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Feedback loops and closed-loop recycling as a driver for dynamics

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Abstract

Material flows play an important role within manufacturing systems in setting the structural interdependences among a set of production processes. Input-Output Analysis (IOA) provides the appropriate computational structure to take these interdependences into account, especially the *feedback loops* which are particularly interesting for modeling issues concerning closed loop recycling. Such an application of IOA at the enterprise or supply chain level is usually a static one, in the extent production is assumed to be instantaneous, transactions are simultaneous and there is no inventory management.

The aim of this paper is to introduce elements of simple dynamics within an environmentally-extended Input-Output technological model which can be used both at the enterprise level and at the supply chain level, in order to accomplish tasks which may range from plain product costing and resource planning to environmental management accounting. An approach will be adopted, which is usually addressed as Activity Level Analysis, in order to discuss how *feedback loops* themselves can be seen as a driver for dynamics within an Input-Output based computational structures, if one does not neglect that the operation of a production process takes time.

Keywords: Enterprise Input-Output Analysis, feedback loops, Environmental management, dynamics

1. Introduction

One of the most important features of an Input-Output-based computational structure is that it allows modelling the reciprocal relationships among the economic sectors of a National Economy as well as among the echelons of a supply chain of production processes. Dealing with interdependences involves managing such a specific aspect of Input-Output Analysis (IOA) as the *feedback loops*, that is the appearance of a commodity as an input among its ancestors. Such feature makes it compelling to solve a system of liner equation, either by using the matrix inversion, whether possible, or the sequential methods. This note will discuss:

- What are the repercussions of representing, in a quantitative way, the behaviour of a network of manufacturing process which are linked by material (and cost) flows, when using an Input-Output-based computational structure; and, in particular
- How *feedback loops* can be seen as a driver for dynamics within such computational structures, if it is not neglected that the operation of a production process takes time.

The discussion is grounded on the following main assumptions:

- IOA is applied at the enterprise level to manage both cost accounting and production planning problems, thus showing peculiarities that make it different from the original macroeconomic leontevian mode, as pointed out by Gambling & Nour (1969).
- The former is intended as a technological model in the sense of Gambling (1968), *i.e.* it is built bottom-upwards from the basic operations it purports to illustrate[1]. The model is common to both the physical environmentally extended accounting and the cost accounting system, consistently with Lin & Polenske (1998).

[1] Further issues arise from this approach. They concern multiproduct processes and the application of allocation procedures in order to determine the cost of producing scrap. However, they will not be considered here, for the sake of simplicity.

- The IO model is constructed as a commodity-by-industry system, in contrast to the original industry-by-industry format of Leontief or the alternative commodity-by-commodity form. As such, it fits in the tradition of the supply-use framework, Von Neumann's model (1945), Koopman's activity analysis (1951), as well as environmental life cycle assessment (Heijungs, 1997).

- The IO model is based on physical input-output tables instead of the usual monetary tables (Weisz & Duchin, 2006). This is done to emphasize the physical causality that is indispensible when discussing the dynamics and temporal hierarchy of production.

- The "activity level analysis" approach, as defined by Heijungs (1997), will be adopted, which implies that the operating time of each of several processes can be "implicitly imposed" according to its contribution to fulfilling an exogenous requirement of commodities or production plan[2].

Section 2 provides the basic interpretation of feedback loops from both a computational and network perspective. Section 3 discusses, from the same perspectives, a possible elimination of cycles as a consequence of the explicit introduction of operation times. Section 4 introduces environmental extensions within such framework, in order to deal with the problem of by-products treatment and their closed loop recycling.

2. The network representation of the balancing procedure

2.1 Formulation of the base problem

Assume that an hypothetical 2×2 manufacturing system produces two commodities (Commodity 1, measured in kg, and Commodity 2, measured in m^3) and consists of two unit processes (producing Commodity 1 and Commodity 2 respectively under the assumption that the *j-th* process produces the *j-th* commodity as its main output) which are mutually linked by material flows. For the representation of this, we can adopt a linear space, the basis vectors representing kg of commodity 1 and m^3 of commodity 2.

[2] Each production process considered will be described in terms of parameters that reasonably approximate its real characteristics and, on this basis, the total activity levels, the amount of resources that must be used to achieve a desired amount of net production and the production costs of both intermediate and final products will be estimated. Such an approach is useful as far as enterprise resource and cost planning is concerned, as can be seen in Boons (1998), Feltham (1970) and Livingstone (1968).

Such a system would be represented in an Input-Output form using a make matrix **V** and a use matrix **U** (both of which are positive semi-definite) as follows:

$$\mathbf{Z} = (\mathbf{V} - \mathbf{U}) = \begin{pmatrix} 100 & 0 \\ 0 & 50 \end{pmatrix} - \begin{pmatrix} 0 & 20 \\ 10 & 0 \end{pmatrix} = \begin{pmatrix} 100 & -20 \\ -10 & 50 \end{pmatrix} \quad (1)$$

Such an hypothetical system is self-contained, as far as the inter-industry flows are concerned, that is, when we leave out factor inputs, environmental extensions, and so on. Matrix **Z** is called the technology matrix (Koopmans, 1951; Heijungs, 1997). Each column represents a technique. The operating time of each process is an aspect which should not be neglected in the analysis. Thus, a vector $\mathbf{c}^T = (6 \quad 1)^T$ is defined whose elements are the operating time basis for each process, say expressed in hours. The system described by matrix **Z** must meet an exogenous set of flows, or final demand. An example would be the amount of final and/or intermediate products that have been planned to be produced in a month. This production plan sets the final demand vector, for example $\mathbf{y}^T = (0 \quad 200)^T$, i.e. 200 m^3 of commodity 2. One can then calculate the activity levels at which the processes are required to operate in order to meet the production plan by solving the following system of simultaneous equations:

$$\begin{cases} 100s_1 - 20s_2 = 0 \\ -10s_1 + 50s_2 = 200 \end{cases}$$

or more generally

$$\mathbf{Zs} = \mathbf{y} \quad (2)$$

which is usually accomplished by post-multiplying the final demand vector by the inverse of the net production matrix, provided that the latter exists[3]:

$$\mathbf{Z}^{-1}\mathbf{y} = \begin{pmatrix} 0{,}0104167 & 0{,}0041667 \\ 0{,}0020833 & 0{,}0208333 \end{pmatrix} \begin{pmatrix} 0 \\ 200 \end{pmatrix} = \begin{pmatrix} s_1 \\ s_2 \end{pmatrix} = \begin{pmatrix} 0{,}83333 \\ 4{,}166667 \end{pmatrix} \quad (3)$$

[3] This topic will not be further discussed here. However, the Hawkins-Simons Condition – whose importance for the significance of the outcomes of a static Input Output model as well as the stability of a dynamic one has been extensively pointed out in Solow (1954) - does not necessarily hold as far as physical transactions are in the focus. Suh and Heijungs (2007) recently discussed how the power series expansion from can be utilized for a general system that includes the physical systems. The authors claimed that this form "unravels a complex network relationship enabling a detailed insight on the structure of the system".

The solution **s** is said to be the vector of operating times . If one denotes the diagonalised scaling vector as $\hat{\mathbf{s}}$, the flows accounted for in **Z** can then be balanced as $\tilde{\mathbf{Z}} = \mathbf{Z}\hat{\mathbf{s}} = \begin{pmatrix} 83{,}33 & -83{,}33 \\ -8{,}33 & 208{,}33 \end{pmatrix}$. This is a way of representing, for example, an inventory problem in Life Cycle Assessment, suggested by Heijungs & Suh (2002).

2.2 Interpretation of feedback loops

From the example above, it can be noted that there is a feedback loop, as previously defined. What comes out from such supplier/customer relationship among the two unit production processes depicted is that there is no need to purchase Commodity 2 from outside the system boundaries considered, since it is supplied by Process 2. From a cost accounting perspective, this mean that the Company which runs both Process 1 and Process 2 will not act as a price taker as far as Commodity 2 is concerned, in the extent the latter will be produced as a main output by Process 2 and will be transferred-into Process 1 at its manufacturing cost, which is to be calculated using the monetary counterpart of the above described Input-Output scheme as described, for example, in Settanni et al. (2007). Heijungs & Suh (2002) exhaustively discussed, in section 4.3, the iterative method to solve the system of linear equations in presence of feedback loops. The authors pointed out that this situation ultimately leads to delineate «a network with recursive relationships as a linear sequence of infinite length». Following such interpretation of feedback loop, the entries of vector **s** can indeed be obtained as the column sum of the results which have been calculated in Tab. 1.

