Expanding Extractions

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ABSTRACT. In this paper we generalize hypothetical extraction techniques. We suggest that the effect of certain economic phenomena can be measured by removing them from an input-output table and by rebalancing the set of input-output accounts. The difference between the two sets of accounts yields the phenomenon's effect (or importance). We suggest that the approach can be used to measure the effect of changes in intermediate output, which are otherwise not easily rationalized within a Leontief framework. Of course, it also can be used to estimate the possible effects of the shutdown of a particular establishment or other identifiable segment of an economy. We demonstrate some properties and potential of the approach using an annual 2006 U.S. input-output accounts.

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1. Introduction

The importance of an industry or set of industries to an economy has been matter of interest for some time. In particular addressing the issue from a development perspective has led to the concept of so-called key sectors (e.g., Rasmussen, 1957; Chenery and Watanabe, 1958). One prominent method used to identify key sectors is a technique called "hypothetical extraction." First put forward by Paelinck, de Caevel, and Degueldre (1965) and Miller (1966), this technique is used by analysts to quantify how much an economy's total output would decrease if a particular industry were not present—hence, the term hypothetical extraction.[1] The quantification is effected using the economy's input-output accounts. One deletes the industry by setting its row and/or column (including final demand) in the accounts to zero. In the new (i.e. hypothetical, constructed) Leontief system, one then calculates the vector of industry gross outputs that satisfies the given vector of industry final demands. The difference before and after extraction, in terms of total gross output volume (i.e. summed over the industries) then indicates the importance of the extracted industry to the entire economy. Typically, large industries and industries that are highly interconnected in the country's or region's production structure are found to be important. An interconnected industry strongly depends on the other industries and vice versa. Its extraction therefore affects many other industries and thus total gross output, adding to its importance.

In any case, it occurred to us that policymakers are frequently interested in estimates of the economic effects of potential decisions that focus upon the fortunes of a particular establishment or segment of an industry. Moreover, as suggested by Ritz and Spaulding (1975), Szyrmer and Walker (1983), and Szyrmer (1986, 1992), policymakers are also often interested in the economywide impacts of industry *changes in the intermediate output* rather than of industry *changes in final demand*, the latter of which comport with assumptions of the standard Leontief model. Finally, rather than identifying key sectors some analysts may prefer to see how such changes in intermediate output

[1] Miller and Lahr (2001) provide a detailed overview of the literature and the various methods that have been used.

might affect just a part of the economy, rather than its entirety—the goal of key sector analysis. Thus, it seemed to us that an approach that generalizes the traditional extraction technique and facilitates, for example, partial extraction of an industry could prove to be as useful as it would be novel.

In Section 2 we start off our investigation by discussing the hypothetical extraction method in some detail. We indicate three shortcomings (i.e. cases that are not covered). Next we show how these can be remedied straightforwardly by generalizing the existing taxonomy of hypothetical extractions. The generalized hypothetical extraction method that we propose allows for a wide range of applications. We focus on two of such applications in Sections 3 and 4. Both deal with partial extraction, and we apply them to the U.S. economy in 2006. Section 5 concludes.

2. Hypothetical Extraction Methods and their Generalization

When applying the hypothetical extraction, analysts have traditionally calculated the decrease in total gross output under the hypothesis that some industry k is no longer present in the system (see Paelinck *et al*., 1965; Miller, 1966; Strassert, 1968). Let there be n industries and denote the interindustry flows by the matrix $\mathbf{Z}$. Denote the vector of industry final demands by $\mathbf{f}$ and the vector of industry gross outputs by $\mathbf{x}$.[2] The accounting equations are given as $\mathbf{x} = \mathbf{Ze} + \mathbf{f}$, where $\mathbf{e}$ is the summation vector consisting of ones. The economy's direct requirements matrix is given by $\mathbf{A}$, where its typical element is defined as $a_{ij} = z_{ij} / x_j$, or $\mathbf{A} = \mathbf{Z}\hat{\mathbf{x}}^{-1}$ using matrix notation. Substitution into the accounting equations gives $\mathbf{x} = \mathbf{Ax} + \mathbf{f}$. For any given final demand vector $\mathbf{f}$, the gross outputs that are necessary to satisfy $\mathbf{f}$ are given by $\mathbf{x} = (\mathbf{I} - \mathbf{A})^{-1}\mathbf{f} = \mathbf{Lf}$, where $\mathbf{I}$ is the identity matrix and $\mathbf{L} \equiv (\mathbf{I} - \mathbf{A})^{-1}$ is the Leontief inverse or multiplier matrix.

Extracting industry k implies that the kth row and column of $\mathbf{A}$ are set equal to zero. Let us indicate this matrix by $\overline{\mathbf{A}}$. The deliveries formerly provided by this industry are presumably met by imports. The same applies to the final demand for the goods and

[2] Matrices are indicated by boldfaced capital letters (e.g. $\mathbf{A}$), vectors are columns by definition and are indicated by boldfaced lowercase letters (e.g. $\mathbf{x}$), and scalars (including elements of matrices or vectors) are indicated by italicized lowercase letters (e.g. c or α). A prime indicates transposition (e.g. $\mathbf{x}'$) and a hat (or circumflex) indicates a diagonal matrix (e.g. $\hat{\mathbf{x}}$) with the elements of a vector (i.e. $\mathbf{x}$) on its main diagonal and all other entries equal to zero.

services provided by industry k. That is, $f_k = 0$ which gives the "new" final demand vector $\bar{\mathbf{f}}$. In the case where the sector is hypothetically eliminated, the vector of industry gross outputs is estimated as $\bar{\mathbf{x}} = (\mathbf{I} - \bar{\mathbf{A}})^{-1}\bar{\mathbf{f}}$. Then to measure the difference after and before extraction one must find the difference $\mathbf{e}'(\bar{\mathbf{x}} - \mathbf{x})$, which is always negative, indicating a reduction.[3]

