variations in response to a unitary final demand change. With a “fuller” technological structure, the management of final demand is more important than it is with a less “full” one.

As an example, consider the case of an economy with just two sectors, in which, for the sake of simplicity, there are only identical inputs:

$$\mathbf{A} = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$$

Table 1 summarizes some possible values for *a* and *b* and the corresponding values for the *spread,* in which it is clear that this increases when the values of *a* and *b* increase.

Homothetic scale and structure effects

As we saw previously, there are two vectors of final demand variations that result in maximum output movement: the vector in which all the final demand components increase and the other vector that is symmetric to this. If we are interested in the vector of increasing output, we will consider the vector $\mathbf{\Delta y^s}$, in which all the components are positive. The corresponding output vector, $\mathbf{\Delta x^s}$**,** is $\mathbf{L\Delta y^s}$**,** and this variation can be decomposed into two components: a scale effect and a structure effect.

Without structural changes, we would have a proportional increase in all sectors

$$\Delta \mathbf{x}^s = \delta_0\, \mathbf{x}^s$$

However, in general, we do not observe this proportional change. On the contrary, $\Delta \mathbf{x}^s$ is a result of the combination of economic expansion in keeping with the existing structure and economic development as given by structural changes in the economy (an identical decomposition can be made for the "optimal" impulse vector of final demand, $\Delta \mathbf{y}^s$).

Formally

$$\Delta \mathbf{x}^s = \mathbf{SC} + \mathbf{ST}$$

where **SC** and **ST** are the *scale vector* and the *structural change vector*. Defining δ such that

$$\delta = \min\left\{\frac{\Delta x_1^s}{x_1^s}, \frac{\Delta x_2^s}{x_2^s}, \cdots, \frac{\Delta x_n^s}{x_n^s}\right\}$$

we have for the scale vector,

$$\mathbf{SC} = \delta \mathbf{x}^s$$

The vector **ST** is then obtained by

$$\mathbf{ST} = \Delta \mathbf{x}^s - \mathbf{SC}$$

Our measures for the scale and the structure effects are then the Euclidean norms of **SC** and **ST**, respectively.

In the empirical application, we present the values for the length of $\Delta \mathbf{x}^s$, **SC** and **ST**, in order to compare the effects produced in terms of scale and structural change with the overall effect.

3. An application to Iberian national and regional (island) I-O tables

In this section, we make an application of the results presented in the previous section to Portugal and Spain (national level) and two respective (island) regions, Azores and Baleares. In each case, these islands represent around 2% of the population and GDP of Portugal and Spain, respectively. For illustration purposes, the input-output tables were aggregated to seven sectors (Table 2).

Table 3 summarizes some results for the national and regional economies. In both cases, the maximum effect is stronger for the national matrix, while the minimum distance is somewhat similar. In other words, the national economy has a larger capacity of reaction to a final demand shock of unitary distance. As a consequence, the spread for the national economy (which is "fuller" than a regional one) is substantially higher than the spread obtained for the islands. Also, in all cases, the effect of structural change is much more important than the scale effect, particularly in the case of Azores, where almost all of the overall effect is originated by this component. This is in accordance with the characteristics of the Azorean economy (low diversification) and other islands, sometimes characterised by important restrictions at the level of productive structures.

4. Conclusions

In this paper, we present a new kind of intersectoral output multipliers that can be used to overcome a serious limitation of the traditional Leontief/Rasmussen multipliers, namely the obligation to consider a fixed (predetermined) structure of final demand.

By solving a properly designed extremum problem, one can calculate the impact on sectoral outputs of a shock in final demand along all vectors at a certain *Euclidean distance* from the initial final demand vector.

An important property of productive structures is the so-called *spread* associated with each technical coefficient matrix, giving the difference between the maximizing and the minimizing impacts.

In the maximizing case, an interesting exercise consists of decomposing the total impact into two effects: a homothetic scale effect, where the economy grows in accordance with the initial structure; a structure effect, shown by the change in structure that is brought about by the maximizing purpose in hand.

An empirical exercise is made in the paper, using Portuguese national and regional (Azores) input-output tables and also Spain and Baleares data. The findings support the idea that, in general, a regional economy has a lower spread than the national economy that includes it. Also, structural changes seem to be much more important that scale changes, particularly in the case of Azores. This may be a characteristic of outermost regions, were the productive structure have severe limitations. The policy implications of these results for outermost regions in Europe must be further investigated, given the practical concern and importance of this regional policy in the European context.

Table 1: *Spread* of a 2x2 matrix A for different values of *a* and *b*

		b									
		0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
a	0	0	0.41	0.87	1.45	2.27	3.56	5.86	10.77	24.69	99.72
	0.1	0	0.56	1.21	2.08	3.41	5.74	10.67	24.61	99.65	
	0.2	0	0.81	1.78	3.17	5.56	10.52	24.49	99.56		
	0.3	0	1.22	2.77	5.25	10.28	24.31	99.41			
	0.4	0	1.96	4.69	9.88	24.00	99.17				
	0.5	0	3.47	9.07	23.44	98.77					
	0.6	0	7.11	22.22	97.96						
	0.7	0	18.75	96.00							
	0.8	0	88.89								
	0.9	0									

Table 2. Sectors used in section 3

1	Agriculture, hunting, forestry and fishing
2	Industry
3	Construction
4	Auto, Hotels and Restaurants
5	Transport and Communications
6	Financial services, real estate services
7	Other services

Table 3. Results for Portugal, Spain, and the islands Azores and Baleares

	Portugal 1999	Azores 1998	Spain 2000	Baleares 2004
λ_{max}	3.11	2.18	3.6	2.52
λ_{min}	0.96	0.93	0.96	0.90
Spread	2.15	1.25	2.63	1.63
SC: scale effect	0.52	0.25	0.56	0.56
ST: structural change effect	1.29	1.29	1.38	1.08
SC+ST	1.81	1.54	1.94	1.64

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