Table 1

In this sense, one can interpret a feedback loop as a situation in which a certain commodity is required *before* it can be produced. This happens in two cases. The first one is the presence of self-consumption, *i.e.* if one considers the *i-th* commodity[4] it happens that the corresponding element of the use matrix **U** is $u_{ii} \neq 0$. The second case is when the system is cyclic, *i.e.* "non decomposable" in the sense of Solow, (1954): if one considers commodities *i* and *j*, with $i \neq j$, then $\left(u^T u\right)_{ij} \neq 0$. In other words,

[4] It is herein assumed that commodities are ordered so that the main output of the *i-th* process is listed as the *i-th* row

Commodity *i* requires Commodity *j* and *vice versa*, just as in the numerical example described above.

Despite the fact the system of linear equations can be solved and even interpreted, how can be possible – from an enterprise resource-planning and cost accounting perspective – to produce Commodity 1 and 2 simultaneously (the former requiring the latter and *vice versa*) while not neglecting that production takes time? If one accounts for transactions within a hindsight-oriented analysis, then it can be assumed that during the time span considered, feedback loops are likely to have had place together with those imports from outside the system which were necessary to activate the production processes in absence of the internally-produced inputs. Whereas, when carrying out a foresight "activity level analysis", such an assumption about *feedback loops* should be reconsidered. At least in the extent that planning for costs is concerned, indeed, it makes sense to make a distinction between externally purchased and internally provided inputs. In the former case, indeed, the company acts as a price taker and the cost structure of the purchased input cannot be influenced unless somehow varying the consumption rates of a certain resource. In the latter case, instead, internally produced input are assumed to be transferred-into the process that requires them and valued according to their (for the moment, assumed variable) manufacturing cost (thus performing some kind of "transfer pricing"), which can be controlled.

2.3 Network representation

Gambling (1968) and Charnes & Cooper (1967) introduced some network characterizations for Input-Output-based cost accounting and planning. Following the approach discussed in Schmidt (2005) and Laurin et al. (2005), the herein considered system's behaviour will be represented by using some basic features of the Petri Nets[5].

Figure 1

In Figure 1, Places (circles) are local states of the system, for example available raw materials, intermediate inputs and output, work-in-process, final outputs

[5] We basically make reference to Reisig and Grzegorz (1998).

and so on[6]. Transitions (rectangles) are active system components, *i.e.* processes. A marking or a state of a Petri net is given by the token distribution on the places of the net. As shown in Figure 1, according to which preconditions are satisfied (*i.e.*, which input places are marked), one can determine what transition is able to fire at a given time[7]. Each run of the system, through the firing of transitions, involves the amounts of commodities indicated in Eq.(1). Then the number of runs of each transition is given by the elements of the scaling vector calculated in Eq.(3).

It appears clearly from Figure 1(a) that the availability of Commodity 2 (place *p3*) is a precondition for the operation of Process 1, *i.e.* the firing of the corresponding transition. One could then say that the time at which such commodity is required as an input differs from the time at which it is produced, as pointed out in Ijiri (1968). Nevertheless, as far as IOA is concerned, Dorfman *et al.* (1958: p.205) write: "For the production of coal, iron is required; for the production of iron, coal is required; no man can say whether the coal industry or the iron industry is earlier or later in the hierarchy of production." So, they seem to deny that a network means temporal order. The following discussion will focus on such aspect, instead.

3. From feedback loops to Input-Output dynamics

3.1 Reformulation of the original scheme.

Now assume that the total active time period of both processes (*i.e.* their "operating" time) can be subdivided into two planning periods. One could denote the first time interval as ζ_1 and the time interval which immediately follows as $\zeta_2 = \zeta_1 + 1$. Assume that the width of these time intervals is the same and amounts to τ time units. It is expected that a process runs, in each period, ρ time units, where $\rho < \tau$. Assuming we are interested in using hours as the appropriate time units, then we can express each interval's width as τ hours. Processes' lead times are also measured in hours. The initial problem in Eq.(1) will be reformulated and extended as follows:

[6] Except the "p-start" place, which serves to initialize the system and does not correspond to any physical flow.
[7] This is done by removing tokens from the input places and adding tokens to the output places.

$$\mathbf{A}=\begin{pmatrix}\mathbf{Z}\\ \hline -\mathbf{M}\end{pmatrix}=\left(\begin{array}{c|c}\mathbf{G} & \mathbf{H}\\ \mathbf{0} & \mathbf{L}\\ \hline -\mathbf{M}^{(1)} & -\mathbf{M}^{(2)}\end{array}\right)=\left(\begin{array}{lr}\text{Commodity 1 (time 1)} & \text{kg}\\ \text{Commodity 2 (time 1)} & \text{m}^3\\ \hline \text{Commodity 1 (time 2)} & \text{kg}\\ \text{Commodity2 (time 2)} & \text{m}^3\\ \hline \text{Externally purchased inputs} & \text{m}^3\end{array}\right)=\left(\begin{array}{cc|cc}100 & -20 & 0 & 0\\ 0 & 50 & -10 & 0\\ \hline 0 & 0 & 100 & -20\\ 0 & 0 & 0 & 50\\ \hline -10 & 0 & 0 & 0\end{array}\right) \quad \textbf{(4)}$$

whereas the matrix of operating times for each process in each period now is $\mathbf{C}=(\text{cycle time} \quad \text{h})=(6 \quad 1 \mid 6 \quad 1)$.

The matrix of externally purchased inputs, $-\mathbf{M}=(-10 \quad 0 \mid 0 \quad 0)$, has been introduced, in order to take into account that Commodity 2 is being purchased from outside the system boundaries in the first period[8]. Thus, matrix **G** represents the technical specifications of both Process 1 and Process 2 considered as active during time interval ζ_1 of width τ. It looks similar to the former matrix **Z**, though no feedback loop has now been taken into account: to get the whole picture for period ζ_1, one should now look at the stacked **G** and $-\mathbf{M}^{(1)}$. Matrix **L** represents the same system considered as active during period ζ_2 that immediately follows ζ_1. Assuming no technology change[9], then **L** = **G**. According to this reformulation of the initial problem, we then consider that the Commodity 1 which is produced and consumed during the first period is different from the Commodity 1 which is produced and consumed in the next period. If such a distinction holds, it is not surprising then to find that **O** is a null matrix. Indeed, it represents flows which go backwards in time. Those flows which, instead, are forwarded from the first to the second time period are accounted for in matrix **H**. The former feedback loop has been now taken into account in matrix **H** because it can be read as if Process 1 operating during the second time period requires a certain amount of Commodity 2 produced by Process 2 operated during the first period.

[8] Assuming just one externally purchased input is obviously unrealistic and only serves the illustrative purposes. The number of economic flows included in **M** depends, instead, upon how the system boundaries have been set, and upon the number of cost drivers chosen to trace the conversion costs to processes. Settanni and Emblemsvåg (2008) discuss this topic. One further distinction should be made within **M** between time period 1 and time period 2. To avoid this, for the sake of simplicity, it has been assumed that the external input price remains the same in the two periods.
[9] This is a strong simplifying assumption. Indeed, some change within the depicted technique is likely to occur, especially as the issue of closed loop recycling will be introduced.

The representation of the network of processes in Figure 1 now changes as depicted in Figure 2. Such network without cycles is called a “process net” in Petri Nets Theory[10].