Over time, several extensions to the afore-described hypothetical extraction method have been introduced, so that it seems reasonable to speak about hypothetical extraction methods. Typically they proposed the extraction of sets of industries or parts thereof. Let the matrix $\mathbf{A}$ be partitioned as follows (see, e.g. Cella, 1984)[4]

$$\mathbf{A} = \begin{bmatrix} \mathbf{A}_{11} & \mathbf{A}_{12} \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{bmatrix} \tag{1}$$

where the submatrices $\mathbf{A}_{11}$ and $\mathbf{A}_{22}$ are square of size $k \times k$ and $(n-k) \times (n-k)$, respectively. The sizes of $\mathbf{A}_{12}$ and $\mathbf{A}_{21}$ therefore are $k \times (n-k)$ and $(n-k) \times k$, respectively. For example, extracting the first k industries yields $\mathbf{A}_{11} = \mathbf{A}_{12} = \mathbf{A}_{21} = 0$, which is case 1 in Miller and Lahr's (2001) overview. They distinguish six other cases: $\mathbf{A}_{12} = \mathbf{A}_{21} = 0$ (case 2a); $\mathbf{A}_{11} = \mathbf{A}_{21} = 0$ (case 2b); $\mathbf{A}_{11} = \mathbf{A}_{12} = 0$ (case 2c); $\mathbf{A}_{12} = 0$ (case 3a); $\mathbf{A}_{21} = 0$ (case 3b); $\mathbf{A}_{11} = 0$ (case 3c).

We see three shortcomings in the currently available set of hypothetical extraction methods. First, the format of the partitioned matrix $\mathbf{A}$ is rather restrictive. For example, in an interregional framework, one might be interested in the importance of the imports bought from the other regions by industry k in region S (see e.g. Dietzenbacher and van der Linden, 1997). The corresponding extraction cannot be expressed by using the partitioned form in (1). An example in a single-economy framework that does not fit the format in (1) is finding the importance of the sales of industry k to industry h. The set of extractions summarized by Miller and Lahr (2001) focuses strictly on each industry's or

[3] Some studies focus on interindustry linkages and measure the effect of extraction on the *other* industries' total gross output. This would yield $\mathbf{e}'(\bar{\mathbf{x}} - \mathbf{x}) + x_k$.

[4] Discussing interregional feedbacks, Miller (1966, 1969) used a similar setting for regions, while Dietzenbacher *et al*. (1993) proposed the regional extraction method.

region's effect on the rest of the economy. The solution to this shortcoming is straightforward. In the first example set $\overline{a}_{ij}^{RS} = 0$ for $j = k$, $i = 1, \ldots, n$ and $R \neq S$, and $\overline{a}_{ij}^{RS} = a_{ij}^{RS}$ otherwise. In the latter example, we would have $\overline{a}_{ij} = 0$ for $i = k$ and $j = h$, and $\overline{a}_{ij} = a_{ij}$ otherwise.

The second shortcoming is that the literature on extractions focuses entirely on nullification. Although this may have been a natural starting point for the early studies (of extracting an entire industry from the economy), it is somewhat surprising that nullification is still the overriding principle. From an analytical point of view, there is no reason why one could not handle a partial extraction. From an economic or policy point of view, there are several cases that call for a partial extraction (in contrast to full extraction, i.e. nullification). In the next two sections we will focus on two such cases. The case in Section 3 reflects the situation were industry k is confronted with capacity constraints. These result in a uniform percentage decrease of the elements in the kth row of the matrix **A**, except for its diagonal element (because the kth column—reflecting the production process—does not alter). The case in Section 4, considers the effect of a partial extraction in industry k that is caused by the full extraction of one of its sub-industries.

Furthermore, from a policy point of view it seems unrealistic to focus on only a single economywide indicator. Often conflicting interests play a role and the gains in terms of one aspect come at the price of a loss in terms of another aspect. For example, for the extremely arid Spanish region of Andalusia, Dietzenbacher and Velázquez (2007) found that nullifying the exports of agricultural products to other countries (but not to other Spanish regions) would induce a substantial reduction in water consumption, whereas the decrease in total value added and total employment would only be very modest.

The third shortcoming in the literature on hypothetical extraction methods is the almost exclusive focus on total gross output.[5] Instead, economic analysts typically tend to

[5] Several authors (c.f., Meller and Marfán 1981; Groenewald, Hagger, and Madden, 1987, 1993) have produced employment outcomes in this context. Note that have typically been those researchers who provided applications in developing nations where concerns of social discord may be close to the surface. As a result, improving employment possibilities is legitimately a more-prominent societal goal than is

prefer considering the change's effects on "the whole economy" in terms of its citizens' welfare, which is typically measured by total value added (and also known as gross domestic product or GDP). Let v_i denote the value added coefficient (i.e. value added in industry *i* per unit of its gross output). Value added multipliers are then defined as $\boldsymbol{\mu}' = \mathbf{v}'\mathbf{L}$, where μ_i gives the (extra) total value added (directly and indirectly) generated by one (extra) unit of final demand of product *i*. For total value added we thus have $VA = \boldsymbol{\mu}'\mathbf{x} = \boldsymbol{\mu}'\mathbf{Lf}$. The same applies to any other overall phenomenon. For example, for the total amount of SO_2 emissions, define v_i as the direct emission coefficient (indicating emissions per unit of gross output *i*) and μ_i as the emission multiplier (indicating the total amount of emissions generated by one unit of final demand *i*). For example, for the total gross output, define $v_i = 1$ for all *i*, then μ_i gives the standard output multiplier.

In conclusion, we advocate expansion of the existing extractions. Our generalized hypothetical extraction method is rather simple. To finding the importance (no matter in which respect) of a phenomenon that can be measured in terms of a transaction or set of transactions, one need only remove those related transactions from the input-output table and/or model (set them to zero), re-run the model, and find the difference between the two sets of computations. That difference is an indicator (or set of indicators) of the importance of the pheomenon. That is, since removing the phenomenon from the table typically changes $\boldsymbol{\mu}$, $\mathbf{L}$ and $\mathbf{f}$ into $\bar{\boldsymbol{\mu}}$, $\bar{\mathbf{L}}$ and $\bar{\mathbf{f}}$, the indicator of the phenomenon's importance is given by $\bar{\boldsymbol{\mu}}'\bar{\mathbf{L}}\bar{\mathbf{f}} - \boldsymbol{\mu}'\mathbf{Lf}$.