Figure 2

Despite the environmental extensions, the balancing problem would still concern only matrix **Z**. Thus, the following is to be solved:

$$\begin{pmatrix} \mathbf{G} & \mathbf{H} \\ \mathbf{0} & \mathbf{L} \end{pmatrix} \begin{pmatrix} \mathbf{s}^{(1)} \\ \mathbf{s}^{(2)} \end{pmatrix} = \begin{pmatrix} \mathbf{y}^{(1)} \\ \mathbf{y}^{(2)} \end{pmatrix} \quad \textbf{(5)}$$

where superscripts denote the time period the variables refer to. Assuming that $\begin{pmatrix} \mathbf{y}^{(1)} \\ \mathbf{y}^{(2)} \end{pmatrix} = \begin{pmatrix} 0 & 0 \mid 0 & 200 \end{pmatrix}^T$, solving Eq.(5) means solving the following system of simultaneous equations

$$\begin{cases} 0 = 100s_1^{(1)} - 20s_2^{(1)} + 0s_1^{(2)} + 0s_2^{(2)} \\ 0 = 0s_1^{(1)} + 50s_2^{(1)} - 10s_1^{(2)} + 0s_2^{(2)} \\ 0 = 0s_1^{(1)} + 0s_2^{(1)} + 100s_1^{(2)} - 20s_2^{(2)} \\ 200 = 0s_1^{(1)} + 0s_2^{(1)} + 0s_1^{(2)} + 50s_2^{(2)} \end{cases} \quad \textbf{(6)}$$

One can solve Eq.(6) by using an iterative method going backward from the last equation. This turns out the results shown in Table 2.

Table 2

However, the operating time vector can be found as usual, by the matrix-inversion, which, interestingly, yields $\mathbf{s} = \begin{pmatrix} 0{,}032 & 0{,}16 & 0{,}8 & 4 \end{pmatrix}^T$ whose elements $s_1^{(1)}, s_2^{(1)}; s_1^{(2)}; s_2^{(2)}$ are such that $s_1^{(1)} + s_1^{(2)} \approx s_1; s_2^{(1)} + s_2^{(2)} \approx s_2$, where s_1 and s_2 are the scaling factors calculated in Eq.(3). All the flows can then be balanced: the ones accounted for within the net output matrix, the externally purchased input and the processes' operating time vector:

[10] Following Reisig and Rozenberg (1998), a process net is a record of all occurrences of events that lead from an initial configuration of a network to a final one, with all conditions involved in these events.

$$\begin{pmatrix} \mathbf{Z} \\ -\mathbf{M} \\ \mathbf{C} \end{pmatrix} \hat{\mathbf{s}} = \begin{pmatrix} \tilde{\mathbf{Z}} \\ -\tilde{\mathbf{M}} \\ \tilde{\mathbf{C}} \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{G}} & \tilde{\mathbf{H}} \\ \mathbf{0} & \tilde{\mathbf{L}} \\ -\tilde{\mathbf{M}}^{(1)} & -\tilde{\mathbf{M}}^{(2)} \\ \tilde{\mathbf{C}}^{(1)} & \mathbf{0} \end{pmatrix} = \left(\begin{array}{cc|cc} 3{,}2 & -3{,}2 & 0 & 0 \\ 0 & 8 & -8 & 0 \\ \hline 0 & 0 & 80 & -80 \\ 0 & 0 & 0 & 200 \\ -0{,}32 & 0 & 0 & 0 \\ 0{,}192 & 0{,}16 & 4{,}8 & 4 \end{array}\right) \quad \textbf{(7)}$$

According to the above results, the main difference between the more compact system of equations (2) which contains feedback loops and the system of equations (5) – as far as **Z** is concerned – which does not contain feedback loops (but explicitly covers a time span) is not mainly in the overall material balance. The material flows in (4), instead, could be intuitively (since no formal demonstration will be provided here) seen as a “temporal decomposition” of the ones in (1). This recalls some basic ideas which have been extensively and formally discussed within the dynamics of input-output systems. Thus, some features of a generalized dynamic input-output model as described by ten Raa (2005, 1986a, 1986b) could be of interest here, in particular, the consequences of considering the time used in the production processes, which defines the time profile of both inputs and outputs.

For the sake of simplicity, such features will be discussed in the following paragraph only with reference to the flows which have been accounted for in the technology matrix **Z**; whereas, the problems which arise when the demand for waste treatment and secondary inputs is explicitly taken into account will be discussed later.

3.2 Elements of dynamic Input-Output and distributed activities

Assume that there is only one process producing only one commodity.

1. A process centered at time 0 transfers inputs $u_0^{(d)}$ at times d $\leq$ 0 into outputs $v_0^{(d)}$ at times d $\geq$ 0;
2. The time profile (d) of input u shows the lead times that must be observed in production. If production is instantaneous, then $u^{(d)} \neq 0$ only for d=0. More

generally $u_{t+p}^{(-p)}$ represents the direct requirements of input p time units prior to the delivery time $t+p$, that is at current time (t).

3. If $s^{(t+p)}$ is the activity level of the process at the delivery time $(t+p)$, due to the presence of different lead times (p = 0 ... ∞) the total amount of input that is to be present at the current time t is $u_t = \sum_{p=0}^{\infty} u_{t+p}^{(-p)} s^{(t+p)}$.

4. The time profile (d) of output v is the lifetime of the product. If $v^{(d)} \neq 0$ only for d=0, then no amount of commodity will be present at times $d>0$. More generally, $v_{t-p}^{(p)}$ is the amount of output whose lifetime is p time units, *i.e.* which has been produced at time $t-p$ and contributes to the output available at time t.

5. If $s^{(t-p)}$ is the activity level of the process at time $(t-p)$, then the contribution to the amount of output available at time t is $v_t = \sum_{p=0}^{\infty} v_{t-p}^{(p)} s^{(t-p)}$. This is said to be the accumulated *capital*.

6. Given the final demand y_t, the basic Input-Output material balance for the system depicted (one process, one commodity) then becomes:

$$\sum_{p=0}^{\infty} v_{t-p}^{(p)} s^{(t-p)} = \sum_{p=0}^{\infty} u_{t+p}^{(-p)} s^{(t+p)} + y_t \quad \textbf{(8)}$$

Following ten Raa (2005), this can also be expressed in terms of a "convolution product" as v*s = u*s + y, where the convolution product between two functions of time f and g is (f*g)(t) = $\sum_{p=-\infty}^{\infty} f^{(p)} g^{(t-p)}$.

The above can be applied to the numerical example adopted so far. The two subsequent periods ζ_1 and ζ_2 are defined as t=1 and t=2 respectively. Instead of using letters $u^{(t)}$ and $v^{(t)}$ to denote the quantities of a single commodity as a function of time, now matrices $\mathbf{U}^{(t)}$ and $\mathbf{V}^{(t)}$ will be used. The process' activity level s and the final demand y are now to be considered as vectors. Both the input and output profile of the system considered can be thought of as dependent on discrete time periods (t). By using equation (7), with a few manipulations, we can now compute Table 3.

Table 3

From Table 3 one can see that for each period (t=1 and t=2), the time horizon is to be extended backward and onward, by subtracting *p* (discrete) time units, where *p* may range from the positives to the negatives. By such an operations, one obtains a time horizon which is made of the period *t* itself (p=0), the periods that come after *t* (namely *t-p* where $p \leq 0$) and the periods that come before *t* (namely *t-p* where $p \geq 0$). Now one shall see the economic activities (production processes, as described by both make matrix **V** and use matrix **U**) as distributed along such time horizon:

Table 4

The assumptions been made, as in ten Raa (2005), that $\mathbf{U}^{(p)} \neq 0$ only if $p \leq 0$. For the moment, assume $\mathbf{V}^{(p)} \neq 0$ only if $p \geq 0$ (otherwise, if $\mathbf{V}^{(p)} \neq 0$ only if $p > 0$ this would mean that each commodity which has been used at time *t* will not produce any output in the same period, but at least one period later). In order to obtain the system's gross production as well as the interindustry demand at time *t*, one needs to respectively sum up the make and use matrices (multiplied by the corresponding activity levels) over time, according to parameter *p*. This can also be graphically represented as in the figure below

Figure 3

For each period *t*, then, the material balance will be calculated taking into account not only the outputs and inputs that are "instantaneously" produced and used up in that period (being $p = 0$). The gross output at time *t* will include, instead, also those output which have been produced one or more periods before period *t* and which contribute, according to their own lifetime, to the amount of commodities available at that time. Thus, processes centered at time *t-p* contribute $\mathbf{V}_{t-p}{}^{(p)}\mathbf{s}^{(t-p)}$ to the stock at time *t*, producing outputs of lifetime *p*. On the other hand, the interindustry demand at time *t* will also include the input requirements of those processes centered at time *t-p* (where $p < 0$) that require $\mathbf{U}_{t-p}{}^{(p)}\mathbf{s}^{(t-p)}$ inputs to be available $|p|$ time units prior to the delivery of output, that is at time *t*.

For the sake of simplicity, now assume that such periods as "-1", "3" are excluded from the planning horizon. Also assume that we cannot see beyond period t=2. Form Tab.3 one obtains that at times t =1 and t =2 the balance equation (7) reads, respectively

$$\begin{aligned} &\mathbf{V}_1 = \mathbf{U}_1 + \mathbf{y}_1 \rightarrow \mathbf{V}_1^{(0)}\mathbf{s}^{(0)} + [\mathbf{V}_0^{(1)} - \mathbf{U}_1^{(0)}]\mathbf{s}^{(1)} - \mathbf{U}_2^{(-1)}\mathbf{s}^{(2)} = \mathbf{y}_1 \\ &\mathbf{V}_2 = \mathbf{U}_2 + \mathbf{y}_2 \rightarrow \mathbf{V}_0^{(2)}\mathbf{s}^{(0)} + \mathbf{V}_1^{(1)}\mathbf{s}^{(1)} + [\mathbf{V}_2^{(0)} - \mathbf{U}_2^{(0)}]\mathbf{s}^{(2)} - \mathbf{U}_3^{(-1)}\mathbf{s}^{(3)} = \mathbf{y}_2 \end{aligned} \quad \textbf{(9)}$$

Because of the microeconomic perspective herein adopted, however, it is unlikely that the process of accumulation of capital takes (computationally) place in the way described above. Indeed, those manufacturing processes whose outputs are durable assets which contribute as fixed capital to the production system under study are hardly ever included within boundaries of the analysis. In other words, such manufacturing processes are assumed not to enter the enterprise production and cost planning. Otherwise, one would be forced to introduce further elements within the analysis such as th depreciation of outputs at each age. Thus, we choose here to put the more restrictive condition such as no capital goods are being produced within the system considered, *i.e.* $\mathbf{V}_t^{(p)} \neq 0$ only if p=0 (t = 1,2).

Although processes within the system boundaries won't provide durable assets to be used as inputs into the other processes, the accumulation of the commodities produced is likely to take place in the form of inventories. The initial stocks of commodities of lifetime p = 1 which comes from period t = 0, preceding the planning period, are assumed as exogenous constants: $\mathbf{K}_1 = \mathbf{V}_0^{(1)}\mathbf{s}^{(0)}$ is a matrix whose elements are constants and represents the initial inventory of commodities available at time t = 1. If $\boldsymbol{e}$ is defined as a unity column vector of appropriate dimensions, one calculates $\mathbf{d}_1 = \mathbf{y}_1 + \mathbf{K}_1\mathbf{e}$ where $\mathbf{K}_1\boldsymbol{e}$ is the row sum of matrix $\mathbf{K}_1$.

The system of equation (9) can now be expressed in a matrix form as

$$\begin{pmatrix} \mathbf{V}_1^{(0)} - \mathbf{U}_1^{(0)} & -\mathbf{U}_2^{(-1)} \\ \mathbf{0} & \mathbf{V}_2^{(0)} - \mathbf{U}_2^{(0)} \end{pmatrix} \begin{pmatrix} \mathbf{s}^{(1)} \\ \mathbf{s}^{(2)} \end{pmatrix} = \begin{pmatrix} \mathbf{d}_1 \\ \mathbf{y}_2 \end{pmatrix} \quad \textbf{(10)}$$

Instead of being introduced as additional exogenous (positive) elements of the final demand, the final inventories of commodities can be thought of as proportional to the increase of the system's activity levels between two periods. "Capital coefficients" must be defined as an additional element of the material balance equation, since they represent the amount of a certain commodity which is needed by a certain process in order to sustain an expected unit increase in its activity levels[11].

Assume that the elements of matrix **B** are such "capital coefficients". In other words, $\mathbf{B}_{t+1}^{(p)}$ is the material requirement at time *t* for inventories usable one period after, that needs an investment to be made $|p|$ (where $p \leq 0$) time units prior to *t*. Eq.(8) can then be reformulated as follows

$$\sum_{p\geq 0}\mathbf{V}_{t-p}^{(p)}\mathbf{s}^{(t-p)} = \sum_{p\leq 0}\mathbf{U}_{t-p}^{(p)}\mathbf{s}^{(t-p)} + \sum_{p\leq 0}\mathbf{B}_{t-p+1}^{(p)}\left(\mathbf{s}^{(t-p+1)} - \mathbf{s}^{(t-p)}\right) + \mathbf{y}_t \qquad \textbf{(11)}$$

From Eq.(11) one can compute Table 5.

Table 5

To illustrate the new element in the mass balance equation (11), capital coefficients matrices will be arranged as two time distributions (limited to time periods *t* =1 and *t* = 2). This is shown in Table 6.

Table 6

Given the assumptions made so far and extending the time horizon to three time periods, we can summarize the mass balance equation as

$$\begin{pmatrix}\mathbf{V}_1^{(0)} & \mathbf{0} & \mathbf{0}\\ \mathbf{0} & \mathbf{V}_2^{(0)} & \mathbf{0}\end{pmatrix}\begin{pmatrix}\mathbf{s}^{(1)}\\ \mathbf{s}^{(2)}\\ \mathbf{s}^{(3)}\end{pmatrix} - \begin{pmatrix}\mathbf{U}_1^{(0)} & \mathbf{U}_2^{(-1)} & \mathbf{U}_3^{(-2)}\\ \mathbf{0} & \mathbf{U}_2^{(0)} & \mathbf{U}_3^{(-1)}\end{pmatrix}\begin{pmatrix}\mathbf{s}^{(1)}\\ \mathbf{s}^{(2)}\\ \mathbf{s}^{(3)}\end{pmatrix} +$$

$$+\begin{pmatrix}\mathbf{B}_2^{(0)} & \mathbf{B}_3^{(-1)} & \mathbf{B}_4^{(-2)}\\ \mathbf{0} & \mathbf{B}_3^{(0)} & \mathbf{B}_4^{(-1)}\end{pmatrix}\begin{pmatrix}\mathbf{s}^{(1)}\\ \mathbf{s}^{(2)}\\ \mathbf{s}^{(3)}\end{pmatrix} - \begin{pmatrix}\mathbf{0} & \mathbf{B}_2^{(0)} & \mathbf{B}_3^{(-1)}\\ \mathbf{0} & \mathbf{0} & \mathbf{B}_3^{(0)}\end{pmatrix}\begin{pmatrix}\mathbf{s}^{(1)}\\ \mathbf{s}^{(2)}\\ \mathbf{s}^{(3)}\end{pmatrix} = \begin{pmatrix}\mathbf{d}_1\\ \mathbf{y}_2\end{pmatrix} \qquad \textbf{(12)}$$

Following ten Raa (1986), now define

for $p = 0$, $\mathbf{G}_t^{(0)} = [\mathbf{V}_t^{(0)} - \mathbf{U}_t^{(0)} + \mathbf{B}_{t+1}^{(0)}]$ (t=1,2)

[11] Duchin and Szyld (1985) referred the investment term in the dynamic system to the increase of productive capacity that has been projected several time periods in advance. The microeconomic model herein outlined, however, will assume that the processes can only provide final stocks of the respective commodities so that the downstream capacities will not be exceeded.

for p ≠ 0, $\mathbf{G}_t^{(p)} = [-\mathbf{U}_{t-p}^{(p)} + \mathbf{B}_{t-p+1}^{(p)} - \mathbf{B}_{t-p}^{(p+1)}]$ (t=1,2)

The first definition is obtained by summing up the elements in the *t*-th row and column of the 2×3 matrices in the left-hand side of equation (11). The second definition is obtained by summing up the elements in the (*t*+*p*) -th row and column of the same matrices. Equation (11) can be adapted to the numerical example and expressed as

$$\sum_{p\le 0} \mathbf{G}_t^{(p)} \mathbf{s}^{(t-p)} = \mathbf{y}_t \qquad (t=1,2) \qquad \textbf{(13)}$$

If we assume that we cannot see beyond $t = 2$, then (10) reduces to

$$\begin{cases} \mathbf{G}_1^{(0)}\mathbf{s}^{(1)} + \mathbf{G}_1^{(1)}\mathbf{s}^{(2)} = \mathbf{y}_1 \\ \quad\mathbf{G}_2^{(0)}\mathbf{s}^{(2)} = \mathbf{y}_2 \end{cases} \qquad \textbf{(14)}$$

The above system can be expressed in matrix form as

$$\begin{pmatrix} \mathbf{G}_1^0 & \mathbf{G}_1^1 \\ \mathbf{0} & \mathbf{G}_2^0 \end{pmatrix} \begin{pmatrix} \mathbf{s}^{(1)} \\ \mathbf{s}^{(2)} \end{pmatrix} = \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix}$$

which is very similar to the traditional dynamic input-output scheme as can be found in Miller and Blair (1985), Leontief (1968), Leontief (1966), though adapted for the use of Make-Use matrices.