3. Partial Extraction in the Case of Capacity Constraints

In this section we will use partial extraction to analyze the effects of capacity constraints. We assume that an industry consists of a number of identical establishments, one of which ceases to exist so that the industry's capacity reduces. The intermediate deliveries sold by this industry (say *k*) then decrease by, say, a fixed percentage α of all deliveries it makes. The deliveries formerly provided by this establishment are presumably either no longer demanded or are met by sources outside of the "local" economy, i.e. they are

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increasing the average wealth of their citizenry, given the ever-increasing income disparities and simultaneous decreasing fortunes of a surprisingly large share of their populations.

imported. Since its output x_k decreases, industry *k*'s input set z_{ik} (for all *i*) also decreases by the same percentage. As a consequence, the *k*th column of the economy's direct requirements matrix **A** remains unchanged. That is, for all *i* = 1,…,*n* we have $\bar{a}_{ik} = \bar{z}_{ik} / \bar{x}_k = (1-\alpha)z_{ik} /(1-\alpha)x_k = a_{ik}$, where (as earlier) an overbar indicates the case of partial extraction. All elements but the diagonal element of the *k*th row of **A**, however, decrease by α. That is, for all *j* = 1,…,*n* ($j \neq k$), $\bar{a}_{kj} = \bar{z}_{kj} / \bar{x}_j = (1-\alpha)z_{kj} / x_j = (1-\alpha)a_{kj}$. Clearly, $0 \le \alpha \le 1$ and note that in the case $\alpha = 1$ we have that $\bar{a}_{kj} = 0$ for all $j \neq k$, which corresponds to case 2c in Miller and Lahr's (2001, p. 418) review of hypothetical extraction methods.[6] In matrix notation we have

$$\overline{\mathbf{A}} = \mathbf{A} - \alpha \mathbf{e}_k \mathbf{b}'_k \tag{2}$$

where $\mathbf{e}_k$ indicates the *k*th unit vector with a one in element *k* and zeros elsewhere. $\mathbf{b}'_k = (a_{1k}, a_{2k}, ..., a_{k,k-1}, 0, a_{k,k+1}, ..., a_{kn})$. We, therefore, have $\mathbf{I} - \overline{\mathbf{A}} = \mathbf{I} - \mathbf{A} + \alpha \mathbf{e}_k \mathbf{b}'_k$. Because $\overline{\mathbf{A}}$ is sum of the prior direct matrix **A** and another matrix, one can readily express its Leontief inverse using techniques—for the inverse of the sum of two matrices—summarized in an excellent review by Henderson and Searle (1981). This yields

$$\overline{\mathbf{L}} = \mathbf{L} - \frac{\alpha \mathbf{L} \mathbf{e}_k \mathbf{b}'_k \mathbf{L}}{1 + \alpha \mathbf{b}'_k \mathbf{L} \mathbf{e}_k} \tag{3}$$

For element (i, j) of the difference between the two Leontief inverses we then have

$$\bar{l}_{ij} - l_{ij} = -\frac{\alpha \mathbf{e}'_i \mathbf{L} \mathbf{e}_k \mathbf{b}'_k \mathbf{L} \mathbf{e}_j}{1 + \alpha \mathbf{b}'_k \mathbf{L} \mathbf{e}_k} = -\frac{\alpha l_{ik} \mathbf{b}'_k \mathbf{L} \mathbf{e}_j}{1 + \alpha \mathbf{b}'_k \mathbf{L} \mathbf{e}_k} \tag{4}$$

[6] This particular extraction case appears to have been first put forward in an internal departmental memo by Ritz and Spaulding (1975). Szyrmer and Walker (1983) and Szyrmer (1986, 1992) independently delivered it to the realm of refereed publications, however.

To further elaborate this expression write the kth row of $\mathbf{A}$ as $\mathbf{a}'_k = (a_{1k}, a_{2k}, ..., a_{kn})$ and note that $\mathbf{a}'_k \mathbf{L} = \mathbf{e}'_k \mathbf{AL} = \mathbf{e}'_k (\mathbf{L} - \mathbf{I})$ the elements of which are given by $l_{kj} - \delta_{kj}$, where $\delta_{kj} = 1$ if $j = k$ and zero otherwise. Next observe that $\mathbf{b}'_k = \mathbf{a}'_k - a_{kk}\mathbf{e}'_k$, then (4) yields

$$\bar{l}_{ij} - l_{ij} = -\frac{\alpha l_{ik}[(1-a_{kk})l_{kj} - \delta_{kj}]}{1+\alpha[(1-a_{kk})l_{kk} - 1]} \tag{5}$$

One can handle final demands in one of two ways. First, they can remain the same. This case corresponds to the situation where one establishment in industry k has ceased to exist and therefore the deliveries of industry k to *any* other industry decrease. The establishment's final demands, however, are somehow met by the remaining establishments in industry k. Recent empirical literature using establishment level panel data by industry highlights how such a scenario might occur. (Tybout, 2003, provides a review.) It turns out that in any given manufacturing industry's establishments for which exports compose a relatively large share of their outputs tend to be less apt to shut down than are equivalently sized domestic establishments. Thus establishments that cater to intermediate industries and that do not deliver to final demand would appear to have a greater propensity to fail. The failure of such establishments would reduce the industry's output and yet maintain the industry's full ability to meet final demand.