This also recalls the numerical example presented in the previous paragraph. In particular, $\mathbf{G}_1^{(0)} = \mathbf{V}_1^{(0)} + \mathrm{U}_1^{(0)} + \mathbf{B}_2^{(0)} = \mathbf{G}$ if $\mathbf{B}_2^{(0)} = 0$ and $\mathbf{G}_2^{(0)} = \mathbf{V}_2^{(0)} + \mathbf{U}_2^{(0)} + \mathbf{B}_3^{(0)} =$ **L= G** if $\mathbf{B}_3^{(0)} = \mathbf{B}_2^{(0)} = \mathbf{0}$ and $\mathbf{V}_1^{(0)} = \mathbf{V}_2^{(0)}$ and $\mathrm{U}_1^{(0)} = +\mathbf{U}_2^{(0)}$ that is there is no technological change between the two periods. Matrix $\mathbf{G}_1^{(1)} = -\mathbf{U}_2^{(-1)} + \mathbf{B}_3^{(-1)} - \mathbf{B}_2^{(0)}$ expresses, instead, the amounts of commodities which are to be produced one period before being used up in production and includes *feedback loops*, - $\mathbf{U}_2^{(-1)}$, as defined at the beginning of this paper. Thus $\mathbf{G}_1^{(1)} = \mathbf{H}$ in the extent $\mathbf{B}_3^{(-1)} - \mathbf{B}_2^{(0)} = \mathbf{0}$

3.3 Introducing environmental extensions

Also some environmental extensions can be considered. It has been assumed that the system also produces two waste types, or secondary products, which either undergo some end-of-pipe treatment or serves as secondary inputs. The net generation of waste k by process j is recorded as the generic element $(\overline{N})_{kj}$ of the matrix $\overline{\mathbf{N}}$, whereas the element $(\underline{N})_{kj}$ of the matrix $\underline{\mathbf{N}}$ represents the input of the same waste into that process. Following Nakamura & Kondo (2006), each process is assumed either to produce a given waste k or to use it as a secondary input, *i.e.* $\forall j, \forall k : (\overline{N})_{kj} \times (\underline{N})_{kj} = 0$. Though this is not the only way to proceed, it seems most convenient for cost accounting purposes. A numerical example is given below.

$$\mathbf{W} = \begin{pmatrix} \mathbf{W}^{(1)} \\ \mathbf{W}^{(2)} \end{pmatrix} = \begin{pmatrix} \overline{\mathbf{N}}^{(1)} & \mathbf{0} \\ \mathbf{0} & -\underline{\mathbf{N}}^{(1)} \\ \hdashline \mathbf{0} & \overline{\mathbf{N}}^{(2)} \\ \mathbf{0} & \mathbf{0} \end{pmatrix} = \begin{pmatrix} \text{waste type 1 (out, time 1)} & \text{kg} \\ \text{waste type 2 (out, time 1)} & \text{kg} \\ \text{waste type 1 (in, time 2)} & \text{kg} \\ \text{waste type 2 (in, time 2)} & \text{kg} \\ \hdashline \text{waste type 1 (out, time 2)} & \text{kg} \\ \text{waste type 2 (out, time 2)} & \text{kg} \\ \text{waste type 1 (in, time 3)} & \text{kg} \\ \text{waste type 2 (in, time 3)} & \text{kg} \end{pmatrix} = \begin{pmatrix} 0 & 0{,}8 & 0 & 0 \\ 4 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0{,}02 \\ \hdashline 0 & 0 & 0 & 0{,}8 \\ 0 & 0 & 4 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \textbf{(15)}$$

These flows can be balanced as follows:

$$\mathbf{W}\hat{\mathbf{s}} = \widetilde{\mathbf{W}} = \begin{pmatrix} 0 & 0{,}128 & 0 & 0 \\ 0{,}128 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -0{,}08 \\ \hdashline 0 & 0 & 0 & 3{,}2 \\ 0 & 0 & 3{,}2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \quad \textbf{(16)}$$

We now turn to the problem of determining the demand for waste treatment by each production process[12]. Consider the above. As a simplifying assumption, the by-products generated in a certain time period can only be used as secondary inputs in the period that immediately follows. The amount of waste that is not recycled internally, determines the demand for its treatment. This must be calculated for each production process and for each period, in order to accurately assign the waste treatment costs.

[12] This is based on the algebra of the Waste IO model developed by Nakamura & Kondo (2002), though adapted for the purposes of Enterprise IOA.

First, the recycling ratio for each waste type k in each period t is determined. With reference to Eq.(7), it reads:

$$\forall t, \forall k : r_k^{(t)} = \sum\nolimits_{j=1}^{n} (\underline{\tilde{N}}^{(t+1)})_{kj} \Big/ \sum\nolimits_{j=1}^{n} (\tilde{\tilde{N}}^{(t)})_{kj} \tag{17}$$

where n is the number of production processes in each period (in our example, n=2). Such ratios can be collected in a vector, which, as far as our numerical example is concerned, reads $\mathbf{r} = (\mathbf{r}^{(1)} \quad \mathbf{r}^{(2)})^T = (0 \quad 0{,}625 \mid 0 \quad 0)^T$. The "sale of waste", *i.e.* the amount of waste produced in the first time period which has been sold within the supply chain of processes considered in the following period, totals $\mathbf{r}^T \begin{pmatrix} \tilde{\tilde{\mathbf{N}}}^{(1)} & 0 \\ 0 & \tilde{\tilde{\mathbf{N}}}^{(2)} \end{pmatrix}$. The percentage of the output of waste which is not recycled within the time horizon considered, instead, is $(\mathbf{I} - \hat{\mathbf{r}})$. Such quantity will undergo some treatment. Assume there is one such treatment process that is run internally. It produces an output that is in fact a treatment service, measured by the physical amount, expressed as weight, of the wastes that undergoes it. If there is not a one-to-one relationship among the waste types and the treatment processes, then a matrix $\mathbf{Q}$ is to be exogenously defined whose element $0 < q_{lk} < 1$ indicates the amount of k-th waste type which undergoes the l-th treatment. For example, $\mathbf{Q} = \begin{pmatrix} \mathbf{Q}^{(1)} & 0 \\ 0 & \mathbf{Q}^{(2)} \end{pmatrix} = \left(\begin{array}{cc|cc} 1 & 1 & 0 & 0 \\ \hline 0 & 0 & 1 & 1 \end{array}\right)$. It should be noted that $\mathbf{e}^T \mathbf{Q} = \mathbf{e}^T$, where $\mathbf{e}$ is a unity column vector of the appropriate dimension. The following

$$\begin{pmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & -\mathbf{Q}^{(1)}(\mathbf{I} - \hat{\mathbf{r}}^{(1)}) & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & -\mathbf{Q}^{(2)}(\mathbf{I} - \hat{\mathbf{r}}^{(2)}) \end{pmatrix} \left(\begin{array}{cc} \tilde{\mathbf{G}} & \tilde{\mathbf{H}} \\ \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{L} \\ \mathbf{0} & \mathbf{0} \\ \hline \tilde{\tilde{\mathbf{N}}}^{(1)} & \mathbf{0} \\ \mathbf{0} & \tilde{\tilde{\mathbf{N}}}^{(2)} \end{array}\right) = \left(\begin{array}{cc} \tilde{\mathbf{G}}_{I,I} & \tilde{\mathbf{H}}_{I,I} \\ \tilde{\mathbf{G}}_{II,I} & \mathbf{0} \\ \hline \mathbf{0} & \tilde{\mathbf{L}}_{I,I} \\ \mathbf{0} & \tilde{\mathbf{L}}_{II,I} \end{array}\right) \tag{18}$$

yields[13] the demand for waste treatment can in each period, respectively[14] $\tilde{\mathbf{G}}_{II,I} = \begin{pmatrix} -0{,}048 & -0{,}128 \end{pmatrix}$ and $\tilde{\mathbf{L}}_{II,I} = \begin{pmatrix} -3{,}2 & -3{,}2 \end{pmatrix}$. The negative figures within such matrices are not to be interpreted as feedback loops, since they have been calculated only as a computational contrivance which allows the assignment of treatment process' costs to production processes.

The treatment process which is run internally, is to be included within the scheme. Input requirements from the production processes are recorded as the entries of matrix $\mathbf{G}_{I,II}$ ($\mathbf{L}_{II,I}$ in the following period). The net output of the treatment process is recorded in matrix $\mathbf{G}_{II,II}$ ($\mathbf{L}_{II,II}$ in the following period). External input requirements and cycle time are specified, respectively, within matrices $-\mathbf{M}_{\bullet,II}$ and $\mathbf{C}_{\bullet,II}$. The treatment processes turns wastes into releases into the environment. The latter are recorded within the matrix of environmental flows $\mathbf{R}$. The system can then be reformulated as follows, under some assumptions[15]:

$$\begin{pmatrix} \mathbf{A} \\ \hline \mathbf{D} \\ \hline \mathbf{R} \\ \hline \mathbf{G}^{(1)} \\ \mathbf{G}^{(2)} \end{pmatrix} = \left(\begin{array}{cc|cc} \tilde{\mathbf{G}}_{I,I} & \mathbf{G}_{I,II} & \tilde{\mathbf{H}}_{I,I} & \mathbf{H}_{I,II} \\ \tilde{\mathbf{G}}_{II,I} & \mathbf{G}_{II,II} & \mathbf{0} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \tilde{\mathbf{L}}_{I,I} & \mathbf{L}_{I,II} \\ \mathbf{0} & \mathbf{0} & \tilde{\mathbf{L}}_{II,I} & \mathbf{L}_{II,II} \\ \hline -\tilde{\mathbf{M}}^{(1)}_{\bullet,I} & -\mathbf{M}^{(1)}_{\bullet,II} & -\tilde{\mathbf{M}}^{(2)}_{\bullet,I} & -\mathbf{M}^{(2)}_{\bullet,II} \\ \tilde{\mathbf{C}}^{(1)}_{\bullet,I} & \mathbf{C}^{(1)}_{\bullet,II} & \tilde{\mathbf{C}}^{(2)}_{\bullet,I} & \mathbf{C}^{(2)}_{\bullet,II} \\ \hline \mathbf{0} & \mathbf{R}^{(1)}_{\bullet,II} & \mathbf{0} & \mathbf{R}^{(2)}_{\bullet,I} \\ \hline \mathbf{r}^{(1)^T}\tilde{\tilde{\mathbf{N}}}^{(1)} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\underline{\tilde{\mathbf{N}}}^{(2)}\mathbf{e} & \mathbf{0} \\ \hline \mathbf{0} & \mathbf{0} & \mathbf{r}^{(2)^T}\tilde{\tilde{\mathbf{N}}}^{(2)} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \end{array}\right) = \left(\begin{array}{ccc|ccc} 3{,}2 & -3{,}2 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & -8 & 0 & 0 \\ -0{,}048 & -0{,}128 & 5 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 18 & -80 & 0 \\ 0 & 0 & 0 & 0 & 200 & 0 \\ 0 & 0 & 0 & -3{,}2 & -3{,}2 & 5 \\ \hline -0{,}32 & 0 & -0{,}1 & 0 & 0 & -0{,}1 \\ 0{,}192 & 0{,}16 & 2 & 4{,}8 & 4 & 2 \\ \hline 0 & 0 & 0{,}5 & 0 & 0 & 0{,}5 \\ \hline 0{,}08 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0{,}8 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \quad \textbf{(19)}$$

where $\mathbf{r}^{(1)^T}\tilde{\tilde{\mathbf{N}}}^{(1)}$ is the "sale of waste" in period t =1. It equals the "input of waste" in the immediately following period. It has been assumed that one cannot see

[13] Some null matrices of dimension 1×2 have been introduced within the outcome of Eq.(7) in order to allow such operation.

[14] The subscripts "I" and "II" have been introduced in order to make a distinction among those rows and columns which refers to the production processes and treatment processes respectively.

[15] Neither the treatment process produce the same output of the production processes nor *vice versa*. The treatment process is not being supplied with production processes' main outputs ($G_{I,II}$ =0, $L_{I,II}$ =0) and it does not require treatment services. It only produces releases into the environment.

beyond period t =2. This prevents one to assign non-zero values to the input of waste in period t =3. The network of processes can now be depicted as in Fig.4[16].

Figure 4

The whole system, including the treatment process, needs to be rescaled. The scaling vector is calculated as $\mathbf{s}_{II} = \mathbf{A}^{-1}\mathbf{y}_{II}$. The reference flows vector $\mathbf{y}_{II}$ includes the final demand for waste treatment, which is set to zero. Assuming $\mathbf{y}_{II} = (0 \quad 0 \quad 0 \mid 0 \quad 200 \quad 0)$, then $\mathbf{s}_{II} = (1 \quad 1 \quad 0{,}0352 \mid 1 \quad 1 \quad 1{,}28)$. This yields

$$\begin{pmatrix} \mathbf{A} \\ \hline \mathbf{D} \\ \hline \mathbf{G}^{(1)} \\ \mathbf{G}^{(2)} \\ \mathbf{R} \end{pmatrix} \hat{\mathbf{s}}_{II} = \begin{pmatrix} \tilde{\mathbf{A}} \\ \hline \tilde{\mathbf{D}} \\ \hline \tilde{\mathbf{G}}^{(1)} \\ \tilde{\mathbf{G}}^{(2)} \\ \tilde{\mathbf{R}} \end{pmatrix} = \left(\begin{array}{ccc|ccc} 3{,}2 & -3{,}2 & 0 & 0 & 0 & 0 \\ 0 & 8 & 0 & -8 & 0 & 0 \\ -0{,}048 & -0{,}128 & 0{,}176 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 18 & -80 & 0 \\ 0 & 0 & 0 & 0 & 200 & 0 \\ 0 & 0 & 0 & -3{,}2 & -3{,}2 & 6{,}4 \\ \hdashline -0{,}32 & 0 & -0{,}0035 & 0 & 0 & -0{,}128 \\ 0{,}192 & 0{,}16 & 0{,}0704 & 4{,}8 & 4 & 2{,}56 \\ \hdashline 0 & 0 & 0{,}0176 & 0 & 0 & 0{,}64 \end{array}\right) \qquad \textbf{(20)}$$

3.3 Applying costs

In a deterministic approach, a vector of standard cost coefficients, expressed in monetary units, can be exogenously defined for each unit of the relevant cost drivers: the externally purchased inputs, the "input of waste" and the cycle times expressed as machine hours[17]. As to the latter, it can be used to assign variable conversion costs to processes according to a predetermined overhead cost rate. Assuming that the prices of externally purchased inputs do not change from t =1 to t = 2, the equivalent of the "value added" vector which is used within the leontevian price system can be calculated as follows:

[16] Instead of representing the waste outflows from processes going into the treatment one, it has been represented the flows of treatment services going from the treatment process into the production ones.
[17] Also the environmental flows **R** can be used as cost drivers and attached a cost rate.