Such a scenario would be expressed as $\bar{\mathbf{x}} - \mathbf{x} = (\bar{\mathbf{L}} - \mathbf{L})\mathbf{f}$ and yields

$$\bar{x}_i - x_i = -\lambda_k l_{ik} = -\frac{\alpha[(1-a_{kk})x_k - f_k]}{1+\alpha[(1-a_{kk})l_{kk} - 1]} l_{ik} \tag{6}$$

This implies that the ratio between the changes in the outputs of two industries equals the ratio between the corresponding elements of the kth column of the Leontief inverse. That is, $\Delta x_i / \Delta x_j = l_{ik} / l_{jk}$ and is independent of the scale of the industry's extraction (i.e. α).[7]

Of course, from a policy perspective there may be but modest interest in the effect of such a change on a single industry's in the value of shipments, which is the nominal

[7] Analyzing the spillovers of a product innovation, Dietzenbacher (2000) arrived at a similar result.

unit of measurement embodied in x_i alone. Other measures are often used to measure impacts of economic change: they are employment (or jobs) and value added—the latter is typically decomposed into a few of its major components, mostly labor income (both including and not including taxes withheld from payrolls) and tax revenues for the economy's various jurisdictional levels.[8] In this paper, we focus upon total value added since this is a measure of economic well being preferred by economists, although often conditions of an analysis may call for any of the other measures as well. To derive the change in total value added, we have

$$\overline{VA} - VA = \Sigma_i v_i (\bar{x}_i - x_i) = -\lambda_k \Sigma v_i l_{ik} = -\lambda_k \mu_k \tag{7}$$

Note that while final demand need not change in the wake of the shuttering of an establishment, it undoubtedly would. This leads us to the second way to handle final demands. In this case, when an establishment in industry *k* that delivers to final demand ceases all production, its deliveries to final demand must also stop. That is, $\bar{f}_k = (1-\alpha) f_k$. The changes in the vector of gross outputs then yield $\bar{\mathbf{x}} - \mathbf{x} = \bar{\mathbf{L}}\bar{\mathbf{f}} - \mathbf{L}\mathbf{f}$, so that (6) is replaced by

$$\bar{x}_i - x_i = -\frac{\alpha[(1-a_{kk})x_k - f_k]}{1+\alpha[(1-a_{kk})l_{kk} - 1]} l_{ik} - \alpha \bar{l}_{ik} f_k \tag{8}$$

Using (5), we have

$$\bar{l}_{ik} = l_{ik} - \frac{\alpha l_{ik}[(1-a_{kk})l_{kk} - 1]}{1+\alpha[(1-a_{kk})l_{kk} - 1]} = l_{ik} \frac{1}{1+\alpha[(1-a_{kk})l_{kk} - 1]}$$

[8] Note that measurement of profit-type income is rather problematic in static economic models. This is because such income is in reality quite volatile from one year to the next since it moves with the business cycle and, thus, sometimes runs negative for certain industries. Indeed, some economic analysts adjust value added (gross domestic product) from government-published values to assure that profit-type income is nonnegative (at least zero) for all industries, save perhaps the passenger transit industry, which is typically subsidized heavily by governments (and for good reasons). This may not be such a strong assumption within input-output models, which paint a long-run view of economic change (McGregor, Swales, and Yin, 1996) since, if they are to survive, industries cannot persistently run on negative profits.

and substituting this into (8) gives

$$\bar{x}_i - x_i = -\tilde{\lambda}_k l_{ik} = -\frac{\alpha[(1-a_{kk})x_k]}{1+\alpha[(1-a_{kk})l_{kk}-1]} l_{ik} \tag{9}$$

For the total value added, we thus get

$$\overline{VA} - VA = \Sigma_i v_i (\bar{x}_i - x_i) = -\tilde{\lambda}_k \Sigma v_i l_{ik} = -\tilde{\lambda}_k \mu_k \tag{10}$$

which is similar to (7) except that λ_k has been replaced by $\tilde{\lambda}_k$. Comparing these two scalars it is obvious that $\tilde{\lambda}_k > \lambda_k$, except when $f_k = 0$ in which case the two assumptions for final demand coincide. This implies that the decrease in gross output in (9)—the case when final demand also is reduced—is larger than the decrease in (6) where final demand remains constant, which fulfills expectations. It is somewhat surprising, however, that reducing final demand as well as intermediate deliveries, ramps us the decrease in output by exactly the same percentage in each industry. That is, the ratio between the two expressions in (9) and (6) is independent of *i* and equals $\tilde{\lambda}_k / \lambda_k$.

Our empirical application is based on the 65-industry (so-called "summary") annual U.S. input-output table for 2006, which we have aggregated for the present purpose to 15 aggregate industries, or "sectors" as the BEA terms them. The results are given in Table 1 as percentage decreases. That is, $-100(\overline{VA} - VA)/VA$ Lowering the intermediate deliveries to other sectors by 10% decreases total value added by as much as 1.3% for Professional and business services (sector 11) and as little as 0.0% for Educational services, health care, and social assistance (sector 12). When also the sector's final demand is decreased by 10%, the reduction in total value added becomes much larger (as alluded to above). The reduction now ranges from 0.1% for Agriculture, forestry, fishing, and hunting (sector 1) at the lower end, to 2.6% for Finance, insurance, real estate, rental, and leasing (sector 10) at the upper end.

INSERT TABLE 1

As is clear from the equations, size matters. For quite a number of sectors we see that their score is similar for both calculations, i.e. excluding (or, not extracting) and including final demand. A 10% extraction in sectors 1 (Agriculture, forestry, fishing, and hunting), 2 (Mining), or 3 (Utilities) yields a small decrease in total value added. In sectors 5 (Manufacturing), 10 (Finance, insurance, real estate, rental, and leasing), and 11 (Professional and business services) the effect is large, while sector 9 (Information) takes an intermediate position for both calculations. At the same time, however, some other sectors exhibit a rather large difference between the two exercises. For example, if the extraction is only with respect to the intermediate deliveries sectors 14 (Other services, except government) and 15 (Government) reach the same 0.1% reduction in total value added. In case also the final demand is reduced by 10%, the value added decrease becomes 0.4% for sector 14 but no less than 1.9% for sector 15. Clearly, it is not just size that matters, but also final demand's share of gross output.