$$\boldsymbol{\omega}^T = \begin{pmatrix}\mathbf{p}_D^T & \overline{\mathbf{p}}^{(1)} & \overline{\mathbf{p}}^{(2)}\end{pmatrix}\begin{pmatrix}\widetilde{\mathbf{D}} \\ \widetilde{\mathbf{G}}^{(1)} \\ \widetilde{\mathbf{G}}^{(2)}\end{pmatrix} = \begin{pmatrix}20{,}3 & 8{,}9 & 4{,}1 & 268{,}8 & 224{,}04 & 150{,}4\end{pmatrix} \qquad \textbf{(21)}$$

where $\mathbf{p}_D^T = \begin{pmatrix}-\mathbf{p}_M^T & \mathbf{p}_C^T\end{pmatrix} = \begin{pmatrix}-30 & 56\end{pmatrix}$: $\mathbf{p}_M^T$ is the standard cost of purchasing raw materials and other process inputs from outside the system; $\mathbf{p}_C^T$ is an overhead cost rate that could be used to trace, by using machine hours as a driver, such costs as machinery depreciation, indirect labour and so on[18]. Each waste type k will be *sold* (and purchased) within the system at exogenous prices $\overline{p}_k$. It is not its "manufacturing cost"[19]; it could also happen, indeed, that $\overline{p}_k \leq 0$. In our example, assume $\begin{pmatrix}\overline{\mathbf{p}}^{(1)} & \overline{\mathbf{p}}^{(2)}\end{pmatrix} = \begin{pmatrix}-0{,}5 & -0{,}5 \mid -0{,}5 & -0{,}5\end{pmatrix}$. The following

$$\mathbf{p}^T = \boldsymbol{\omega}^T \mathbf{A}^{-1} = \begin{pmatrix}6{,}7 & 4{,}2 \;\vdots\; 23{,}5 \mid 4{,}7 & 3{,}38 \;\vdots\; 23{,}5\end{pmatrix} \qquad \textbf{(22)}$$

yields , the unit production cost for the output of each stage of the supply chain considered in each time period. It includes those cost incurred to run the treatment processes internally, which have been assigned according to the demand for such treatment. The following

$$\hat{\mathbf{p}}\widetilde{\mathbf{A}} = {}_p\widetilde{\mathbf{A}} = \left(\begin{array}{cc:c|cc:c} 21{,}44 & -21{,}44 & 0 & 0 & 0 & 0 \\ 0 & 33{,}40 & 0 & -33{,}40 & 0 & 0 \\ -1{,}13 & -3{,}01 & 4{,}14 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 377{,}41 & -377{,}41 & 0 \\ 0 & 0 & 0 & 0 & 676{,}65 & 0 \\ 0 & 0 & 0 & -75{,}2 & -75{,}2 & 150{,}4 \end{array}\right) \qquad \textbf{(23)}$$

yields the monetary counterpart of the balanced systems' net output matrix $\widetilde{\mathbf{A}}$. The following

$${}_p\widetilde{\mathbf{A}}\mathbf{e} = \hat{\mathbf{p}}\mathbf{y}_{II} = \begin{pmatrix}0 & 0 & 0 & 0 & 676{,}64 \mid 0\end{pmatrix} \qquad \textbf{(24)}$$

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[18] If the conversion costs are not common to all processes, then each cycle time should be recorded in a separate row. This allows different overhead cost rates to be applied to different processes.

[19] The manufacturing cost of producing a waste can be determined applying some joint product costing. The computational procedure would be similar to the allocation one as described by Heijungs and Suh (2002) as to LCA. According to the cause-effect principle, the cost of producing a waste together with a given main output, once determined, should be transferred into the process requiring the latter.

turns out to be the total manufacturing cost of the net output that meets the final demand.

3.4 A comparison between steady state and two-periods model

If we had run all the previous calculations using a steady-state representation of the system, we would have obtained $\mathbf{p}^T = (4{,}6 \quad 3{,}3 \mid 23{,}5)$, wich can be compared to Eq.(20). The total manufacturing cost of the net output that meets the final demand would have read ${}_p\mathbf{y}_{II} = (0 \quad 668 \quad 0)^T$.

which is slightly less than the one obtained in Eq.(21), basically because, *ceteris paribus*, using an externally purchased input in period t =1 in Process 1 instead of the internally produced Commodity 2 is more expensive.

4. Conclusions

The present note have discussed some simple issues of dynamics which are induced by a different perspective on feedback loops as might be of interest for Input-Output based enterprise resource and cost planning. The main point has been that if one does not neglect that production takes time, then feedback loops themselves may induce some form of dynamics within an Input-Output computational structure which recalls the kind of dynamics that is quite often described when dealing with IOA.

Also the formal repercussions of such view on a simple hypothetical environmental extension has been illustrated. Many aspects of the analysis, however, have not been discussed within the present note which would have entailed focusing on much more formal aspects. Yet, the aim has been that of contributing to the debate about the issue of dynamics namely within an environmental management tool like Life Cycle Assessment (LCA), as discussed in Udo de Haes et al. (2004), and especially as far as its integration with Cost Accounting is concerned. The latter has been discussed in Huppes et al. (2004) and Norris (2001).

The basic assumption here has been that, as pointed out in Settanni et al., (2007), in the extent the computational structure of both LCA and its economic counterpart (Life Cycle Costing - LCC) is to be somehow consistent, it must be input-output-based in both cases, and the latter is to include the appropriate environmental extensions. Issues of dynamics must then be considered, in dealing with such topics, consistently with the theory of Input-Output Analysis.

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Tables:

Table 1

Run	S_1	S_2
1	0,8	4
2	0,032	0,16
3	0,00128	0,0064
	0,83328	4,1164

Iterative method to solve the system of linear equations in presence of feedback loops

Table 2

Run	$S_1^{(1)}$	$S_2^{(1)}$	$S_1^{(2)}$	$S_2^{(2)}$
1	0,032	0,16	0,8	4

Iterative method to solve the system of linear equations without feedback loops

Table 3

t	p	t-p	$U_{t-p}^{(p)}s^{(t-p)}$	$U_t=\sum U_{t-p}^{(p)}s^{(t-p)}$	t-p	$V_{t-p}^{(p)}s^{(t-p)}$	$V_t=\sum V_{t-p}^{(p)}s^{(t-p)}$
1	-1	2	$U_2^{(-1)}s^{(2)}$		2	$V_2^{(-1)}s^{(2)}=0$ by assumption	
1	0	1	$U_1^{(0)}s^{(1)}$		1	$V_1^{(0)}s^{(1)}$	
1	1	0	$U_0^{(1)}s^{(0)}=0$ by assumption		0	$V_0^{(1)}s^{(0)}$	
				$U_1=U_1^{(0)}s^{(1)}+U_2^{(-1)}s^{(2)}$			$V_1=V_1^{(0)}s^{(1)}+V_0^{(1)}s^{(0)}$
2	-1	3	$U_3^{(-1)}s^{(3)}$		3	$V_3^{(-1)}s^{(3)}= 0$ by assumption	
2	0	2	$U_2^{(0)}s^{(2)}$		2	$V_2^{(0)}s^{(2)}$	
2	1	1	$U_1^{(1)}s^{(1)}=0$ by assumption		1	$V_1^{(1)}s^{(1)}$	
2	2	0	$U_0^{(2)}s^{(0)}=0$ by assumption		0	$V_0^{(2)}s^{(0)}$	
				$U_2=U_2^{(0)}s^{(2)}+U_3^{(-1)}s^{(3)}$			$V_2=V_2^{(0)}s^{(2)}+V_1^{(1)}s^{(1)}+V_0^{(2)}s^{(0)}$

Time values of make and use matrices

Table 4

					t-|p|	t	t+|p|		
			(p>0)	-1	0	1	2	3	... (p<0)
Use Matrix	t	1		-	-	$U_1^{(0)}s^{(1)}$	$U_2^{(-1)}s^{(2)}$	$U_3^{(-2)}s^{(3)}$	$U_{1-p}^{(p)}s^{(1-p)}$
		2		-	-	-	$U_2^{(0)}s^{(2)}$	$U_3^{(-1)}s^{(3)}$	$U_{2-p}^{(p)}s^{(2-p)}$
Make Matrix	t	1	$V_{1-p}^{(p)}s^{(1-p)}$	$V_{-1}^{(2)}s^{(-1)}$	$V_0^{(1)}s^{(0)}$	$V_1^{(0)}s^{(1)}$	-	-	
		2	$V_{2-p}^{(p)}s^{(2-p)}$	$V_{-1}^{(3)}s^{(-1)}$	$V_0^{(2)}s^{(0)}$	$V_1^{(1)}s^{(1)}$	$V_2^{(0)}s^{(2)}$	-	

Table 5

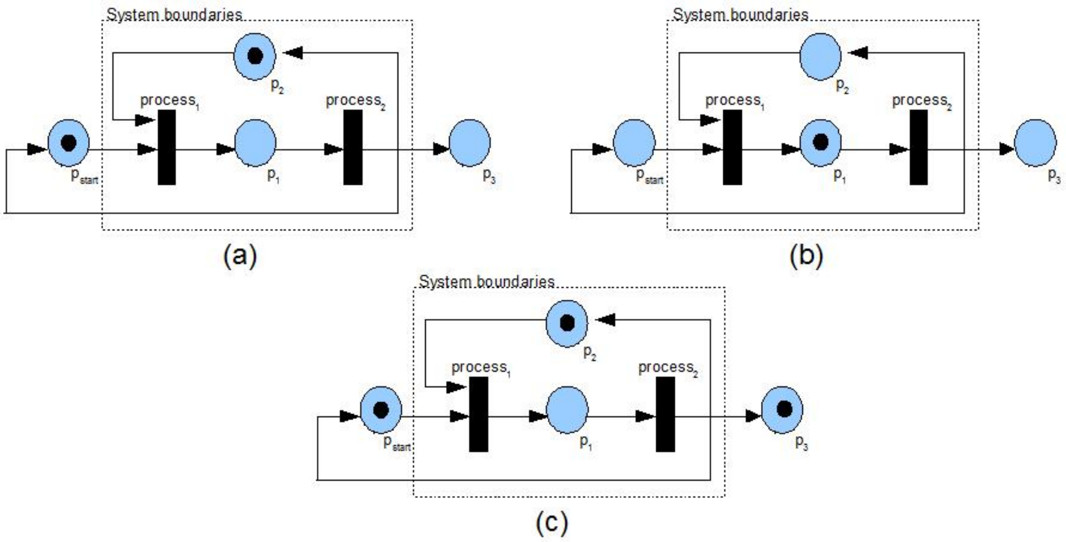
t	p	t-p	p+1	$U_{t-p}^{(p)}s^{(t-p)}$	$U_t=\sum U_{t-p}^{(p)}s^{(t-p)}$	$B_t=\sum[B_{t-p+1}^{(p)}s^{(t-p+1)}-B_{t-p}^{(p+1)}s^{(t-p)}]$	$V_{t-p}^{(p)}s^{(t-p)}$	$V_t=\sum V_{t-p}^{(p)}s^{(t-p)}$
1	-1	2	0	$U_2^{(-1)}s^{(2)}$		$B_3^{(-1)}s^{(3)}-B_2^{(0)}s^{(2)}$	$V_2^{(-1)}s^{(2)}=0$ by assumption	
1	0	1	1	$U_1^{(0)}s^{(1)}$		$B_2^{(0)}s^{(2)}-B_1^{(1)}s^{(1)}=B_2^{(0)}s^{(2)}$ by assumption	$V_1^{(0)}s^{(1)}$	
1	1	0	2	$U_0^{(1)}s^{(0)}=0$ by assumption		$B_1^{(1)}s^{(1)}-B_0^{(2)}s^{(0)}=0$ by assumption	$V_0^{(1)}s^{(0)}=0$ by assumption	
					$U_1=U_1^{(0)}s^{(1)}+U_2^{(-1)}s^{(2)}$	$B_1=B_3^{(-1)}s^{(3)}-B_2^{(0)}s^{(2)}+B_2^{(0)}s^{(2)}$		$V_1=V_0^{(1)}s^{(0)}$
2	-1	3	0	$U_3^{(-1)}s^{(3)}$		$B_4^{(-1)}s^{(4)}-B_3^{(0)}s^{(3)}$	$V_3^{(-1)}s^{(3)}=0$ by assumption	
2	0	2	1	$U_2^{(0)}s^{(2)}$		$B_3^{(0)}s^{(3)}-B_2^{(1)}s^{(2)}=B_3^{(0)}s^{(2)}$ by assum.	$V_2^{(0)}s^{(2)}$	
2	1	1	2	$U_1^{(1)}s^{(1)}=0$ by assumption		$B_2^{(1)}s^{(2)}-B_1^{(2)}s^{(1)}=0$ by assumption	$V_1^{(1)}s^{(1)}=0$ by assumption	
2	2	0	3	$U_0^{(2)}s^{(0)}=0$ by assumption		$B_1^{(2)}s^{(1)}-B_0^{(3)}s^{(0)}=0$ by assumption	$V_0^{(2)}s^{(0)}=0$ by assumption	
					$U_2=U_2^{(0)}s^{(2)}+U_3^{(-1)}s^{(3)}$			$V_2=V_2^{(0)}s^{(2)}+V_1^{(1)}s^{(1)}+V_0^{(2)}s^{(0)}$

Table 6

			t+|p|, (p=0...-∞)		
			1	2	... (p<0)
$B_{1-p+1}^{(p)}$	t	1	$B_2^{(0)}s^{(2)}$	$B_3^{(-1)}s^{(3)}$	$B_{1-p+1}^{(p)}s^{(1-p+1)}$
		2	$B_2^{(1)}s^{(2)}$	$B_3^{(0)}s^{(3)}$	$B_{2-p+1}^{(p)}s^{(2-p+1)}$
$B_{1-p}^{(p+1)}$	t	1	$B_1^{(1)}s^{(1)}$	$B_2^{(0)}s^{(2)}$	$B_{1-p}^{(p+1)}s^{(1-p)}$
		2	$B_1^{(2)}s^{(1)}$	$B_2^{(1)}s^{(2)}$	$B_{2-p}^{(p+1)}s^{(2-p)}$

Figures:

Figure 1



The initial system state (a), after firing *process1* (b) and after firing *process2* (c).

Figure 2

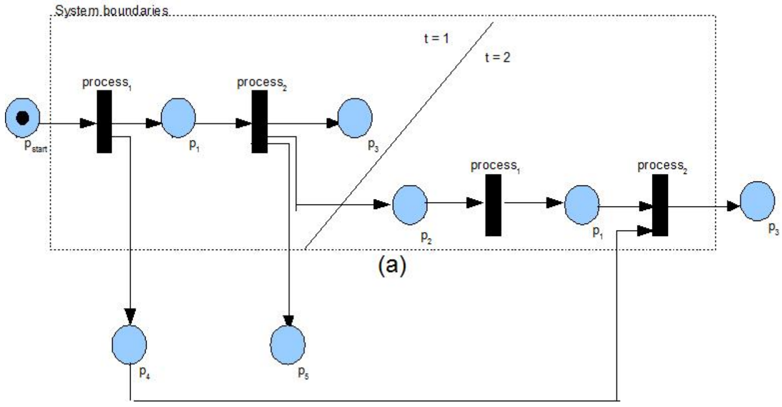


Figure 3

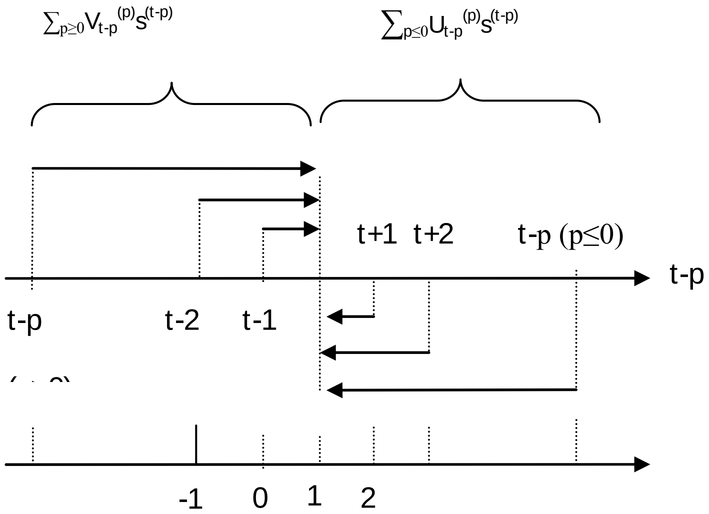


Figure 4