The ratio between the two outcomes is given by

$$\frac{(\overline{VA}-VA)_{including}}{(\overline{VA}-VA)_{excluding}}=\frac{\widetilde{\lambda}_k}{\lambda_k}=\frac{(1-a_{kk})x_k}{(1-a_{kk})x_k-f_k}=1+\frac{1}{(1-a_{kk})(x_k/f_k)-1} \tag{11}$$

where the denominator in the last term is always positive because $x_k=\Sigma_j a_{kj}x_j+f_k>$ $a_{kk}x_k+f_k$ (unless $a_{kj}=0$ for all $j\neq k$, in which case there is nothing to extract). It then follows that sectors that sell a relat8ively large share of their output as intermediate deliveries, x_k/f_k is also relatively large and the difference between the two calculations is relatively small. Calculating the ratio in (11), the smallest outcome is 1.58/1.26 for sector 11 (Professional and business services), which is a sector that largely focuses on delivering its services to other intermediate sectors. The largest outcome (1.11/0.04) is for sector 12 (Educational services, health care, and social assistance), which delivers services that are consumed almost entirely by households.

The results obtained from equations (7) and (10) differ for different values of α and those in Table 1 are for α = 0.1. Figure 1 gives the decrease in total value added when a α•100% extraction takes place in Manufacturing (sector 5). Note that $0 \le \alpha \le 1$, where $\alpha = 0$ indicates no extraction (and hence no decrease in total value added) and $\alpha = 1$ corresponds to full extraction of the entire sector. In Figure 1, α = 0, 0.01, 0.02, …, 0.99, 1.

INSERT FIGURE 1

The two graphs in Figure 1 are upward sloping—as might have been expected—indicating that larger capacity constraints lead to larger reductions in total value added. Somewhat surprising, however, is that the graphs suggest a linear relationship between α and the percentage decrease in total value added. meanwhile, equations (7) and (10) clearly demonstrate that the relationship is nonlinear. The crucial part that determines the nature of the relationship is given by

$$\frac{\alpha}{1+\alpha[(1-a_{kk})l_{kk}-1]} = \frac{\alpha}{1+0.04\alpha} \approx \alpha$$

where we have used the data for sector 5, i.e. $a_{55} = 0.2493$ and $l_{55} = 1.3829$, and $0 \le \alpha \le 1$. So, although the relationship between α and the percentage decrease in total value added is nonlinear, it is *very nearly* linear. It thus follows that in this application the result for partial extraction can well be estimated from the result for full extraction. That is, if full extraction yields $\overline{VA} - VA = \beta$, then partial extraction of a share α yields $\overline{VA} - VA = \alpha\beta$.

4. Partial Extraction in the Case of Fully Extracting a Sub-industry

In this section we consider the effects of partial extraction for the case where detailed information by establishment is available. Of course, establishments in most industries are rather heterogeneous. As a result, when an establishment ceases to exist, the row for arbitrary industry *k* in the matrix **A** will not then decrease by a fixed percentage as suggested in the last section of this paper. Moreover, because establishments have different production processes, changes in the *k*th column of **A** also must occur when an establishment shuts down. To mimic this situation, we start from the 65-industry input-output table and its 15-sector aggregate. For example in the 15-sector rendition, sector 2 (Mining) consists of three industries (2.1: Oil and gas extraction, 2.1: Mining, except oil and gas, and 2.3: Support activities for mining). We successively extract each of the industries 2.1, 2.2, and 2.3 from the accounts. When we extract industry 2.1, economy-wide total value added is calculated from the 65-industry model as $VA^{(2.1)} = (\boldsymbol{\mu}^{(2.1)})'\mathbf{L}^{(2.1)}\mathbf{f}^{(2.1)}$, where $\boldsymbol{\mu}^{(2.1)}$ is the vector with value added coefficients in which the element for industry 2.1 has been set to zero, $\mathbf{L}^{(2.1)} = (\mathbf{I} - \mathbf{A}^{(2.1)})^{-1}$ with $\mathbf{A}^{(2.1)}$ the input matrix in which the row and column for industry 2.1 have been set to zero, and $\mathbf{f}^{(2.1)}$ is the final demand vector in which the element for industry 2.1 has been set to zero.

Next we aggregate the 65-industry matrix $\mathbf{A}^{(2.1)}$ and vectors $\boldsymbol{\mu}^{(2.1)}$ and $\mathbf{f}^{(2.1)}$ to 15 sectors. Let the aggregates be denoted by $\overline{\mathbf{A}}^{(2.1)}$, $\overline{\boldsymbol{\mu}}^{(2.1)}$ and $\overline{\mathbf{f}}^{(2.1)}$, respectively. Note that the full extraction of industry 2.1 (Oil and gas extraction) at the 65-industry level means a partial extraction of sector 2 (Mining) at the aggregated 15-sector level. Using the 15-sector level model, the total value added is given by $\overline{VA}^{(2.1)} = (\overline{\boldsymbol{\mu}}^{(2.1)})'\overline{\mathbf{L}}^{(2.1)}\,\overline{\mathrm{f}}^{(2.1)}$. This means that for partial extraction of sector 2, we have 3 cases: fully extracting industry 2.1, fully extracting industry 2.2, and fully extracting 2.3.

In Table 2, we present the percentage decrease in total value added caused by each successive extraction of the 65 industries and see its effects on both the 65-industry model and the 15-sector model. That is, at the 65-industry level we have calculated, for example, $-100(VA^{(2.1)} - VA)/VA$ and at the 15-sector level $-100(\overline{VA}^{(2.1)} - VA)/VA$. First, note that corresponding results across the two agregations are surprisingly close. This indicates that in moving from a U.S. economy model of 65 industries to one of 15 sectors, the results from partial extraction do not suffer much from aggregation bias. Typically, performing calculations on input-output tables at a detailed level (with 65 industries in our case) and aggregating afterwards (so as to obtain values added for the 15 sectors) yields a substantially different answer than aggregating first (to 15 sectors in our case) and then calculating the value added for each sector. For Table 2 the two sets of results (with values added for 15 sectors) have been further aggregated to yield the overall differences in value added. It seems that positive and negative biases from aggregation cancel each other.

INSERT TABLE 2

The second observation is the large variability in results. Extracting Apparel and leather and allied products (5.3), Transit and ground passenger transportation (8.5) or Pipeline transportation (8.6) reduces GDP only by 0.2%, whereas extracting Real estate (10.5) yields a 15.4% reduction.[9] The amount of decrease reflects the importance of an industry for the economy in terms of—in this paper—its GDP. The extraction method (no matter whether partial or full) takes two relevant aspects into consideration. On the one hand, it accounts for the size of the industry, as denoted by its gross output. That is, extracting large industries tends to lead to larger reductions in value added than do extractions of industries with lower levels of gross output. On the other hand, it also accounts for the interconnectedness of the industry in the economy. The more intrinsically connected an industry is with the rest of the economy, the more it depends on the fortunes of other industries and vice versa. As a consequence, the more an industry is connected to the rest of the economy, the larger is the set of reductions its demise will induce in the form of lost value added in other industries.

Industries whose extraction yields a decrease in GDP larger than 10% are: Real estate (industry 10.5 with 15.4%) and State and local government (industry 15.4 with 11.5%). Industries where extraction induces overall GDP reductions between 5 and 10% are: Retail trade (sector 7 with 9.7%), Construction (sector 4 with 8.5%), Wholesale trade (sector 6 with 8.0%), Miscellaneous professional, scientific and technical services (industry 11.2 with 6.6%), Federal general government (industry 15.2 with 5.6%), and

[9] It is undoubtedly worthwhile to note here that extraction of some industries makes little sense. In this case, what does it mean to extract the real estate sector? Outside of outlawing such a sector or of ceding central government ownership of all land, it is not clear how the real estate sector would be eliminated from an economy. Regardless, it is fairly clear that at least the management of properties, if not the sale of it as well, will continue with or without the existence of a formal real estate sector.

Federal Reserve banks, credit intermediation, and related activities (industry 10.1 with 5.3%).

A third observation is the large variability of the results, also within a sector. Recall that the full extraction of an industry means a partial extraction of the sector (unless the sector consists of only one industry, which is the case for 5 sectors: Utilities, Construction, Wholesale trade, Retail trade, and Other services except government). The total value added decrease from partial extraction of a given sector due to the full extraction of one industry may differ largely from the result due to the full extraction of another industry. The most significant sectors in this respect are: Government (sector 15) with total value added decrease ranging from 0.7% to 11.5%; Finance, insurance, real estate, rental, and leasing (sector 10, range 0.8−15.4%); and Educational services, health care, and social assistance (sector 11, range 0.4−6.6%).

5. Conclusion

The body of literature on the traditional hypothetical extraction method focuses on calculations of the total gross output lost when one of the economy's industries ceases to exist after some fashion. The variants of possible hypothetical extractions covered by the literature cut a rather large swath of possibilities. Nonetheless, we feel that they remain overly restrictive. In this paper, we have noted three shortcomings of the usual extraction methods. First, array of possibilities presented to date yield implications of industry extractions only to the rest of the economy. Their mathematical formulations suggest that other choices are inappropriate or not possible. Second, extraction methods are implemented by nullifying certain coefficients and variables. The nullification of sectors in the techniques is complete, thus they cannot deal with the possible partial elimination of a sector. Although somewhat less problematic, the third shortcoming is that, except for certain cases, the applications of the various extractions have focused on the resulting decrease in the economy's total gross output as an indicator of sectoral importance. Often, however, one is more interested in an indicator based on the economy's total value added (or GDP), employment, emissions, or energy use, or even a set of such indicators.

As a remedy to these three shortcomings, we have proposed a generalized hypothetical extraction method. In a way, our generalization returns us to the roots of the hypothetical extraction method as espoused by Paelinck, de Caevel, and Degueldre (1965). In that piece, the authors measured the importance of a certain phenomenon by removing the phenomenon from the input-output table and/or model and then simply recalculating. The difference between the two sets of computations (the modified and the original) is an indicator (or set of indicators) of the phenomenon's importance. We also add that gross output is a rather non-useful measure and that analysts should use one appropriate to their application, e.g., value added, employment, labor income, tax revenues, pollution emissions, fuel usage, and water usage.

We focused our application on partial extractions. In the first exercise we examined how the U.S. economy might when industries faced proportional reductions in their productive capacities. In the second exercise, we considered the repercussions to the U.S. economy if a particular industry within a sector (i.e. an aggregate industry) would cease to exist. While we perform these exercises on industries and with U.S. input-output tables, we should be clear that the method is very general and covers many types of applications. For all intents and purposes, for example, the industries in our applications could be construed as establishments, and the broader sectors as industries. Moreover, this generalized hypothetical extraction approach would certainly answer the problem that led Oosterhaven and Stelder, (2002) to develop so-called "net multipliers," which have raised an unusually lively debate.[10]

References

Cella, Guido (1984) "The Input-Output Measurement of Interindustry Linkages," *Oxford Bulletin of Economics and Statistics*, 46, 73-84.

Chenery, Hollis B. and Tsunehiko Watanabe (1958) "International Comparisons of the Structure of Production," *Econometrica*, 26, 487-521.

Dietzenbacher, Erik (2000) "Spillovers of Innovation Effects," *Journal of Policy Modeling*, 22, 27-42.

_____ (2005) "More on Multipliers," *Journal of Regional Science*, 45, 421-426.

Dietzenbacher, Erik and Jan A. van der Linden (1997) "Sectoral and Spatial Linkages in the EC Production Structure," *Journal of Regional Science*, 37, 235-257.

Dietzenbacher, Erik and Esther Velázquez (2007) "Analysing Andalusian Virtual Water Trade in an Input-Output Framework," *Regional Studies*, 41, 185-196.

[10] See de Mesnard (2002, 2007a, 2007b), Dietzenbacher (2005), and Oosterhaven (2007).

Dietzenbacher, Erik, Jan A. van der Linden and Albert E. Steenge (1993) "The Regional Extraction Method: EC Input-Output Comparisons," *Economic Systems Research*, 5, 185-206.

Groenewold, Nicolaas, Alfred J. Hagger and John R. Madden (1987) "The Measurement of Industry Employment Contribution in an Input-Output Model," *Regional Studies*, 21, 255-263.

_____ (1993) "Measuring Industry Importance: An Australian Application," *Annals of Regional Science*, 27, 175-182.

de Mesnard, Louis (2002) "Note about the Concept of 'Net Multipliers'," *Journal of Regional Science*, 42, 545-548.

_____ (2007a) "A Critical Comment on Oosterhaven-Stelder Net Multipliers," *Annals of Regional Science*, 41, 249-271.

_____ (2007b) "Reply to Oosterhaven's: The Net Multiplier is a New Key Sector Indicator," *Annals of Regional Science*, 41, 285-296.

McGregor, Peter G., J. Kim Swales and Ya Ping Yin, 1996, "A Long-Run Interpretation of Regional Input-Output Analysis," *Journal of Regional Science* 36, 3, 479-500.

Meller, Patricio and Manuel Marfán (1981) "Small and Large Industry: Employment Generation, Linkages, and Key Sectors," *Economic Development and Cultural Change,* 29, 263-274.

Miller, Ronald E. (1966) "Interregional Feedbacks in Input-Output Models: Some Preliminary Results," *Papers, Regional Science Association*, 17, 105-125.

_____ (1969) "Interregional Feedbacks in Input-Output Models: Some Experimental Results," *Western Economic Journal*, 7, 41-50.

Miller, Ronald E. and Michael L. Lahr (2001) "A Taxonomy of Extractions," in: Michael L. Lahr and Ronald E. Miller (eds), *Regional Science Perspectives in Economic Analysis* (Amsterdam: Elsevier Science), pp. 407-441.

Oosterhaven, Jan (2007) "The Net Multiplier is a New Key Sector Indicator: Reply to De Mesnard's Comment," *Annals of Regional Science*, 41, 273-283.

Oosterhaven, Jan and Dirk Stelder (2002) "Net Multipliers Avoid Exaggerating Impacts: With a Bi-regional Illustration for the Dutch Transportation Sector," *Journal of Regional Science*, 42, 533-543.

Paelinck, Jean, J. de Caevel and J. Degueldre (1965) "Analyse Quantitative de Certaines Phénomènes du Développement Régional Polarisé: Essai de Simulation Statique d'Itéraires de Propogation," in: Bibliothèque de l'Institut de Science Économique, No. 7, *Problèmes de Conversion Économique: Analyses Théoretiques et Études Appliquées* (Paris: M.-Th. Génin), pp. 341-387.

Rasmussen, P. Nøregaard (1957) *Studies in Inter-Sectoral Relations* (Amsterdam: North-Holland).

Ritz, Philip M. and Elizabeth Spaulding. (1975) "Basic I-O Terminology," mimeo, U.S. Department of Commerce, Bureau of Economic Analysis, Interindustry Economics Division, February 25.

Strassert, Günter (1968) "Zur Bestimmung strategischer Sektoren mit Hilfe von input-Output Modellen," *Jahrbücher für Nationalökonomie und Statistik*, 182(3), 211-215.

Szyrmer, Janusz M. (1986) "Measuring Connectedness of Input-Output Models: 2. Total Flow Concept," *Environment and Planning A*, 18, 107-121.

_____ (1992) "Input-Output Coefficients and Multipliers from a Total Flow Perspective," *Environment and Planning A*, 24, 921-937.

Szyrmer, Janusz M. and Robert T. Walker (1983) "Interregional Total Flow: A Concept and Application to a U.S. Input-Output Model," *Review of Regional Studies*, 13(2), 12-21.

Tybout, James R. 2003. "Plant and Firm-Level Evidence on New Trade Theories," in: Eun Kwan Choi and James Harrigan (eds), *Handbook of International Economics.* (Oxford: Basil-Blackwell), pp. 388-415.

Table 1. Percentage decrease in value added due to a 10% decrease in the sector's deliveries

	Sector	Final demand	
		excluding, eq (7)	including, eq (10)
1	Agriculture, forestry, fishing, and hunting	0.11	0.17
2	Mining	0.19	0.27
3	Utilities	0.13	0.28
4	Construction	0.11	0.85
5	Manufacturing	0.76	2.20
6	Wholesale trade	0.37	0.80
7	Retail trade	0.14	0.98
8	Transportation and warehousing	0.26	0.45
9	Information	0.38	0.72
10	Finance, insurance, real estate, rental, and leasing	0.85	2.59
11	Professional and business services	1.26	1.58
12	Educational services, health care, and social assistance	0.04	1.11
13	Arts, entertainment, recreation, accommodation, and food services	0.14	0.70
14	Other services, except government	0.13	0.39
15	Government	0.13	1.90

Table 2. Percentage decrease in total value added due to full extraction of an industry

	Extracted industry	Full 65-level	Partial 15-level
1	*Agriculture, forestry, fishing, and hunting*		
1.1	Farms	1.42	1.43
1.2	Forestry, fishing, and related activities	0.31	0.31
2	*Mining*		
2.1	Oil and gas extraction	1.53	1.54
2.2	Mining, except oil and gas	0.45	0.45
2.3	Support activities for mining	0.77	0.78
3	*Utilities*	2.74	2.76
4	*Construction*	8.53	8.48
5	*Manufacturing*		
5.1	Food and beverage and tobacco products	3.83	3.82
5.2	Textile mills and textile product mills	0.36	0.36
5.3	Apparel and leather and allied products	0.20	0.20
5.4	Wood products	0.58	0.58
5.5	Paper products	0.86	0.86
5.6	Printing and related support activities	0.64	0.64
5.7	Petroleum and coal products	2.09	2.10
5.8	Chemical products	3.15	3.19
5.9	Plastics and rubber products	1.21	1.20
5.10	Nonmetallic mineral products	0.78	0.77
5.11	Primary metals	1.00	1.00
5.12	Fabricated metal products	1.76	1.78
5.13	Machinery	1.84	1.84
5.14	Computer and electronic products	2.13	2.13
5.15	Electrical equipment, appliances, and components	0.68	0.68
5.16	Motor vehicles, bodies and trailers, and parts	2.09	2.10
5.17	Other transportation equipment	1.14	1.13
5.18	Furniture and related products	0.54	0.54
5.19	Miscellaneous manufacturing	0.95	0.95
6	*Wholesale trade*	7.96	7.95
7	*Retail trade*	9.73	9.72
8	*Transportation and warehousing*		
8.1	Air transportation	0.83	0.86
8.2	Rail transportation	0.44	0.44
8.3	Water transportation	0.24	0.24
8.4	Truck transportation	1.58	1.60
8.5	Transit and ground passenger transportation	0.21	0.21
8.6	Pipeline transportation	0.19	0.20
8.7	Other transportation and support activities	0.90	0.91
8.8	Warehousing and storage	0.36	0.36
9	*Information*		
9.1	Publishing industries (includes software)	1.90	1.89
9.2	Motion picture and sound recording industries	0.52	0.52
9.3	Broadcasting and telecommunications	4.03	4.02
9.4	Information and data processing services	1.06	1.06

Table 2. Continued

	Extracted industry	Full 65-level	Partial 15-level
10	*Finance, insurance, real estate, rental, and leasing*		
10.1	Federal Reserve banks, credit intermediation, and related activities	5.24	5.26
10.2	Securities, commodity contracts, and investments	2.51	2.51
10.3	Insurance carriers and related activities	2.96	2.96
10.4	Funds, trusts, and other financial vehicles	0.79	0.80
10.5	Real estate	15.43	15.39
10.6	Rental and leasing services and lessors of intangible assets	1.84	1.83
11	*Educational services, health care, and social assistance*		
11.1	Legal services	1.90	1.89
11.2	Miscellaneous professional, scientific and technical services	6.66	6.64
11.3	Computer systems design and related services	1.41	1.41
11.4	Management of companies and enterprises	2.89	2.88
11.5	Administrative and support services	3.91	3.91
11.6	Waste management and remediation services	0.43	0.44
12	*Educational services, health care, and social assistance*		
12.1	Educational services	1.49	1.49
12.2	Ambulatory health care services	4.96	4.95
12.3	Hospitals and nursing and residential care facilities	4.66	4.65
13	*Arts, entertainment, recreation, accommodation, and food services*		
13.1	Social assistance	0.94	0.93
13.2	Performing arts, spectator sports, museums, and related activities	0.57	0.57
13.3	Amusements, gambling, and recreation industries	0.76	0.76
13.4	Accommodation	1.32	1.32
13.5	Food services and drinking places	3.47	3.43
14	*Other services, except government*	3.89	3.89
15	*Government*		
15.1	Federal government enterprises	0.67	0.68
15.2	Federal general government	5.63	5.64
15.3	State and local government enterprises	1.39	1.40
15.4	State and local general government	11.44	11.49

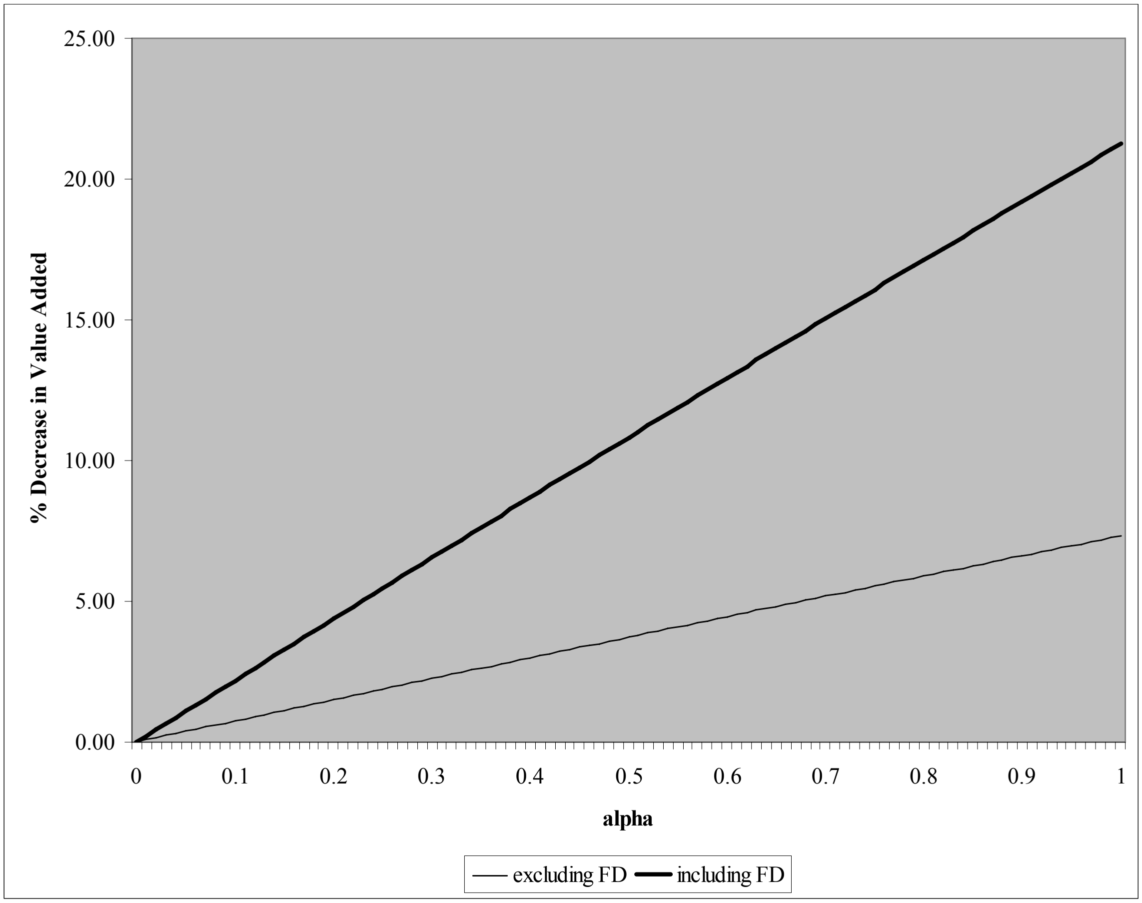

Figure 1. The relationship between the extent of partial extraction (alpha) and the percentage decrease in total value added